First Edition

POST-TENSIONED BUILDINGS
Design and Construction

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POST-TENSIONED BUILDINGS
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ABOUT THIS BOOK

Over the last 32 years, I have been involved in the design and observation of post-tensioned projects in more than 35 countries worldwide. I have also had the opportunity to meet thousands of others whose work encompasses the design, construction, review, and approval of post-tensioned buildings and bridges. Through observing and discussing concepts and design procedures with other engineers, I have learned a great deal; this book is my attempt to organize the critical aspects of what I have learned, and present it in a lucid and simple manner.

Although the book is primarily written for practicing engineers who design post-tensioned buildings, it will also serve those who want to learn more about the practice of post-tensioning. Contractors, building officials, plan checkers, students, and researchers will all find the book of value. The topics covered are treated in depth and taken to the point of practical application.

Post-tensioning in building construction—a practice over half a century old in the US—is relatively new in many other parts of the world. In the US, the practice started and primarily developed as an "art" and is still greatly influenced by the early practice. When combined with the science of engineering, post-tensioning offers economic advantages and the potential of superior performance. These economic and performance advantages have been the driving force in the worldwide acceptance of post-tensioning. The US practice, along with the long history of satisfactory performance of post-tensioned buildings, served as a guide for the initial practice in other countries. However, the worldwide practice developed and matured in different ways. To a large extent, the practice still remains an "art," supported with different degrees of engineering science, depending on the country of practice.

Through my interactions with other engineers, I have come to understand the topics that we engineers master well, and those that we often have conceptual difficulties in either mastering or applying. In many instances, the difficulty is the result of the material not having been taught as part of the course work of the underlying engineering degree, or of not being well explained in the available textbooks. For some engineers, these difficulties coupled with their lack of experience in post-tensioning design and the "art" component of the practice has surrounded post-tensioning with a halo of mystery and undeserved complexity. It is my hope that this book will remove this mystery.

I have used pictures and diagrams as well as text to explain the concepts and procedures. In particular, I have emphasized the topics that I have identified as stumbling blocks for many engineers. I have addressed these topics in several ways, at different locations in the book. Some topics may appear obvious, even trivial to experienced designers, yet they are discussed in detail—then repeated, and repeated again. Repetition, while irritating, can go a long way in ensuring that a concept is understood.

The book assumes a basic knowledge of conventionally reinforced concrete design. Founded on this knowledge, the material presented covers the full range of post-tensioning principles, including the principles necessary for efficient design. The focus of the book is on the science of engineering, rather than the "art" of post-tensioning design; thus there is more emphasis on the ultimate objectives of "serviceability" and "safety" rather than strict adherence to local or traditional practice. The objective is to benefit a larger number of my colleagues, as well as plan checkers and reviewers, and to make it easier to follow a design and move it through the approval process less painfully. Having mastered the basic concepts, it becomes easier to accept that there is more than one way of designing a post-tensioned structure that meets serviceability and safety requirements.

The parameters and bounds of design are ultimately defined by building codes. Commercial construction in the US is governed by the International Building Code (IBC); the concrete requirements are based on ACI 318: Building Code Requirements for Structural Concrete. With respect to post-tensioned concrete, ACI 318 primarily reflects current design practice in the US; as with other aspects of our lives, the ACI 318 requirements are heavily influenced by special interests, particularly those of the post-tensioning material suppliers. This becomes apparent when the ACI requirements are compared to those of other countries. For example, the European Code EC2[1], having had to address the interest of a larger number of countries, enjoys a greater component of engineering science. Where applicable, this book attempts to cover the topics of ACI and EC2 side by side. The objective is to emphasize that there is more than one way of arriving at a "safe" and "serviceable" design, and that the designs under different codes can be quite different.

The book comes in two versions: a US edition, and an International edition. The US edition uses the US system of units (lb, in) that is common in US construction, along with the equivalent values in SI units (N, mm). It covers both ACI/IBC and EC2, which in addition to being mandatory in a large number of European countries is being used more and more as a basis for other building codes.

The International edition of the book covers the same topics according to both ACI/IBC and EC2, in the SI (N, mm) system of units. In addition, where applicable, it includes the recommendations of TR43, Post-Tensioned Concrete Floors Design Handbook. TR43 is a publication of the UK Concrete Society that provides recommendations for design and construction of post-tensioned buildings

A second goal of this book is to address the widespread use of software in design. We no longer need to learn how to calculate deflections or determine the value of the moment at a given section — our software does it for us. The ability to do longhand calculations of moments, shears, and deflections has become obsolete, just as the ability to use slide rules became obsolete when hand-held calculators were introduced. Many of us rely entirely on software- viewed as a black box — to provide us with the values we need to accomplish our design tasks.

Further, many of us have come to place our faith fully on the output of the "black box" in our office. This development presents two issues. First, the design engineer is still responsible for the design. The design must result a serviceable and safe structure; it should also be economical.

Second, the "black boxes" that are currently available are not all the same. While

[1] EC2 EN 1992-1-1:2004

the input and output of different software programs may seem similar, there are significant differences in the assumptions, simplifications, and procedures used in the internal workings of the software. The former required skill of knowing how to calculate deflections and bending moments has been replaced by the need to recognize the assumptions and procedures that a given software program uses in processing the input data, as well as the accuracy and reliability of the solutions it produces.

Drawing on my experience extending over three decades as the lead engineer in the development of software for the analysis and design of concrete structures and specifically post-tensioned concrete structures—namely ADAPT [2]—I have tried to shed light in the book on several critical aspects of software evaluation.

I acknowledge that some readers may not have had a course in finite elements. Since finite element analysis forms the basis of concrete floor design, a general understanding is necessary to evaluate the suitability of design software. I have therefore devoted a section of the book to this topic. In simple, yet precise, words I have explained the finite element concepts that design engineers need to know when using currently available design software. I have followed the explanation with examples that illustrate how different software packages may not go through the same internal steps and thus may not produce the same results for a particular set of input data.

The book includes two detailed, longhand numerical examples. One example is for a column-supported floor system and the other is for a beam frame. The examples reflect real-life conditions, and the calculations are done according to both ACI and EC2 requirements. The International edition also goes through the examples using TR43.

In recent years, there has been much progress in Building Information Modeling (BIM). However there is still a stumbling block when it comes to integrating the work of structural engineers into the otherwise smooth flow of the BIM process. The problem arises from the necessity of having to create an "analytical" model from the architect's "physical" model of the concrete frame. The BIM model reflects the actual geometries of a building - the "physical" model. The practice of structural engineers to date has been to simplify the physical model to an analytical model created from intersecting centroidal lines. Switching from the physical to analytical model disrupts the smooth flow of information through the BIM process; the results of the structural analysis cannot be transferred directly to the BIM model. Section 8.2 of the book examines this problem and offers a feasible and practical solution.

While I have downplayed the requirement for longhand calculation of deflections and moments, I cannot over-emphasize the importance of our first course in structural engineering — statics — and the ability to draw a complete free-body diagram. Estimating the applicable loads correctly, identifying a load path, and making sure that the design values are in "static equilibrium" with the applied loads go a long way in structural design—irrespective of how complex the structure is. Likewise, it is important to think about "ductility." Throughout the book, the emphasis for safety is on (i) selection of an uninterrupted load path, (ii) static equilibrium of the applied loads with the design forces of the load path, and (iii) adequate ductility. Satisfaction of these conditions will result in a safe structure, even when the design conflicts with the results of a widely-used analysis and design software.

Over the course of one's career, there may be specific events, often unforeseen, with long-lasting effects. I owe my love of post-tensioning and lifelong commitment to it to

[2] www.adaptsoft.com

two individuals – Philip French[3] and Rene Friedrich[4]. After the 1979 revolution in Iran, I moved to Germany working at Darmstadt University. Philip, an old college friend, was head of a precast and post-tensioning firm in Hawaii and arranged for me to meet Rene, who was on his yearly ski trip to Switzerland. Rene, the US manager of VSL[5] Corporation, led me to post-tensioning by offering me a position in VSL's California headquarters.

The book benefited from the contribution of Ms. Roshni Malyiakkal, an exceptionally bright student, and later long-time colleague of mine at ADAPT Corporation, who has gone through the material meticulously to verify its accuracy.

I dedicate this book to my mother, a lifelong teacher, who lost her husband when she was 28, but devoted her life to hard work and single-handedly supporting and ensuring the education of her three children. I am also indebted to my wife, Ingrid, whose unwavering support I have enjoyed since 1958.

Bijan O. Aalami
Palo Alto, January 2014

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3 Philip French; www.confab-precast.com
4 Rene Friedrich; www.avarconstruction.com
5 VSL Corporation; www.vsl.net

CHAPTER 1

INTRODUCTION

View of a Post-Tensioned Tower in Honolulu
(Phoenix P516)

1.1 PURPOSE AND OUTLINE

This book is the culmination of over 30 years of experience in post-tensioning, gathered through design practice, teaching, observation, and close interaction with post-tensioning designers worldwide. The book is an attempt to demystify post-tensioning and bring its concepts and in-depth principles to the point of everyday practice on both common and complex structures. No attempt is made to exhaust the material on prestressing that is readily available in many good textbooks, such as [Collins et al, 1997; Nawy 1997]. Rather, the focus is on what practicing engineers need to know in order to produce a good design or to evaluate an existing structure.

Unlike conventionally reinforced concrete, where the determination of design forces and the associated reinforcement is fairly straightforward, a successful post-tensioning design requires a designer's experience and input beyond commonly available textbook formulas. Veteran post-tensioning designers often refer to the "art" of post-tensioning "design," as opposed strictly to its underlying mechanical theories. Post-tensioning design requires experience-based judgment from the designer before the number-crunching can start. This book is intended to help the post-tensioning engineer to develop or strengthen the required know-how and experience that is prerequisite of a good design.

This Chapter offers a brief review of historical development of post-tensioning in building construction. **Chapter 2** describes the technique of post-tensioning, with focus on the commonly available systems and hardware, as well as the economics of post-tensioning, and estimate of quantities.

Chapter 3 outlines the design concepts and procedures for post-tensioned floor systems. It explains the steps for breaking a three-dimensional model of a floor system into design strips for the purpose of its structural design, or its compliance with the governing building codes. It covers both the in-service (Serviceability Limit State—SLS) and safety requirements (Ultimate Limit State—ULS), to determine the necessary reinforcement and the associated detailing. The outlined procedure is valid for both conventionally reinforced and prestressed floor systems. The procedure forms the basis of contemporary automated algorithms implemented in major design software.

Chapter 4 reviews the items that you need to fully understand and master in order to achieve a good design. It is a compilation of different topics, each addressing one aspect of prestressing concept and design. The explanations unravel the ambiguity that generally surrounds the post-tensioning design.

Chapter 5 lists the 10 steps that you would follow, when you manually design a post-tensioned floor system or a beam frame.

In **Chapter 6**, the basic concepts and procedures presented in Chapter 4 and the 10 steps for the design a post-tensioned floor system suggested in Chapter 5 are brought together to help you manually design a column-supported two-way floor system. Unlike most textbooks, where simple examples are presented, the floor system selected for Chapter 6 is intended to reflect the realistic conditions of construction, where design parameters are oftentimes neither regular nor simple. The longhand calculation presented navigates through scenarios that you are likely to encounter in design of real structures. The US edition of the book follows the ACI 318/IBC [1] codes as well as the European code EC2[2], with the emphasis on American system of units. The international edition of the book includes additional design steps using TR4[3], with emphasis on SI system of units.

While we recognize that today few engineers are likely to design a floor system using longhand calculations, the information in Chapter 6 is essential to the understanding of design process. It helps to build up the necessary design skills as well as validation of designs obtained through automated procedures. In addition, it provides a reference point for comparison of design outcomes using different building codes.

In **Chapter 7** we design a beam frame, applying the procedure outlined in Chapter 6 for design of a two-way column-supported floor system and, using the 10 steps outlined in Chapter 4. We follow the same building codes used in Chapter 6. In Chapter 7, we focus on crystallizing the differences between design of a beam frame and that of a two-way floor system.

Few engineers today are likely to engage in hand calculation for design of routine floor systems in the environments of consulting offices. Computer programs have universally replaced the hand calculations that used to be common. **Chapter 8** covers the automated design of post-tensioned floors through presentation of the modeling and design of a post-tensioned floor system, using leading commercially available software.[4]

More and more often around the world, commercial and residential multi-story buildings are constructed with post-tensioned floor systems. In most cases, design engineers perform the post-tensioning analysis by extracting each floor level of a multi-story building, and treating the extracted floor in isolation. The impact of post-tensioning on the frame of a multi-story building is reviewed in **Chapter 9.** This Chapter addresses the changes in column and wall

1 ACI 318-11,14; International Building Code (IBC, 2012)
2 EC2 EN1992-1-1;2004
3 TR43, 2005
4 Builder – ADAPT Floor Pro www.adaptsoft.com

reactions resulting from the application of post-tensioning in the floors and diversion of precompression into walls, along with a brief reference to effects of floor shortening due to post-tensioning.[5]

Stress losses in prestressing tendons and allowance for them in design are basic in design of post-tensioned members. **Chapter 10** offers a clear outline of the sources of prestressing losses, along with a well-established procedure for their calculation. Several numerical examples illustrate the application of the material.

The treatment of post-tensioning tendons for structural analysis has evolved extensively. From the simple load-balancing procedure introduced in early 1960s to analytical modeling of tendons as discretized finite elements, there has been a significant leap in the modeling and analysis of post-tensioning tendons. **Chapter 11** walks you through each stage of development and presents the state of the art in tendon modeling and analysis.

Design of a member for bending is a routine step in practically every post-tensioning design. For completeness and building code-based protocols, **Chapter 12** is devoted to design of post-tensioned members in bending. It covers the design aspects encountered in practical structures.

1.2 BRIEF HISTORY OF POST-TENSIONING IN BUILDING CONSTRUCTION

Prestressing is simply the application of forces tending to bend and compress a concrete element. Typically, prestressing is applied in order to counteract the bending and tensile stresses which result from other loads. The principle of prestressing is simple, and is briefly outlined in Section 2.1. A short outline of the historical development of prestressing in building construction is given in reference [Aalami, 2007].

A. Early Attempts: In 1872, P.H. Jackson [Billington, 2004], a San Francisco engineer, obtained a patent for post-tensioning. He inserted steel rods into masonry units and stressed them with a threaded device. He applied the counteracting forces correctly, but because of the properties of steels available in 1886, these forces did not remain effective for very long. His effort was followed in 1888 by C.W. Doehring, who obtained a patent in Germany for prestressing slabs with metal wires. Because early steel had a relatively low yield stress (Fig. 1.2A-1), none of these early attempts were successful. Low initial jacking stress, combined with high creep and shrinkage of the concrete, eroded the bulk of the prestressing force applied to the structure, leaving the steel practically ineffective.

A dramatic increase in the effective stress in the prestressing strands after all stress losses was the first critical milestone in making prestressing a practical proposition. Figure 1.2A-1 illustrates the significant gain in the effective stress of today's most commonly used strands in comparison to early steel. The same figure includes higher strength strands that are gradually replacing the current popular prestressing steel in building construction.

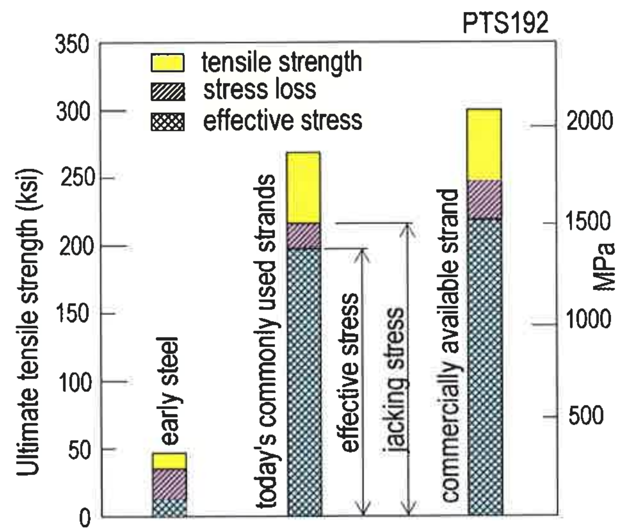


FIGURE 1.2A-1 Steel Strength and Prestress Losses

B. Practical Hardware: From 1926 to 1928, Eugene Freyssinet recognized the effects of long-term stress losses in prestressing and used a new high-strength steel to successfully construct prestressed members in France. In 1940, he introduced the well-known and well-accepted Freyssinet system, comprised of conical wedge anchors for 12 wire tendons (Fig. 1.2B-1). Developments in high-strength steel, coupled with the invention of prestressing hardware, proved to be another critical breakthrough in the effective application of prestressing. Although many prominent

5 Shortening effects and the mitigation of their adverse impacts are addressed in detail in ADAPT TN241 and ADAPT TN242

(a) View of interior cone (b) Diagram of anchorage assembly

FIGURE 1.2B-1 Freyssinet's Early Anchorage Device (P128a; P128b)

engineers - including Magnel in Belgium, Guyon in France, Leonhardt in Germany, and Mikhilove in Russia- continued to develop prestressing technologies, the focus of prestressing activity continued to be in bridge construction and special structures. Early developers paid little attention to opportunities of post-tensioning techniques in building construction.

C. Post-Tensioned Buildings: It was not until the early 1950s and the introduction of lift slab construction in the US that pioneering engineers revisited the application of prestressing to eliminate cracks and reduce deflections in thin flat slabs in buildings. While credit is due to these innovators for the introduction of prestressing, the principal design instrument for its application was put forward by T.Y. Lin [Lin, 1963] through the concept of "load balancing." In its basic form, load balancing allows the engineer to view the effects of post-tensioning as a reduction in the design dead load applied to a slab--a design condition that engineers could well understand and handle (Fig. 1.2C-1). The economy made possible by post-tensioning and the simplicity of load balancing allowed pioneering engineers and contractors to drive the growth of post-tensioning construction in the US.

Basic load balancing is described in detail in Section 4.8.1B. Simply, it is based on the premise that axial force and bending effects exerted on a member from prestressing can be decoupled and analyzed separately, followed by superposition of the effects of each. Further, the prestressing force is assumed constant over the full length of a tendon. Finally, the elevation of the member centroid with respect to the line of action of precompression force remains unchanged along a member's length.

Refer to Fig. 1.2C-1. Part (a) of the figure shows a member of uniform thickness subjected to an applied dead load (1) and post-tensioned with a continuous tendon. In Part (b) of the figure the tendon is assumed to have been removed from the member and replaced with an equivalent load (2). The equivalent load is equal to the force the tendon was exerting upon the member, when it was in place. In part (c) the net effect, load (1) minus load (2), is a reduced load used in combination with traditional methods for design of the member. Note that the concentrated forces resulting from discontinuities in the tendon [see load (2)] are transferred directly to the supports and do not affect the member. The axial load P shown in part (b) of the figure results in a uniform compressive stress (precompression) which is added to the effects of bending to complete the stress analysis.

A major drawback in simple load balancing outlined above was its limitation to slabs of uniform thickness. Real floor systems often feature members of different thickness, and can include changes in elevation, beam and slab construction, and other geometric features that violate the basic premise of constant

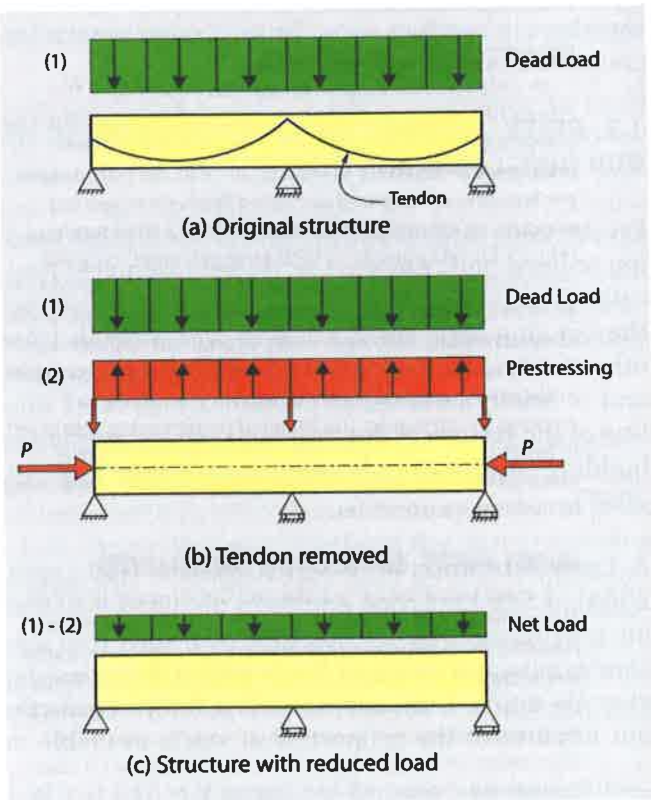


FIGURE 1.2C-1 Simple Load Balancing Viewing the Post-Tensioning as a Reduction in Dead Load (P640)

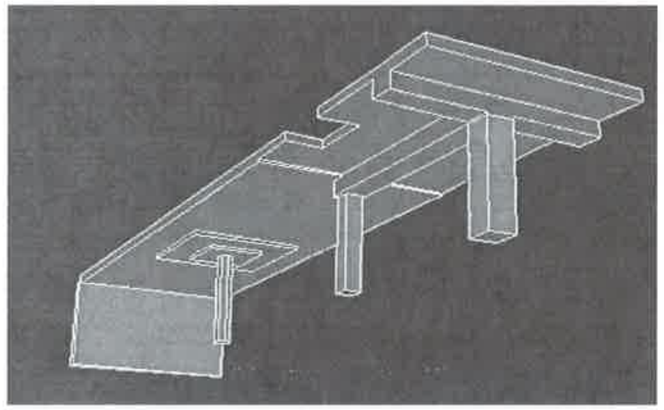


FIGURE 1.2C-2 View of a Design Strip with Non-Uniform Geometry (P129)

eccentricity of line of precompression with respect to a member's centroidal axis. Figure 1.2C-2 illustrates a design strip from a floor slab, where changes in the centroidal axis of the member along its length invalidate the application of simple load balancing.

Extended load balancing, introduced by Aalami [Aalami, 1990] and detailed in Chapter 4, generalizes the application of "load balancing" to practical floor systems. Briefly, extended load balancing covers the analysis of prestressing members where the distance between the line of precompression and the member's centroidal axis is not constant. In effect, the extended load balancing among other features accounts for changes in thickness of a post-tensioned member.

Refer to Fig. 1.2C-3. Part (a) illustrates a post-tensioned member of non-uniform thickness. In Part (b) the tendon is assumed to be removed and replaced with equivalent loads that consist of distributed uplift forces due to the parabolic tendon profiles and concentrated axial loads at the ends of the tendons. Lack of alignment of the axial forces results in additional bending of the beam. In Part (c), in order to maintain the premise of decoupling of axial and bending effects, a moment is introduced at the change in member geometry (step in the member). In part (d) the vertical forces from tendon geometry (1) and the moment(s) introduced at the change in member geometry (2) result in reaction at the supports (3). Note that the concentrated loads resulting from discontinuities in the tendon as well as the moment due to the change in geometry affect the member reactions at supports (3). The concept and procedure is explained in greater detail in Chapter 4.

D. Early Design Tools, Detailing and Field Procedures: The introduction of personal computers in 1980s led to the development of first-generation software that mimicked the prevailing longhand calculation having been based on "isolated treatment of design strips" as outlined in Chapter 3. Widespread availability and ease of use of such analysis and design software, such as PTdata[6] and ADAPT-PT[7], led to an accelerated growth of post-tensioned building construction.

The computational know-how and tools were reinforced by the introduction of practical mono-strand hardware for stressing and anchoring single strand tendons--a necessity for the thin slab construction advanced by Edward K. Rice, founder of Atlas Prestressing Corp. Other important factors in the adoption and wide use of post-tensioning are development of extruded plastic-coated tendons, creation of the Post-Tensioning Institute (PTI) pioneered by

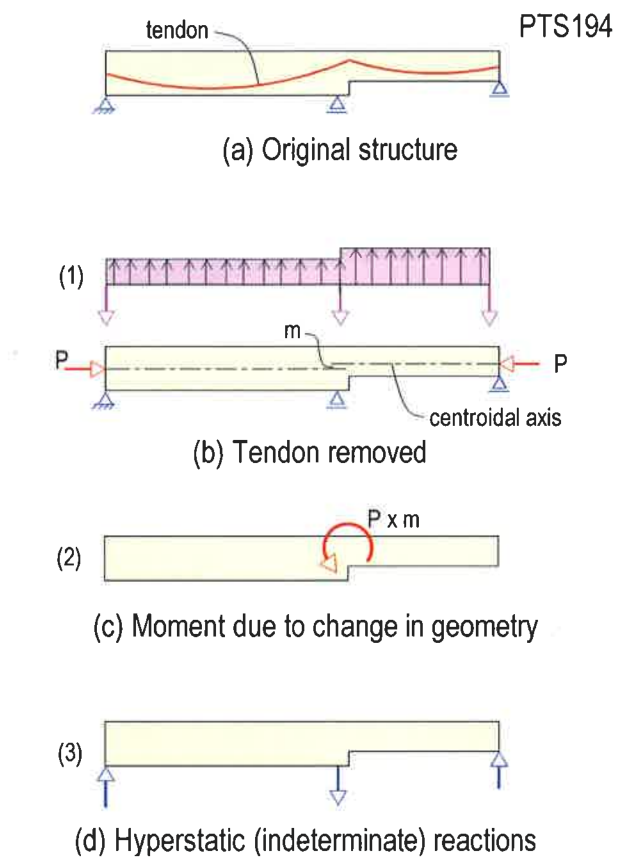


FIGURE 1.2C-3 Illustration of Extended Load Balancing

6 Structural Data Incorporated, Huntington Beach CA
7 ADAPT-PT www.adaptsoft.com

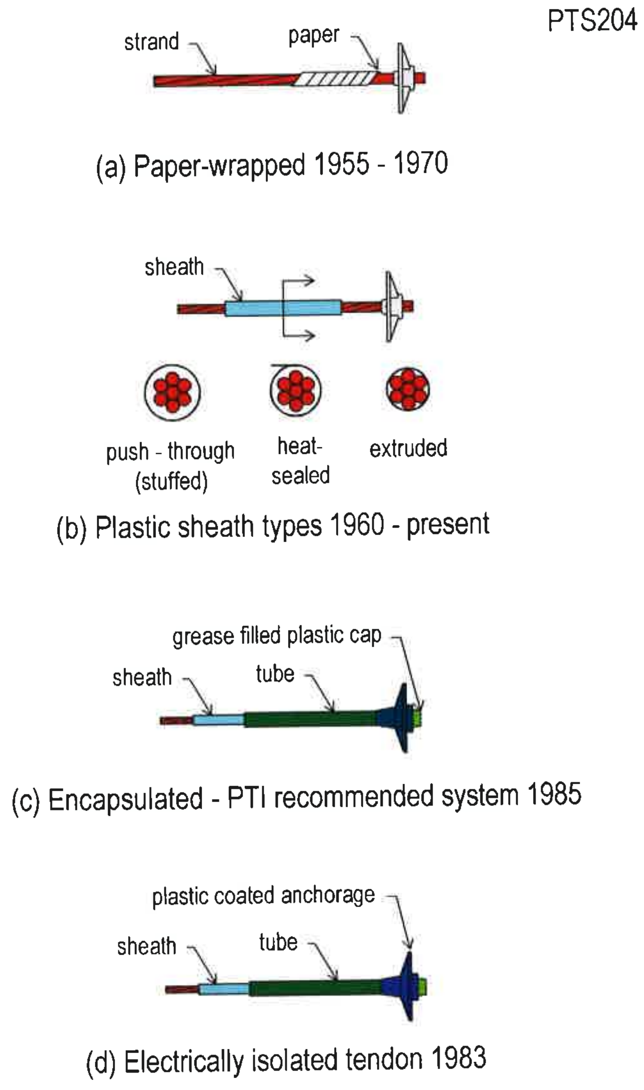


FIGURE 1.2D-1 Development of Single Strand Unbonded Tendons in USA (Courtesy Morris Shupack)

Clifford Fryermuth, and the development of Fryermuth's many educational seminars in the US and publications and guidelines on post-tensioning in building construction.

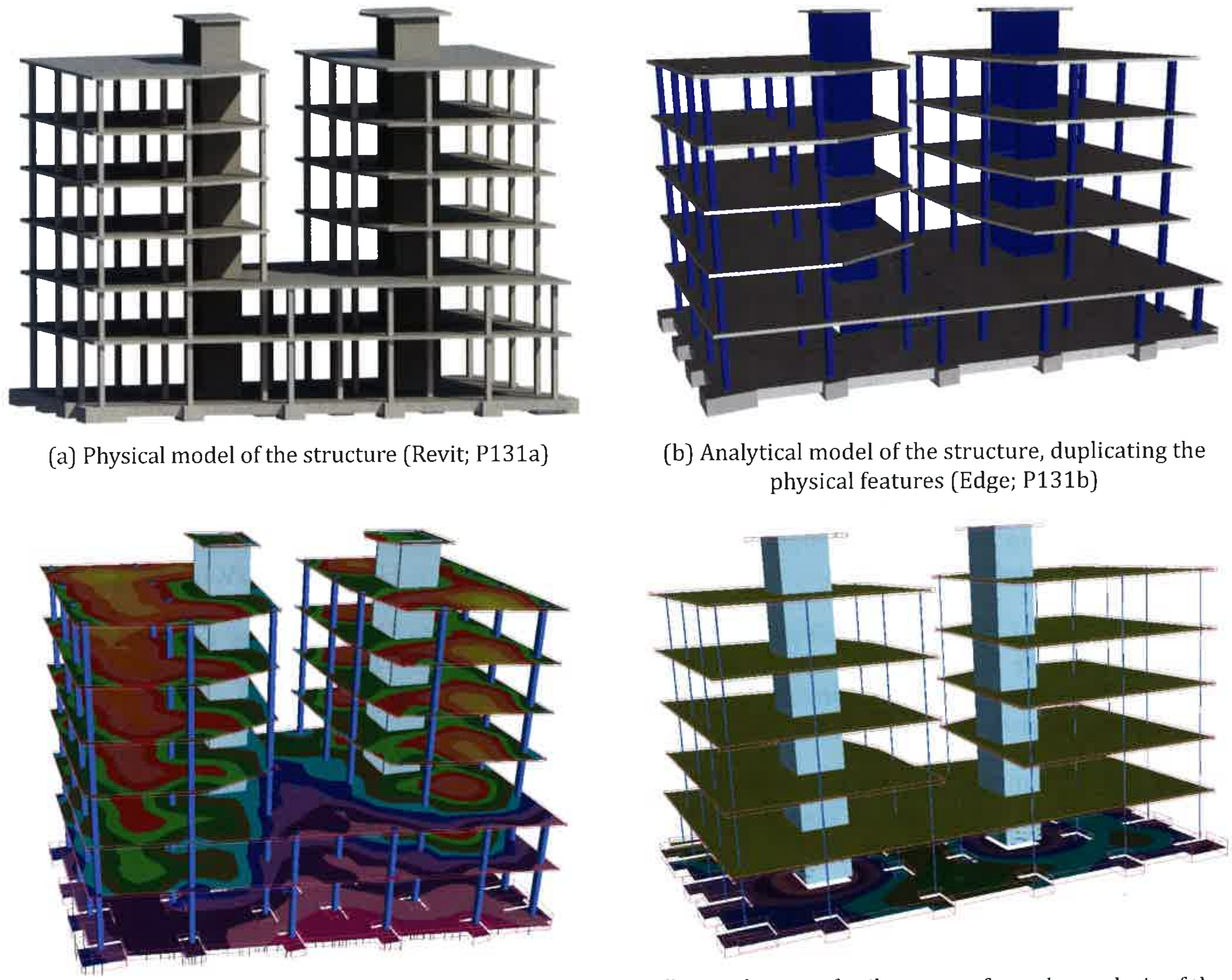
Figure 1.2D-1 illustrates the development of mono-strand unbonded tendons in the US, followed in Fig. 1.2D-2 by a sample of a modern extruded and encapsulated stressing device for a corrosion-resistant application.

E. Modern Design Tools; Integration with BIM: Automation in the design process, from inception to construction, has culminated in the development of BIM (Building Information Modeling). As expounded in Chapter 8, BIM attempts to integrate the entirety

FIGURE 1.2D-2 Example of a Single Strand Corrosion Resistant Encapsulated Anchorage (P130)

of design and construction into a single seamless process, with capability of information interchange among the participating trades and processes. BIM posed a major challenge to the seamless integration of structural engineers' work into the remainder of building information package. The extraction of information from a 3D construction model (Fig. 1.2E-1) to the structural engineer's idealized analysis model (Fig. 1.2E-2); to be used for analysis and design, and the subsequent transfer of structural design information back to the 3D construction model is a formidable challenge. Recent developments in structural engineering and software technology have met the challenge and successfully resolved the obstacle through the introduction of virtual analysis space, as outlined in Chapter 8.

Propelled by BIM, the modeling and integrated design of a building as a whole is likely to be the preferred choice in major construction. For the design of post-tensioned floors and beam frames, the application of design tools based on isolated design strips will continue. As expounded in Chapter 3, the same does not apply to post-tensioning, unlike conventionally reinforced concrete, where unique and automated designs can be achieved once the geometry, boundary conditions, material properties and loads are known. For post-tensioning to achieve economical designs and reach its optimization, the design engineer's knowledge and experience along with specific inputs are required. To date, software based on design strips such as that shown in Fig. 1.2E-2 provide the optimization not available through the integrated 3D design tools.



(a) Physical model of the structure (Revit; P131a)

(b) Analytical model of the structure, duplicating the physical features (Edge; P131b)

(c) Deformation contour, including foundation (Edge; P131c)

(d) Distribution of soil pressure from the analysis of the entire building (Edge; P131d)

FIGURE 1.2E-1 Multi-Story Building and its Analytical Model for Structural Analysis (P131)

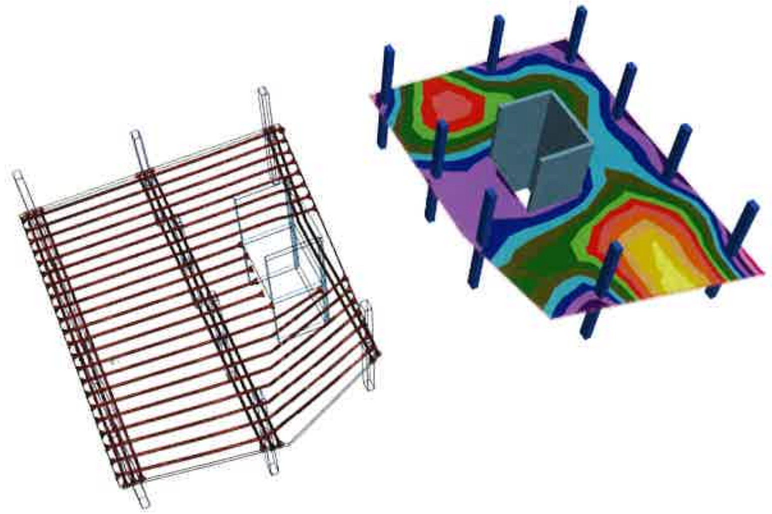


FIGURE 1.2E-2 Tendon Layout and Deformation of a Single Level Extracted from the Entire Building Model (Edge; P132)

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CHAPTER 2

POST-TENSIONING

View of a Post-Tensioned Building
(Seattle, USA, P133)

2.1 BRIEF DESCRIPTION OF PRESTRESSING

Post-tensioning is a method of prestressing concrete whereby the prestressing tendons are pulled and anchored after the concrete that they are embedded has developed sufficient strength. Prestressing concepts and the underlying theoretical background have been fully covered in a number of good text books. Leonhard [1964] covers the early works. Collins and Mitchell [Collins et al, 1997] offer an in-depth and comprehensive coverage of the subject. The book by Ed Nawy [Nawy, 1997] is a good university course text on the subject.

2.1.1 Prestressing Options

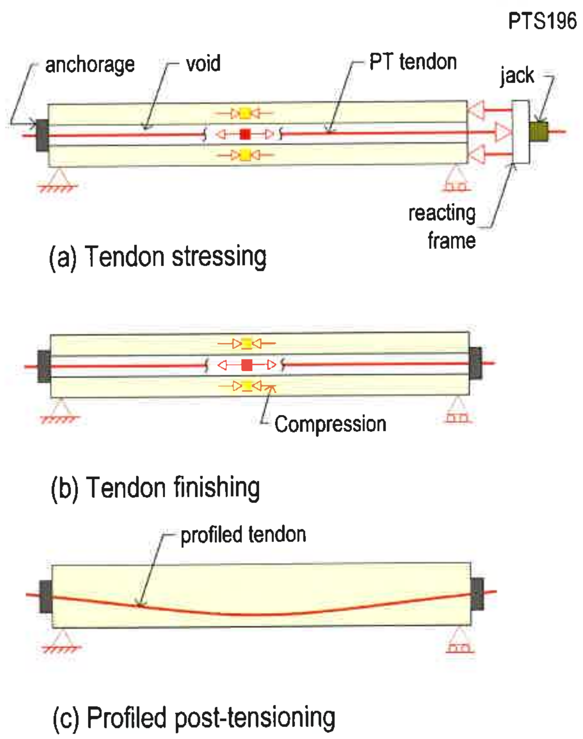
Quoting from Collins et al, "the basic concept of reinforced concrete, for both prestressed and non-prestressed construction, is that steel reinforcement is placed in those locations of a structure where tensile stresses are likely to occur. In prestressed concrete construction, high-strength reinforcement is used. This reinforcement is tensioned prior to the application of external loads. This initial tensioning of the reinforcement precompresses the surrounding concrete, giving it the ability to resist higher loads prior to cracking. "

A. Post-Tensioning: There are two types of prestressing: post-tensioning and pre-tensioning. Post-tensioning was developed first. In post-tensioning, the concrete is cast with a duct or sleeve that creates a void for the post-tensioning steel (Fig. 2.1.1A-1a). The post-tensioning steel is either placed in the duct or sleeve before the concrete is cast, or inserted through it afterwards. After the concrete has gained adequate strength, a stressing jack pulls the steel strand while reacting against the body of the concrete member. The tension in the steel imparts an equal compression in the surrounding concrete. Once the force in the steel reaches its design value, the tendon is anchored against the body of concrete, locking off the tension in the steel and the compression in the concrete.

Over time, a fraction of the initial force in the concrete is lost due to both creep and shrinkage of the concrete and relaxation in the prestressing steel. With today's material and practices, the loss ranges from 10% to 15% of the initial stress. The sources and computation of stress losses are detailed in Chapter 10.

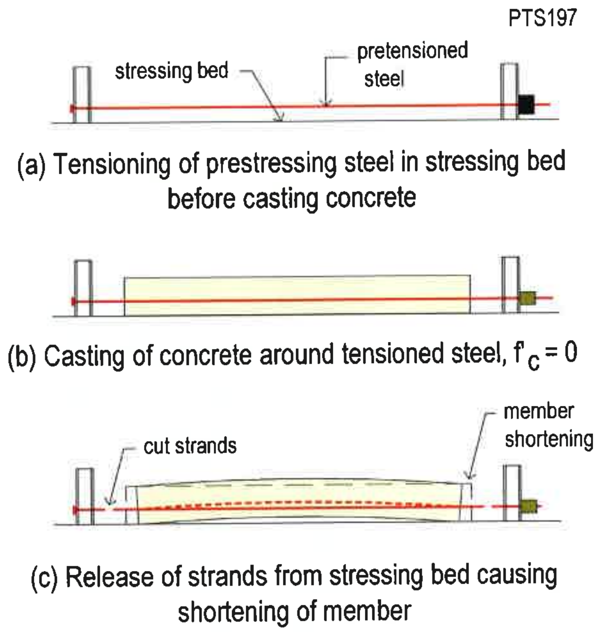
In practice, post-tensioning is rarely configured as illustrated in Fig. 2.1.1A-1a. Because the prestressing force is not applied until the concrete has cured, the tendon can be profiled as shown in part (c) of the figure. Profiling the tendon provides an additional benefit because the tendon tends to straighten under the tension and thus applies lateral- upward—forces to the concrete. The tendon profile can be designed so that its lateral forces counteract the loads—such as selfweight—that the post-tensioned member is expected to carry (part c of the figure).

B. Pre-tensioning: Practical application of pre-tensioning was developed by the German engineer E.



Basics of Post-tensioning Construction

FIGURE 2.1.1A-1



Basics of Pre-Tensioning Steps

FIGURE 2.1.1B-1

Hoyer [Hoyer, E. 1938]. As illustrated in Fig 2.1.1B-1, the prestressing steel is first stressed and anchored against external bulkheads (part a); concrete is then cast over the stressed steel (part b). Once the concrete has developed adequate strength, the tendons are released from the bulkhead. The tendency of the stretched tendons to shorten pre-compresses the concrete (part c).

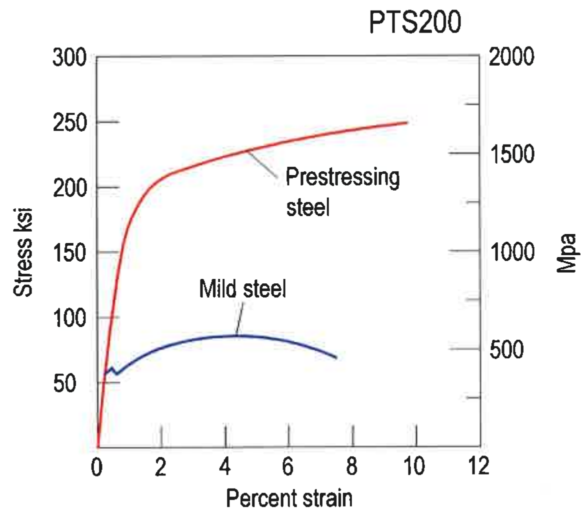
In the majority of pre-tensioning applications, the stressed tendons are straight. Occasionally, the prestressing steel is forced down (harped) at one or more locations along its path to create a more structurally advantageous profile.

2.2 DISTINGUISHING FEATURES AND ADVANTAGES OF POST-TENSIONED CONSTRUCTION

In addition to the direct economic advantages of post-tensioning that can be achieved through savings in construction cost and maintenance, post-tensioned construction has several additional advantages when compared to conventionally reinforced alternatives. This Section reviews some of the important features of post-tensioned construction.

A. Use of Less Steel to Provide Safety; Lower Overall Carbon Footprint: The markedly higher tensile strength of prestressing steel currently used in con-

struction provides four and half times the capacity of the steel used for conventional reinforcement (Fig. 2.2A-1). Consequently, replacing the conventional reinforcement required for strength with post-tensioning can reduce the weight of the reinforcement by over four times. Note that the 4.5 times advantage is achieved using grouted (bonded) systems. For unbonded systems, the ratio in savings is from 3.4 to 4 times[1]—still a significant advantage, especially where reduction in a building's carbon footprint is a concern. Hayek and Khalil address the structural efficiency of Post-Tensioned buildings from a sustainability perspective in reference [Hayek et al, 2012]. Later in this Chapter, the differences between the bonded and unbonded methods of post-tensioning construction are explained.



Stress - Strain Diagrams

FIGURE 2.2A-1 Strength Values of Prestressing and Conventional Steel

B. Use of Less Steel through Eliminating Reinforcement for Shrinkage; Temperature and Diaphragm Action: Building codes require a minimum amount of reinforcement in slabs to control shrinkage and temperature cracking.[2] The reinforcement must be distributed uniformly throughout the slab, at a spacing that is typically between one and one and one-half times the thickness of the slab. This requirement has led to the common practice of providing a mesh of reinforcement over the entire slab. At

[1] Designs based on ACI 318-11; the ratio is different in other building codes

[2] The shrinkage, and temperature reinforcement is 0.002 times the concrete cross-section (ACI 318-11 Section 7-12)

(a) View of a conventionally reinforced slab (P138)

(b) View of a post-tensioned slab (P139)

FIGURE 2.2B-1 Views of Slab Construction

best, only a fraction of the mesh reinforcement can be considered when designing the floor system for strength. Neither a mesh nor closely spaced bars are required in a post-tensioned slab. The precompression imparted by the tendons spreads rapidly through the slab from the anchorages and is sufficient to prevent cracking from shrinkage and temperature changes.[3] Figure 2.2B-1 compares a conventionally reinforced slab with a post-tensioned slab.

Seismic forces generated by a floor's mass must be distributed to the structure's lateral force-resisting system, such as the shear walls, seismic frames, and columns. The floor system acts as a diaphragm to transfer these forces to the members that support

[3] 100 psi precompression is considered adequate to meet shrinkage and temperature effects (ACI 318-11 Section 7.12)

the floor. In addition to generating forces at each level, the floors participate in the distribution of the seismic forces from one level to the next among the lateral force-resisting members of a concrete frame. The level-to-level re-distribution of seismic forces, as well as the distribution of seismic forces generated by a floor's mass at each level, requires a floor system to act as a diaphragm. Building codes[4] require a minimum area of reinforcement to be distributed throughout a slab in both directions, so that the slab will have the necessary in-plane strength for diaphragm action. The precompression provided by post-tensioning tendons under service conditions is generally more than the minimum reinforcement required for diaphragm action[5] —thus eliminating the requirement for added reinforcement

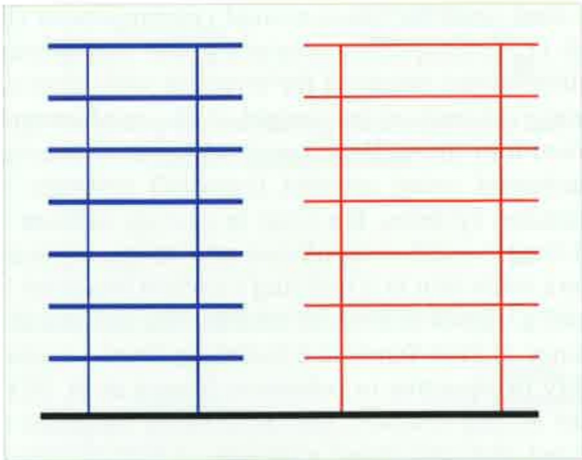
C. Thinner Slabs—Equals Less Concrete: Once span length exceeds a threshold value of about 16 ft (5 m), a post-tensioned slab will be approximately one-third thinner than a reinforced concrete slab designed for the same loading. The reduction in thickness means less concrete material is required for the slab as well as for supporting columns, walls, and foundation. In addition to reducing material costs, post-tensioning thus significantly reduces the carbon footprint of the construction.

D. Longer Spans: Post-tensioned slabs can span greater distances than conventionally reinforced slabs of the same thickness. In addition to providing larger open spaces, the longer spans allow a reduction in the number of supports, with associated savings. The following figure (Fig. 2.2D-1) schematically represents the comparison of a conventionally reinforced concrete frame with one that has been post-tensioned. The figure highlights the advantage of using longer spans and thinner slabs.

E. Simple Forms; Elimination of Beams: In many cases, judicious application of post-tensioning can allow beams to be eliminated. Flat-slab construction, illustrated in Fig. 2.2E-1, reduces the cost of forming. In the US, forming costs can be as much as one-third of a floor system's cost; simplification in forming is thus a great advantage.

F. Ability to Better Span Irregular Support Arrangements: Today's architectural aspirations, coupled with improved material properties and advances

PTS198



(a) RC (b) PT

FIGURE 2.2D-1 Conventionally Reinforced and Post-Tensioned Concrete Frames—Span and Thickness Comparison

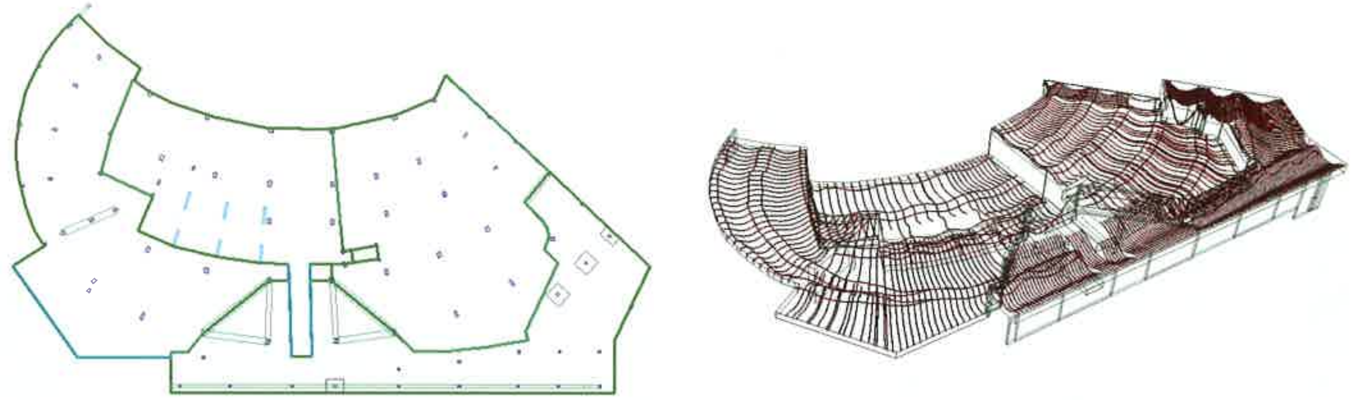
in structural analysis and design, produce daring floor layouts with large open spaces, as well as irregular arrangement of supports and plan geometry. Post-tensioned flat slabs that do not have to rely on the beam-and-slab framing common in conventionally reinforced concrete are particularly adaptable to irregular geometry. Figure 2.2F-1 is an example of an irregular post-tensioned slab constructed in Florida.

G. Lighter Concrete Frames; Lower Seismic Demand: A post-tensioned concrete frame is generally one-third lighter than a comparable conventionally

FIGURE 2.2E-1 Beamless Flat Slab Construction Using Post-Tensioning (P134)

[4] ACI 318-11; Section 7.12

[5] ACI 318-11; Section 21.11.7



(a) Plan showing irregular support layout wihout beams (P135a)

(b) View of tendon layout (P135b)

FIGURE 2.2F-1 Example of an Irregular Floor Geometry and Support Layout (Alta Hospital Podium, USA)

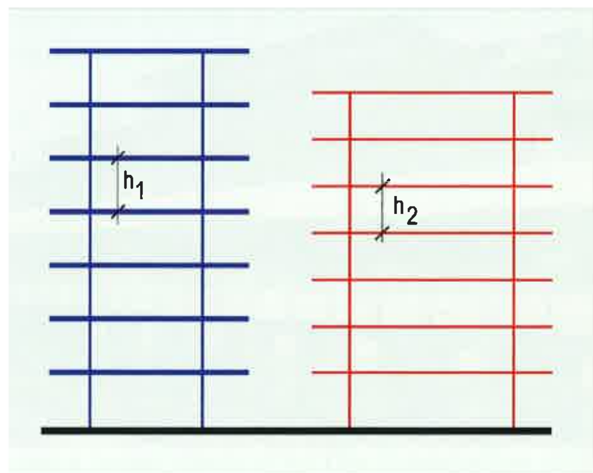
reinforced design when the supports and foundations take advantage of the lighter floor system. However, in some cases, the post-tensioning enters the design through "value engineering" after the construction contract is awarded. The value engineering may not take full advantage of the reduction of weight achieved by converting the slabs to post-tensioning because to do so would require redesigning the columns, walls and foundations. Greater benefit is achieved when the construction is designed to be post-tensioned from the start, and the entire framing is designed in recognition of the reduced floor weights.

In California, builders often use lightweight concrete for the floor systems, and regular concrete for the columns and walls. This combination can reduce the weight of a building by one-half—a major advantage in regions of high seismic risk.

H. Shorter Concrete Frames: Reduced slab thickness and elimination of beams result in shorter floor-to-floor height, and consequently, a reduction in the total height of a building (Fig. 2.2H-1). For a typical twenty-five story residential building, this can allow another floor to be added without increasing the building height. Alternatively, there is an advantage to reducing the façade surface and its cost. A structural design benefit of a shorter building is that the lever arm for the overturning moment created by seismic or wind forces is smaller.

I. Greater Ability to Resist Concentrated Forces: One of the salient properties of post-tensioning is that when stressed, a tendon exerts force on the structure. When the building is in service, the post-tensioning acts as an active load system, in the same sense as other loads, such as dead and live loads. The applied force from post-tensioning is generally configured to counteract the externally applied forces, thus reducing their undesirable effects. Consider the application of a column as shown in Fig. 2.2I-1. When conventionally reinforced concrete is used, the slab needs to deflect and crack before the reinforcement beneath a concentrated load is mobilized to resist the effects of the load. In a post-tensioned slab, the tendons can be profiled to apply an upward force in an amount that, when combined with the precompression from the tendons, will counteract the ap-

PTS199



(a) RC (b) PT

Figure 2.2H-1 Reduction in Height Resulting from Thinner Floors and Elimination of Beams

(a) Base plate of steel columns planted on PT slab (P136a)

(b) Steel frames planted on post-tensioned slab (P136b)

FIGURE 2.2I-1 Concentrated Load Supported on Post-Tensioned Slab

plied force without undue deflection and without the need for local cracking. This is why post-tensioning is viewed as "active" reinforcement versus conventional reinforcement bars, which are viewed as "passive" reinforcement.

When a change in occupancy from one level to the next calls for different arrangements of columns on different levels, the active force of post-tensioning can be used to effectively handle the load transfer in the slab from the columns that terminate from the framing above. In the framing of the office building in Fig. 2.2I-2, the steel columns shown are positioned to suit the layout of the office space above, while the space below is laid out for vehicle parking. Flat slab construction with column drops was used to handle the requirement; the columns below are fewer in number, farther apart, and arranged to suit an optimum parking layout.

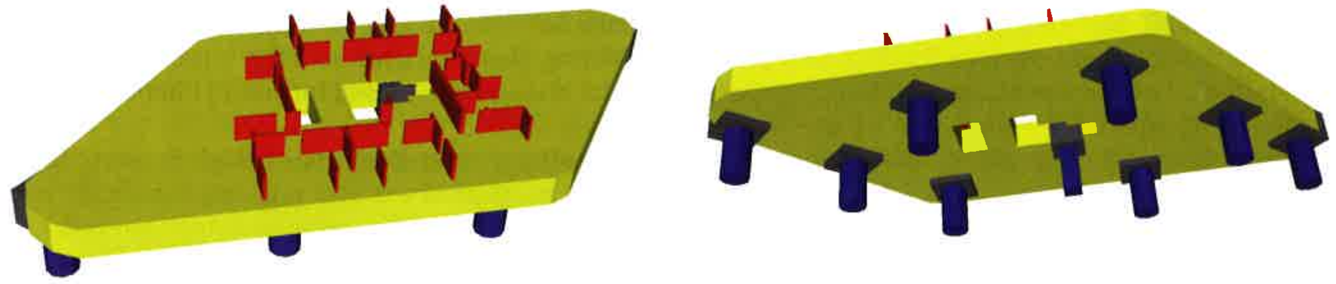
A fitting application of this concept is seen in transfer plate construction in the foyers of commercial buildings, where multiple levels of superstructure are supported on a slab that rests on a few supports to provide open space (Fig. 2.2I-3).

J. Reduced Deflections: Floor systems reinforced with post-tensioning generally deflect less than com-

(a) Office building constructed with steel framing on a post-tensioned slab (P137a)

(b) Level below the steel frame superstructure featuring support arrangement for parking (P137b)

FIGURE 2.2I-2 PT Slab Between Parking Level and Office Levels Above Affords Rearrangement of Columns to Suit Each Occupancy (Irwin Plaza; San Rafael, California)



(a) Top view of transfer plate (P140a)

(b) Underside view of transfer plate (P140b)

FIGURE 2.2I-3 Analysis Model of a 2.5 m Thick (8' 2") Transfer Plate, Supporting a Multi-Level Tower Over an Open Foyer (Macau; Courtesy of Freyssinet)

parable conventionally reinforced designs; reduced deflection has been one of the driving forces in the increased use of post-tensioning in the US. There are two reasons for the reduced deflections. First, much of loading on a concrete floor is self-weight. Post-tensioning can be designed to provide an upward force that balances a high fraction of a floor's self weight—thus reducing the net downward force that causes deflection. Second, because cracking can be reduced, or even eliminated, post-tensioned floors have greater flexural stiffness than comparable reinforced concrete floors in service.

K. Reduced Cracking: Because ACI 318 imposes a low limit on the allowable in-service tensile stresses, two-way post-tensioned floor slabs designed using ACI 318 will be essentially crack-free under service conditions. When using the European Code EC2[6], the designer selects the extent of allowable cracking and a "design crack width", based on the anticipated in-service conditions of the floor system. The designer's choice becomes the amount of cracking and design crack width, as opposed to the elimination of cracking that results when using ACI 318.

L. Improved Resistance to Water Penetration: The concrete used commercially for construction of buildings, whether conventionally reinforced or prestressed, is not watertight. Additives to concrete mix and/or other measures are necessary if ingress of water is to be avoided. Post-tensioning provides a greater resistance to water penetration, however, because post-tensioned slabs have fewer cracks than conventionally reinforced floors.

M. Perception and Acceptability of Vibration: Vibration is a serviceability consideration. Within a certain frequency range (mostly between 4 and 8 Hz), vibration coupled with high acceleration (0.1–0.3% of gravitational acceleration), can be perceived by occupants and experienced as undesirable [ADAPT TN 290, 2010]. Foot fall on large areas supported on thin slabs can trigger unacceptable vibration; cracking will exacerbate the problem because it lowers the natural frequency of the slab. Post-tensioned slabs are generally thinner than their conventionally reinforced counterparts and have longer spans, thus they are more prone to unacceptable vibrations. However, two benefits of post-tensioning help to reduce the susceptibility of post-tensioned floors to objectionable vibration. One is a reduction in weight (mass) because the slabs are thinner, and the other is a larger relative stiffness because there is less cracking. Both of these features help to increase the natural frequency of vibration and improve the design. On the other hand, the longer spans used in post-tensioned structures tend to lower the natural frequencies and aggravate the perception of vibration. For these reasons, the vibration of post-tensioned slabs under foot fall should generally be investigated where spans are relatively large.

2.3 APPLICATION OF POST-TENSIONING IN BUILDING CONSTRUCTION

Post-tensioning was first used for slabs in the US in the mid-1950s. Most post-tensioned slabs constructed at that time were associated with the lift-slab method of construction; post-tensioning was introduced to solve the weight, deflection, and cracking problems that arose with conventionally reinforced lift-slabs. Since then, post-tensioned slabs have become a major element in the construction of floor systems of all types of commercial and residential buildings. In their most popular form, namely two-way flat slabs, they have been proven to perform well and be economical for both high- and low-rise

[6] EC2 – 1992 -1 -1:2004

buildings. Post-tensioning is widely used in the US; in selected other areas, such as Panama City, Panama, almost all of the significant buildings are post-tensioned. Figure 2.3-1 shows one of the highest concentrations of post-tensioned buildings in a city skyline.

FIGURE 2.3-1 Panama City, Panama. Practically all Notable Buildings are Post-Tensioned (P340)

Post-tensioning is not limited to building construction. However, given the scope of this book, the following list is confined to examples of its use in buildings.

2.3.1 Floor Systems—Flat Slab Construction
Flat slabs are a type of construction wherein a floor resists the applied loads without beams; the slab itself is the primary structural element of the floor system. A flat slab can include steps, openings, and local thickenings around the columns, such as column drops, drop panels, or slab bands. A computer model of a typical flat slab in a multi-story building is shown in Fig. 2.3.1-1. In most optimum designs for residential and commercial buildings, flat slabs have

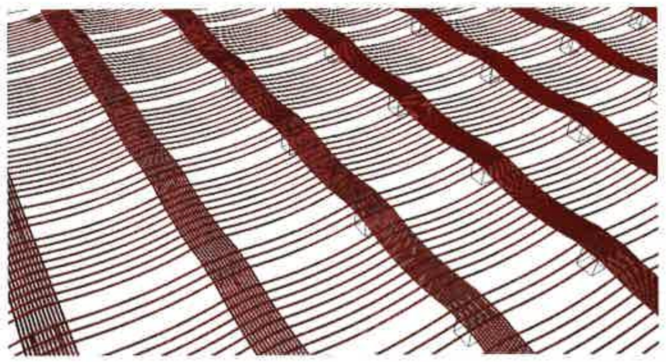


FIGURE 2.3.1-1 Partial Plan of Tendon Layout of a Water Tank Roof (figure shows column drops; columns not shown for clarity; Hefner, VSL grouted system; P518)

spans between 26 and 33 ft. (8 to 10 m). The load-resisting characteristics and the structural features of flat slabs are discussed further in Chapter 3.

A. Application in Regions of High Seismic Risk: Buildings constructed with post-tensioning are particularly well suited for regions of high-seismic-risk because of their lower weight and height, and the improved diaphragm action that results from the precompression. Figures 2.3.1A-1 and 2 illustrate the use of post-tensioning in San Francisco and other regions of California that are exposed to the highest seismic risks in the US.

Post-tensioning is not limited to high- rise buildings. The following (Figs 2.3.1A-3 and 4) are several examples of mid-rise buildings in San Francisco and other high- seismic- risk regions of California.

FIGURE 2.3.1A-1 Post-Tensioned Building at 301 Mission Street, San Francisco, California (P342)

B. Application in Regions of High Wind Forces: Where high wind forces such as hurricanes govern the lateral force-resisting design of a building, post-tensioned buildings have performed well. Florida

FIGURE 2.3.1A-2 W-Hotel, San Francisco, California (P345)

Figure 2.3.1A-4 Ocean Villa Apartments, Southern California (P348)

FIGURE 2.3.1A-3 Red Lion Inn, Modesto, California (P347)

FIGURE 2.3.1B-1 Post-Tensioned Building Constructed to Resist Wind Forces (Four Seasons Hotel, Florida, P349)

abounds with examples of post-tensioned high-rise buildings designed to resist extreme wind forces (Fig. 2.3.1B-1).

C. General Building Applications: Post-tensioned buildings need not be located in high seismic or high wind regions to be economical and perform well. There are numerous successful applications in different environmental and loading conditions, from the harsh weather conditions of Saudi Arabia and the Persian Gulf to the mild and pleasant environment of Buenos Aires, Argentina. Figures 2.3.1C-1 and 2 are examples in Norway and London.

FIGURE 2.3.1C-1 Post-Tensioned Building Aslund (Norway) Cantilevering over Promenade (P353)

FIGURE 2.3.1C-2 Post-Tensioned Building (Knightsbridge, London; P354)

2.3.2 Floor Systems—Beam and Slab Construction

When the aspect ratio of a slab panel exceeds two, it is often more economical to use beam-and-slab construction rather than a flat slab. The parking structure layout common in the US lends itself to such construction; Figure 2.3.2-1 is an example. Other examples are illustrated in Figs. 2.3.2-2 and 2.3.2-3.

The parking structure in Fig. 2.3.2-3 is constructed with an unbonded post-tensioned beam and slab system and is located at the San Jose International Airport (San Jose, California), in one of the highest seismic-risk regions of the country. One criterion for the selection of the construction scheme was the

FIGURE 2.3.2-1 View of Rostam Parking Structure (P355)

FIGURE 2.3.2-2 Interior View of a Beam and Slab Parking Structure (San Francisco Airport; P356)

FIGURE 2.3.2-3 San Jose California International Airport Parking Structure (P357)

City of San Jose's requirement that the building remain functional subsequent to the anticipated "big" earthquake in the San Francisco Bay Area; the intent was to construct a facility that could be used for the distribution of medical and food supplies. The structure has performed extremely well to date; it withstood the 1989 Loma Prieta earthquake [Aalami, et al, 1990] unscathed.

2.3.3 Podium Slabs in Low Rise Buildings

A frequent practice in the US for buildings up to five levels is to take advantage of the ability of post-tensioned slabs to resist concentrated loads from the posts and walls of the upper levels without requiring a support immediately below each load. This application — referred to as a podium slab — is common in buildings where the lowest level will be used for parking or retail applications which require a support layout that is different from the residential or office levels above.

Figure 2.3.3-1 shows a post-tensioned podium slab supporting four levels of light-framed superstructure under construction. Figure 2.3.3-2 shows a completed structure, in which the upper levels rest on a post-tensioned slab that forms the ceiling of the ground-level retail shops.

In addition to the building superstructure, a podium slab may support a landscaped plaza with changes in elevation and high landscaping loads, making the configuration of such slabs more complex. The flexibility of post-tensioned flat-slab construction allows even the most complicated geometry and loading to be handled with relative ease; in addition, the thinner slabs and absence of beams prove an advantage where height limits are a concern. Three-dimensional modeling of the geometry and post-tensioning

FIGURE 2.3.3-1 Low Rise Building with Post-Tensioned Podium Slab under Construction (California; P359)

FIGURE 2.3.3-2 Low Rise Building Constructed on Top of a Post-Tensioned Podium Slab. The Podium Slab Spans the Retail Space on the Ground Level (Redwood City, California; P360)

tendon layout is generally necessary for complex podium slabs, to better estimate the force distribution and arrive at an optimum design. Figure 2.3.3-3 illustrates the computer model of a podium slab with irregular geometry.

2.3.4 Transfer Plates

Where open space is required at the ground floor level of a high-rise building, one solution is to terminate the supports of the upper floors on a slab referred to as a transfer plate. The transfer plate receives the loads from the column and wall supports of the superstructure, and transfers them to a lim-

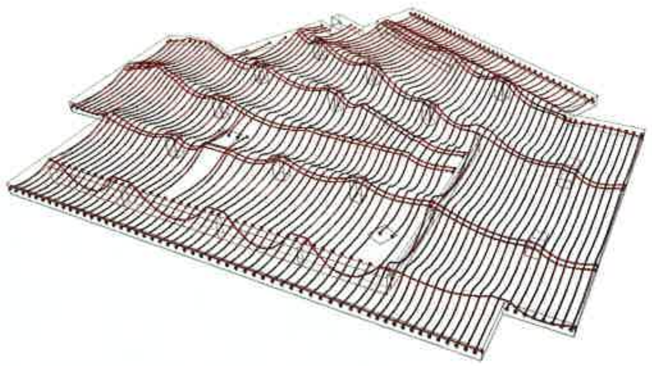


FIGURE 2.3.3-3 Three Dimensional Computer Model of a Podium Slab for a Low-Rise Building (P361)

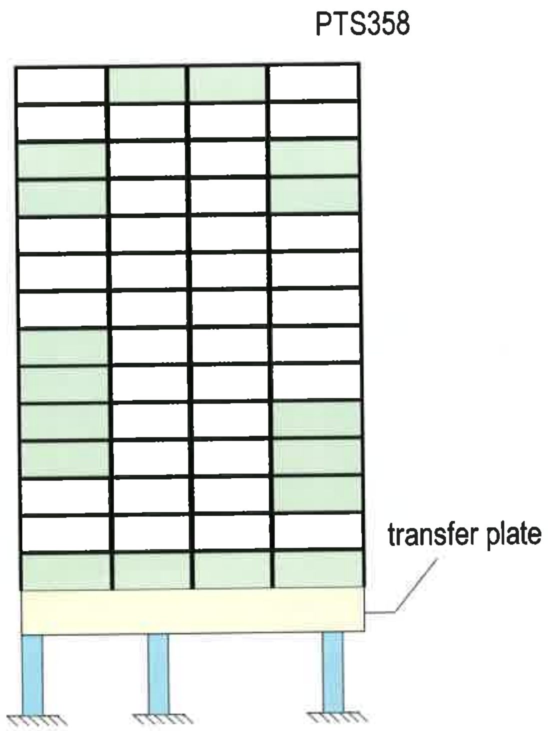


FIGURE 2.3.4-1 Symbolic View of Application of a Transfer Plate

ited number of generally widely spaced supports. A symbolic representation of a transfer plate is shown in Fig. 2.3.4-1; Fig. 2.3.4-2 shows an example where a transfer plate was used to span a viaduct.

Figure 2.3.4-3 illustrates the application of a transfer plate in a building in Macau. The more than 80-story building is resting on a 3.5 m (11'-6") post-tensioned transfer plate with column drops that bring the total slab thickness to 5.5 m (18') at the supports. The computer model generated for the analysis and design of the slab shows the large number of walls

FIGURE 2.3.4-2 View of a Transfer Plate that Straddles a Viaduct (New York; P492)

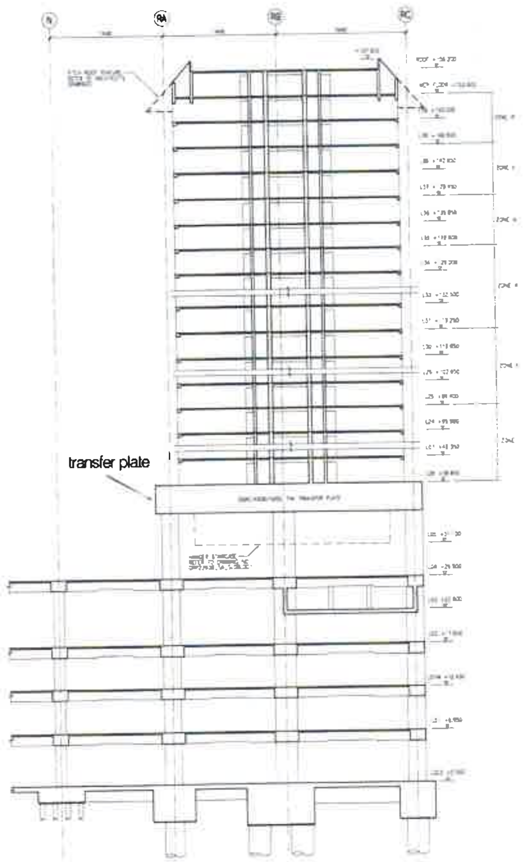


FIGURE 2.3.4-3 Transfer Slab of Service Apartments in Macau (Courtesy of Freyssinet HK; 362)

from the superstructure terminating on the transfer plate with only a few, widely-spaced supports below (Fig. 2.3.4-4).

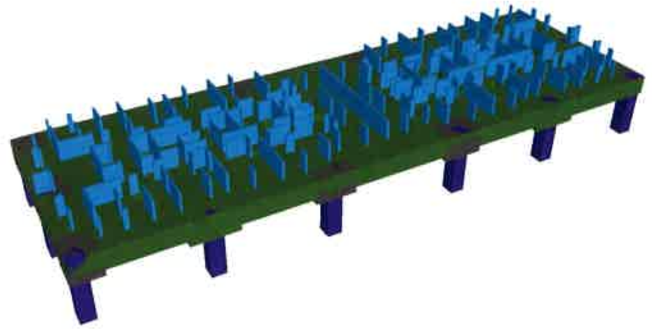


FIGURE 2.3.4-4 Computer Model of Service Apartments in Macau (Freyssinet HK; ADAPT Corp, P363)

To facilitate the construction of this transfer plate, which was located over 30 m (98 ft) above grade; two-layer construction was used. The bottom 1m (3' 3") thick layer was cast first and designed to carry the weight of the upper 2.5 m (8' 2") layer. The combined thickness was designed to resist the loads from the tower above. As indicated in Fig. 2.3.4-5, the post-tensioning tendons of both the lower and upper layers were installed before the lower layer was cast and stressed. Figure 2.3.4-6 shows a technician securing one of the ducts for the multi-strand tendons for the 3.5 m (11' 6") transfer plate.

2.3.5 Mat/Raft Foundation

When the allowable bearing pressure is not adequate to resist peak stresses below walls and columns, but the total area of the soil below the footprint of a structure is large enough to resist the total load, a mat/raft foundation can be a viable solution.

FIGURE 2.3.4-5 Transfer Plate Showing Both the 1 m Bottom Layer Tendons and the Full Height Tendons (P364)

FIGURE 2.3.4-6 Technician Adjusting a Post-Tensioning Tendon for the Full 3.5 m (11' 6") Height (P365)

FIGURE 2.3.4-7 Construction View of a Transfer Plate 1.5 m (5 ft) Thick (Courtesy Freyssinet Gulf; ADAPT; Jabal Omar, KSA; P366)

A mat foundation is a slab of mostly uniform thickness that often extends over the structure's entire footprint; its function is to distribute the loads, so that the soil pressure is reduced to allowable values. Figure 2.3.5-1 illustrates the point and the application of post-tensioning.

In conventionally reinforced concrete construction, when the mat/raft thickness is not adequate, the load from above does not distribute adequately over the entire mat surface. If the concentration of pressure below the loads exceeds the allowable bearing pressure (Fig. 2.3.5-1a), the mat thickness must be increased to achieve adequate distribution of the

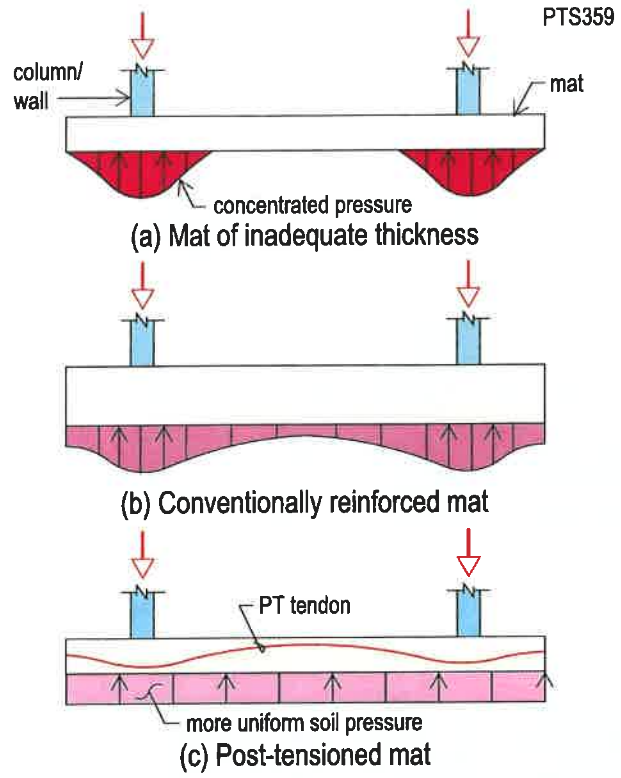


FIGURE 2.3.5-1 Load Transfer of Conventionally Reinforced and Post-Tensioned Mat Foundations

FIGURE 2.3.5-2 Post-Tensioned Mat Foundation, (Fortaleta, Brazil, courtesy Impacto; P367)

load (Fig. 2.3.5-1b). For a thirty-story building, depending on the support configuration, the required mat thickness in the San Francisco Bay Area[7] can be as high as 8 to 10 ft (2.4–3 m). Alternatively, if the mat is post-tensioned, the tendons can be configured to counteract the applied loads and distribute them over a larger area (Fig. 2.3.5-1c). A well-designed post-tensioned mat can be as much as 40 percent thinner than a reinforced concrete mat designed for the same loads.

Examples of post-tensioned mat foundations using unbonded tendons are shown in Figs. 2.3.5-2 and 3; an example using grouted tendons is shown in Fig. 2.3.5-4.

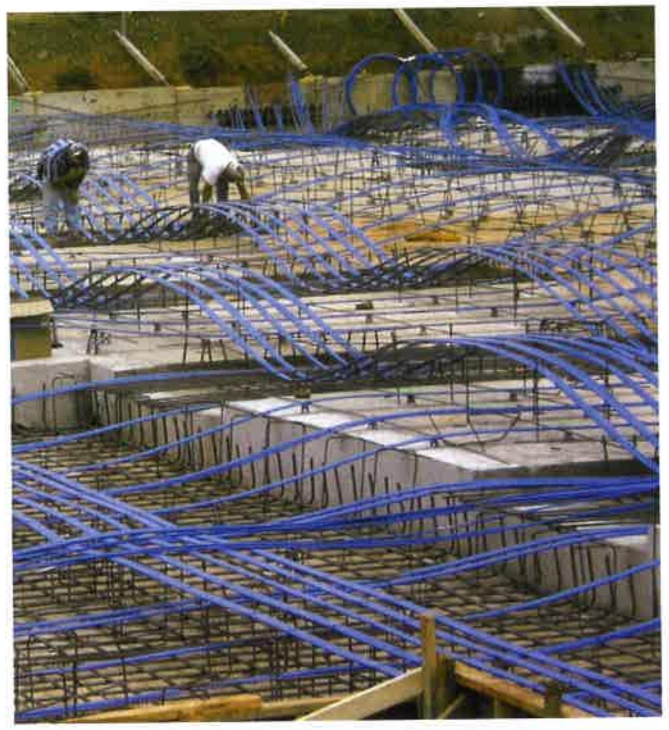


FIGURE 2.3.5-3 Post-Tensioned Mat Foundation Using Unbonded Tendons (VSL; P368)

FIGURE 2.3.5-4 Example of a Mat Foundation in KSA Using Grouted Tendons (Freyssinet Gulf; P369)

[7] Particularly in the areas around the Bay that rest on the soil known as Bay Mud.

FIGURE 2.3.6-1 Industrial Floor Slab with Storage Stacks Served by Forklift Trucks (Courtesy ADAPT; Moscow; P717)

Post-tensioned mat foundations can be highly economical. Besides reducing the slab thickness and the amount of reinforcement required, post-tensioned foundations require less excavation and hauling.

2.3.6 Industrial Ground-Supported Slabs

In industrial storage areas, such as the ones shown in Figs. 2.3.6-1 and 2, forklifts are used to stock and retrieve loads from multi-level stacks. A flat, crack-free floor surface is of paramount importance for the safe operation of forklifts. Joint-less post-tensioned industrial ground-supported slabs can be used to provide the smooth ride necessary for forklifts and other loading equipment.

A conventionally reinforced industrial slab, besides being subject to potential shrinkage cracks, cannot accommodate the changes in the underlying soil with the flexibility of a post-tensioned alternative. The formation of cracks in conventionally reinforced slabs as shown in Fig. 2.3.6-3 can impair the smooth operation of loading equipment on a slab surface.

FIGURE 2.3.6-2 View of an Industrial Warehouse with Multilevel Storage Stacks Served by Forklift Trucks (P371)

FIGURE 2.3.6-3 Example of Conventionally Reinforced Industrial Floor Slab with Extensive Cracking (P372)

Construction of a post-tensioned ground-supported industrial slab involves preparing the upper layer of the underlying soil to a design-specified bulk modulus, covering it with two layers of plastic sheets (Fig. 2.3.6-4), and dividing the slab area into segments. The post-tensioning tendons are then laid out (Fig. 2.3.6-5), the concrete is cast for each segment, and the tendons are stressed. An important contribution of the plastic sheets is the reduction of friction between the slab and the underlying soil when the tendons are stressed. To provide continuity of precompression across the construction joints, the tendons must either be overlapped at the joints or made continuous across the joints by couplers or intermediate stressing anchorage devices. Figure 2.3.6-5 is an industrial complex with a post-tensioned floor.

FIGURE 2.3.6-4 Industrial Floor Foundation Being Prepared with Friction-Reducing Moisture Barrier (P373)

Figure 2.3.6-5 Industrial Building Provided with a Post-Tensioned Ground-Supported Slab (Brazil; Aereas; P375)

2.3.7 Slab-on- Grade SOG; Residential and Light Industrial

The largest application of post-tensioning in building construction in the US is for the foundations of residential and light industrial buildings on expansive soils. Over half the tonnage of post-tensioning strands used in the US is for such foundations; the next most common application is in building construction (Fig. 2.3.7-1). Traditionally, in most other parts of the world, the primary application of post-tensioning has been in bridge construction, followed by special applications, and finally residential and commercial buildings. However, this is rapidly changing as contractors and owners become aware of the advantages of using post-tensioning in buildings.

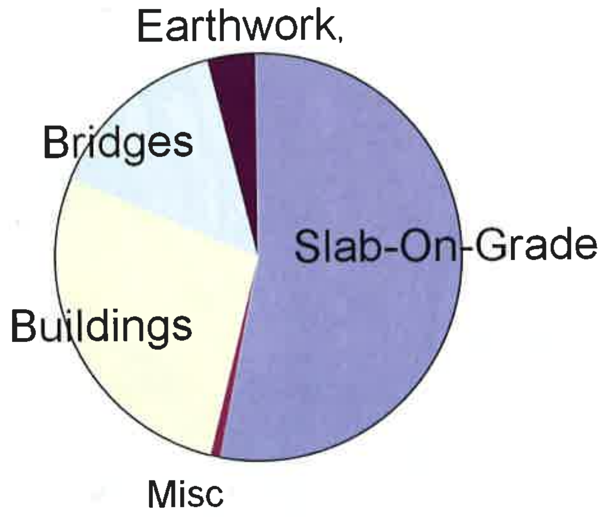


FIGURE 2.3.7-1 Typical Tonnage Breakdown of Post-Tensioning Application in the US (P376)

Post-tensioned slabs-on-grade (SOG) are often used in regions where variations in seasonal moisture coupled with expansive clays result in significant seasonal changes in the volume of the soil, as shown in Fig. 2.3.7-2.

Figure 2.3.7-3 illustrates the difference between the permanent differential settlement caused by consolidation of the underlying soil, and the seasonal change in soil volume at the perimeter of the slab.

Light buildings typically have shallow foundations that are susceptible to volumetric changes in the supporting soil; post-tensioned SOGs can be designed to limit the effects of seasonal changes on these buildings by limiting the foundation movement. The objective is generally to limit the deformation in the structure to an amount that does not impair its serviceability.

FIGURE 2.3.7-2 Example of Expansive Soil Subject to Seasonal Changes in Volume (P377)

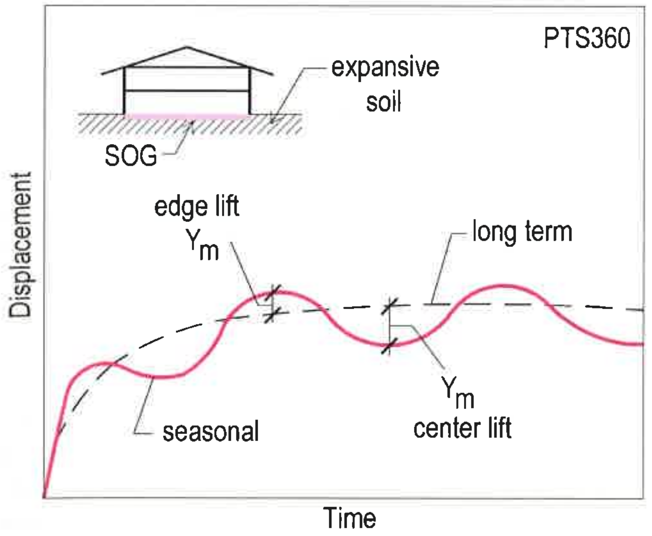


FIGURE 2.3.7-3 Sustained and Seasonal Movement of a Foundation on Expansive Soil

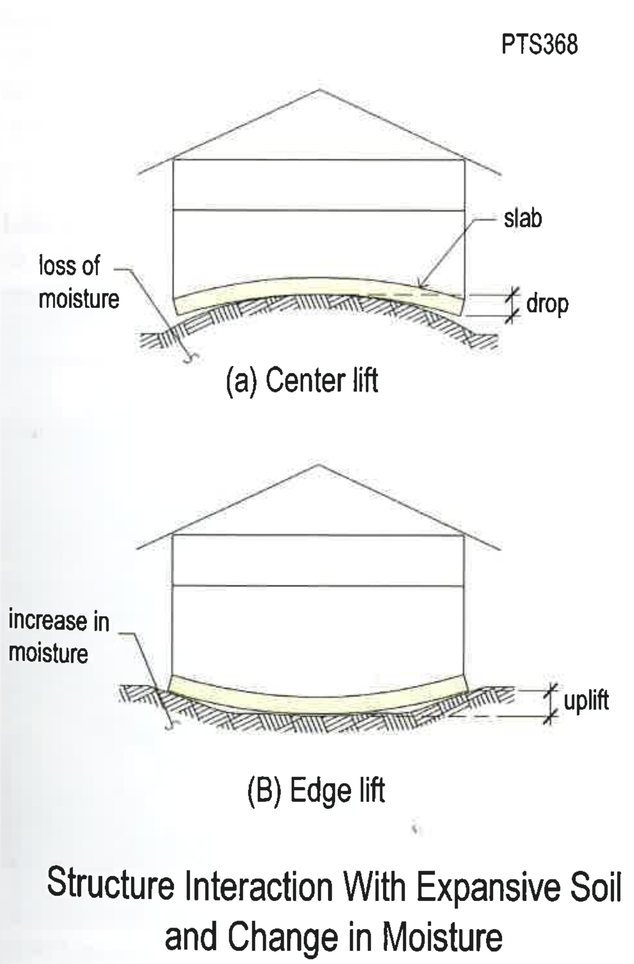


FIGURE 2.3.7-4 Impact of Expansive Soil Movement on a Building

The two extreme conditions for a light building on expansive soil are illustrated in Fig. 2.3.7-4 and 2.3.7-5. Center lift (more properly called edge-drop) occurs when loss of moisture causes the soil around the perimeter of the slab to subside; this is illustrated in Part (a) of the figure. Edge lift occurs during the wet season, when the increase in moisture content causes the soil around the perimeter of the slab to swell. Soil movement between the two extreme conditions can exceed 4" (100 mm). Such conditions are not uncommon in parts of Texas, Colorado and California.

The project's geotechnical engineer must determine estimated values for these movements. For the edge lift condition, the engineer must design the slab to resist the expected uplift. For the center lift condition, the engineer must also estimate how far in from the slab edge there is likely to be a separation between the slab and the soil, and design the slab to cantilever over this distance.

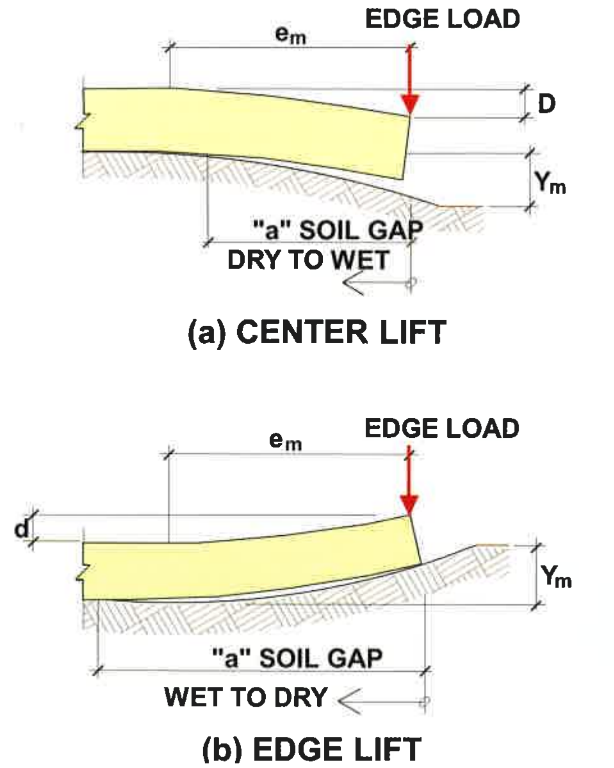


FIGURE 2.3.7-5 Extreme SOG Design Conditions (P641)

It should be noted that there is considerable disagreement within the profession as to the underlying engineering concepts and appropriate practices for SOG design [Martter, 2013]. The design is best to be based on basic engineering fundamentals, using the displacement values.

The preferred construction practice in California is a slab of uniform thickness with a shallow perimeter beam (Fig. 2.3.7-6). In other parts of the country, thinner slabs combined with interior beams are used (Fig. 2.3.7-7). In both designs, the primary reinforcement is generally limited to post-tensioning

FIGURE 2.3.7-6 Example of a Post-Tensioned SOG Ready to be Cast, California
(Common dimensions are 7 to 8 in (180 to 200 mm) for the uniform slab thickness with perimeter beams 12 in (300 mm) deep and 10 in (250 mm) wide; P378a)

FIGURE 2.3.7-7 Example of a Post-Tensioned SOG Foundation in Texas.
(Slab 4 to 5 in thick (100 to 130 mm), beams 12 to 15 ft (3.6 to 4.5 m) apart tendons 4 to 5 ft spacing (1,200 to 1,500 mm)) (P379)

tendons as shown in the figures — there is neither a mesh, nor any other reinforcement in the slab. Nonprestressed reinforcing bars must be added at discontinuities, however.

An alternative sometimes used in California is the waffle slab.[8] This option uses a single 0.5" (13 mm) straight strand in each waffle stem (Fig. 2.3.7-8). The voids below the waffles are considered to accommodate the expansion of the underlying soil better; the ribs provide added stiffness.

2.3.8 Retrofit through External Post-Tensioning

Post-tensioning has been used effectively to correct both strength and deflection deficiencies in buildings. Post-tensioning distinguishes itself from other alternatives—such as externally applied synthetic fibers and metal strips—in several respects. In addition to adding strength, post-tensioning applies force to the structure and affects its in-service condition, for example by reducing deflections—a unique feature that synthetic fibers and metal strips cannot offer. In addition, when post-tensioning is applied judiciously, its active force can reconfigure the probable failure mode of a structure, thus enhancing the structure's level of safety. The following offers an example for each application.

A. Post-Tensioning Counteracts Deflection and Provides Strength: The schematic of Fig 2.3.8A-1 illustrates the basic principle of how an externally applied post-tensioned tendon can be used to exert an upward force and reduce the deflection of a member.

FIGURE 2.3.7-8 Post-Tensioned Waffle Slab Foundation System (Courtesy of CONCO, California; P380)

8 www.conconow.com

This type of application distinguishes itself from the alternative of externally applied synthetic fibers in that it reduces in-service deflections and can close or reduce existing cracks.

A notable application of externally applied post-tensioning is the retrofit of the seven-story parking structure at Pier 39 in San Francisco [Aalami et al. 1989], (Figs 2.3.8A2, and 4) where post-tensioning was used to restore the in-service requirements of the structure and provide the strength necessary to comply with the governing code. The unbonded

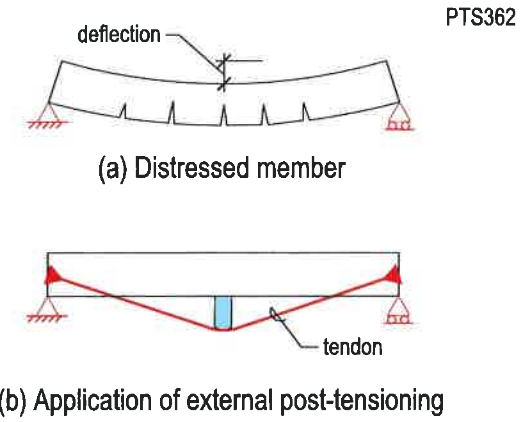


FIGURE 2.3.8A-1 Deformation and Strength Correction Through Externally Applied Post-Tensioning

FIGURE 2.3.8A-2 Pier 39 Parking Structure
Blocks at the outer face of the columns are anchorages of the externally applied tendons (San Francisco, California; P381)

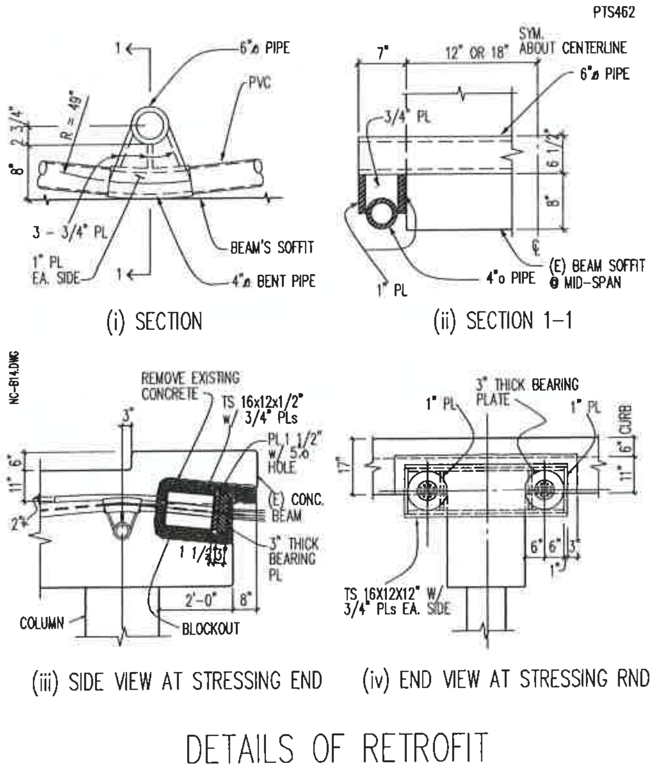


FIGURE 2.3.8A-3 Tendon Deviator and End Block Details (Pier 39 Parking Structure; San Francisco)

strands used for the retrofit were galvanized and encased in precast hollow members to provide fire protection (Fig. 2.3.8A-4). The details of the deviators and the end block for stressing are shown in Fig. 2.3.8A-3.

B. Retrofit through External Post-Tensioning to Change Failure Mode: The "active force" of external post-tensioning can be used to enhance the strength capacity of a floor system, and bring it into compliance with the prevailing building code. The concept rests on mobilizing more of the existing reinforcement in a floor system to resist design loads; it has been successfully applied to retrofit floors that are deficient in strength [Aalami et al, 1995]. The concept is described next, followed by an example of its application.

A typical floor layout of a parking level constructed using a column-supported two-way floor system is shown in Fig. 2.3.8B-1. With increasing load, the structure will develop the failure mechanism shown in 2.3.8B-2a and b. The applied load will be resisted by internal moments developed along the hinge lines as shown. There will be cracking along these hinge lines, at the top of the slab over the columns and at

FIGURE 2.3.8A-4 External Tendon for a Continuous Beam
(Pier 39 Parking Structure, San Francisco; P490)

the bottom of the slab at the mid-point between the columns. This failure mechanism draws its resistance from the reinforcement that crosses the hinge lines but does not mobilize the reinforcement parallel to the hinge lines. Hence, the resistance only uses half of the reinforcement in the slab.

By providing the right amount of externally applied force along the column lines, at the midpoint between columns (marked by Xs in Fig. 2.3.8B-2c), it is possible to force the slab to use the reinforcement in both directions prior to failure. The mobilization of reinforcement in both directions greatly increases the load- resisting capacity of the floor system. In the example discussed next, external post-tensioning was used to achieve this objective.

Figure 2.3.8B-3 shows the plan of a podium slab, and the formation of strength cracks prior to its retrofit. Due to an error, the slab was designed without any allowance for the four-level superstructure that the slab was intended to support. During construction of the superstructure, the floor slab exhibited large deflections, accompanied by cracking as shown in the figure. The location and formation of the cracks were a clear indication of inadequate strength. By installing external tendons below the slab along the column lines, and profiling them as shown in the construction details of Fig. 2.3.8B-4, the right amount of external forces were generated to force the failure mode of the floor from a "one-way" configuration (part a, Fig. 2.3.8B-2) to a "two-way" configuration shown in the same figure. The available reinforce-

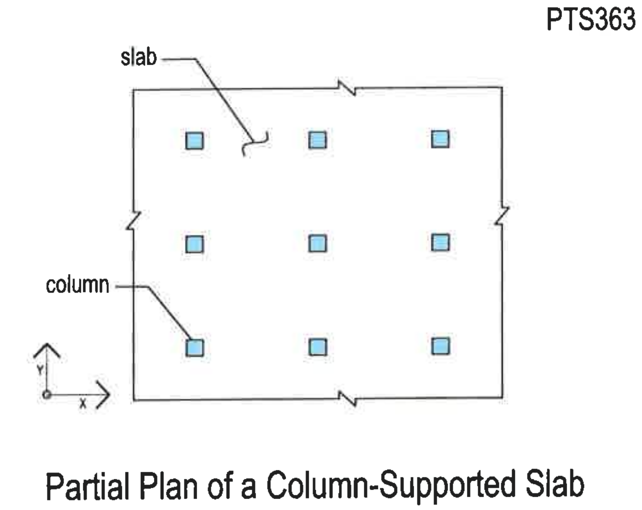


FIGURE 2.3.8B-1 Typical Support Layout of a Two-Way Parking Floor Slab

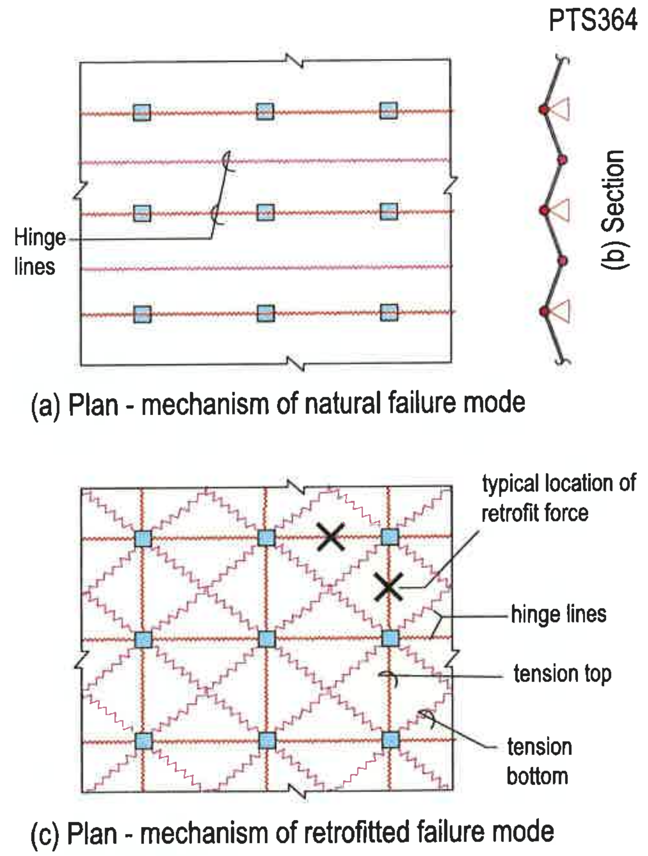


FIGURE 2.3.8B-2 Failure Mechanisms of a Two-Way Floor System before and after the Application of Externally Applied Retrofit Forces

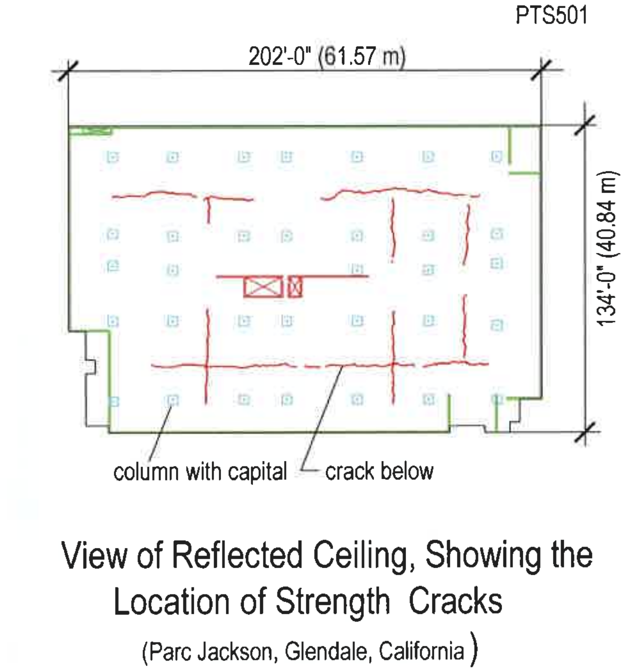
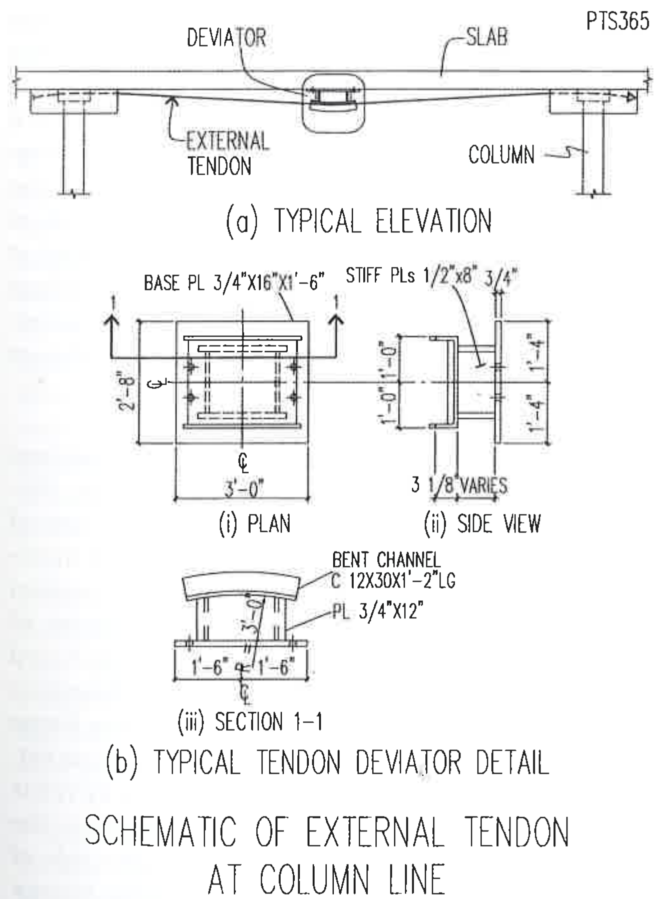


FIGURE 2.3.8B-3



Figure, 2.3.8B-4 Details of Externally Applied Post-Tensioning

ment in the slab proved to be adequate once the failure mode was changed.[9]

2.3.9 Post-Tensioning to Restore Geometry in Seismic Frames

In regions of high seismic risk, such as much of California, buildings are designed to undergo post-elastic deformation; this helps dissipate the seismic energy and reduces the demand on resistance from the building's frame. While buildings are designed to prevent collapse under anticipated seismic forces, they are expected to sustain damage. Observations from the 1994 Northridge earthquake in Southern California revealed that multi-story buildings that have experienced post-elastic deformation may not return to their original plumb position. The residual tilt in the building can cause both operational and maintenance problems.

Post-tensioning can be used as a means of restoring a building closer to its original position after post-

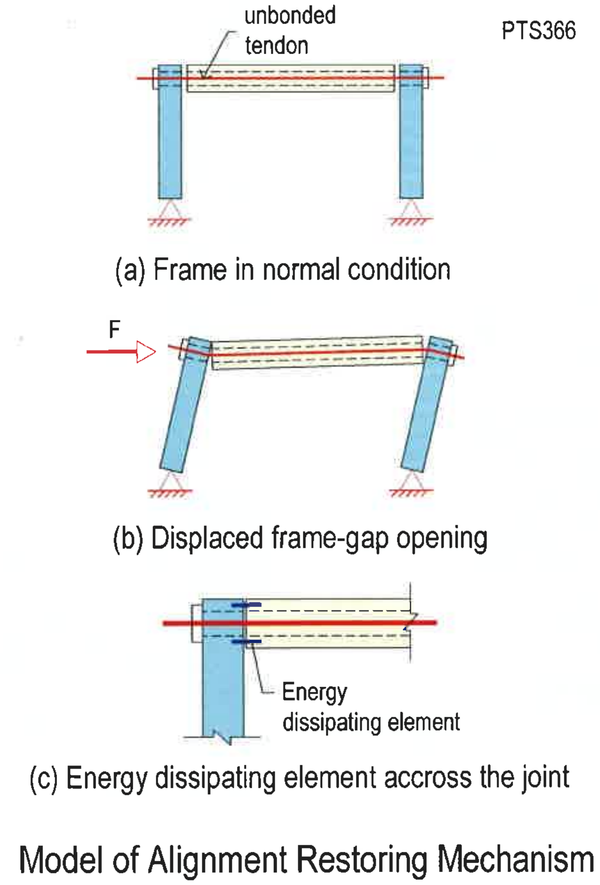


FIGURE 2.3.9-1 Model of Frame with Corrective Post-Tensioning

[9] Courtesy of Dr Markar Grigorian, MGA, Glendale, CA

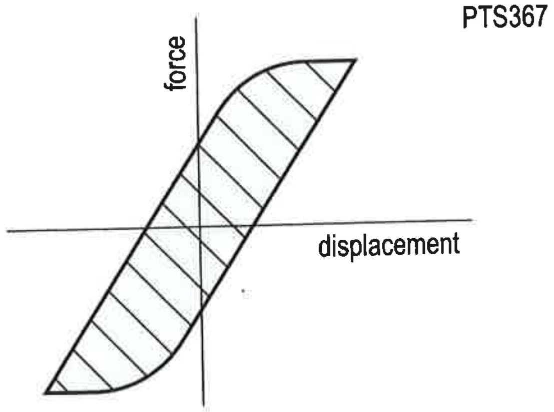


FIGURE 2.3.9-2 Idealized Illustration of Energy Dissipation at Joints

FIGURE 2.3.9-3 Paramount Building
(San Francisco, California; P385)

elastic deformation from an earthquake. This is done by directing and controlling the post-elastic deformation to designated locations, and using the force of prestressing tendons to restore the building to its original plumb position.

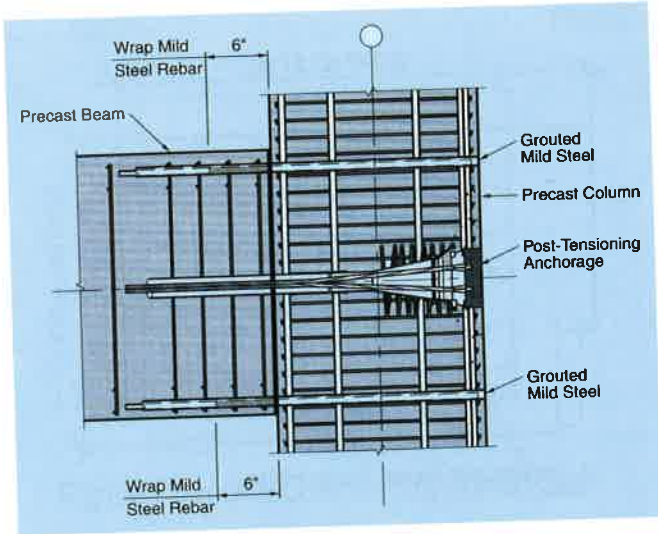


FIGURE 2.3.9-4 Detail of the Beam-Column Joint
(P386)

Figure 2.3.9-1 illustrates the concept. Consider three distinct, rigid members assembled to form a frame. The connection is achieved through an elastic element stretched through an oversized tube and anchored at the outer face of the vertical members (Part a). A horizontal force F will displace the frame as shown in Part (b) of the figure. The applied force is counteracted by the extension and increase in force in the elastic element. Once the external force F is removed, the tension in the elastic element will restore the frame to its original geometry. Because the interface between the members opens and closes as the force varies (Part c), a device can be placed at the interface to absorb part of the energy and reduce the displacement. The energy-dissipating characteristics of the device are generally of the form shown in Fig. 2.3.9-2.

Figure 2.3.9-3 shows a building in San Francisco that was constructed using the above concept. [Englekirk, 2002]. The designated seismic frames of the building are made up of precast concrete members. A multi-strand unbonded tendon was passed through an oversized duct cast into the center of each beam; the tendon was stressed and anchored at the outer end of the corner columns. It functions as the elastic element designed to restore the frame to its original position, after a lateral displacement. Energy dissipation at the joints is provided by rebar that extends through the beam-column interface; the rebar is debonded over a distance on either side of the interface (Fig. 2.3.9-4). The post-elastic energy dissipation is intended to be limited to these designated bars.

(a) Partially assembled frame (P387a)

(b) Close up of frame showing tendon duct (P387b)

FIGURE 2.3.9-5 Parking Structure Frame with Geometry Restoring Feature

The concept has been used in several parking structures constructed in California. Figure 2.3.9-5 illustrates the frame of a parking structure with this displacement-restoring feature.

2.3.10 Post-Tensioning in Walls

Application of post-tensioning in walls is not common. However, where the lateral (horizontal) forces along the length of a wall are large and the vertical axial force is not adequate to prevent a wall from excessive tension and possibly overturning, post-tensioning along the height of the wall in the vertical direction can be used to reduce tension and keep the wall in position. Such scenarios are likely to be applicable in regions of high seismic risk. Application of post-tensioned walls is discussed in [Perez et al. 2004], [Stevenson et al. 2008].

2.3.11 Post-Tensioning in Columns

Post-tensioning reduces the axial capacity of a column. Hence, post-tensioning is only beneficial when a column is likely to be subjected to significant bending and its design will be governed by the bending effects of the applied loads.

Pre-tensioning is often used to counteract the stresses that develop in precast columns or piles during transportation and installation.

2.3.12 Special Applications of Post-Tensioning

Long span exhibition halls, tension structures, and shell structures often feature tensioned cables in one form or another. These special applications are beyond the scope of this book.

2.4 POST-TENSIONING MATERIAL AND HARDWARE

Post-tensioned buildings are constructed with the same materials that are used for conventionally reinforced concrete buildings, along with post-tensioning tendons and the hardware associated with placing, stressing, and finishing the tendons. Figures 2.4-1 through 2.4-3 show typical examples of post-tensioning in building construction.

The materials and equipment unique to post-tensioned construction are:

❖ prestressing steel, typically seven-wire strand, encased in either a duct or sheathing;
❖ anchorage devices, consisting of an anchor block (plate) and wedges; and a device for the dead end, if the tendon is stressed at one end only;
❖ tendon support rebar or chairs to create the design profile of the tendons at installation;
❖ pocket formers to create a recess at the edge or surface of the concrete member for the nose of the stressing jack;

FIGURE 2.4-1 Post-Tensioned Slab Reinforced with Grouted Tendons, Ready to Receive Concrete
(VSL; P141)

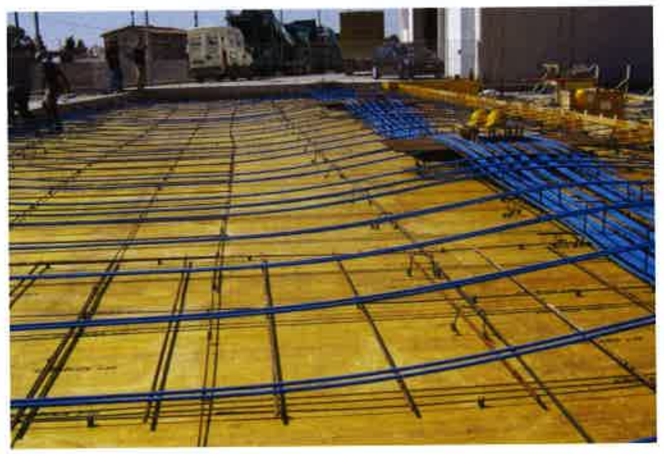


FIGURE 2.4-2 Post-Tensioned Slab, Reinforced with Unbonded Tendons, Ready to Receive Concrete
(Courtesy of GRANDI STRUCTURE S.r.l. Italy;P142)

FIGURE 2.4-3 Multi-Strand Construction for a Department Store, Dubai
(Courtesy of Freyssinet Gulf; P143)

- a stressing jack; and
- a grout mixer and pump (for bonded systems).

There are two types of post-tensioning tendons: "unbonded" and "bonded," also referred to as "grouted." If a tendon is to be unbonded, the prestressing steel is coated with a corrosion-inhibiting grease, and then encased in a plastic sheathing. The grease minimizes the friction at stressing, and in combination with the sheathing provides long-term protection to the steel.

If a tendon is to be bonded, the strands are placed inside a duct that will be pressure injected with a cementitious grout after the strands are stressed. The grout bonds the prestressing steel to the surrounding concrete and also provides corrosion protection to the steel. The two tendon systems differ widely in design, construction and response to the applied load. However, both can be designed and constructed to meet the code, or design-stipulated serviceability and safety requirements.

Figures 2.4-4 shows the components of an unbonded post-tensioning system. The reference [Kelley, 2003] discusses the components of an unbonded tendon in detail.

Figure 2.4-5 is a schematic view of an unbonded tendon that extends through a construction joint. The figure shows a dead end anchorage, an anchorage for intermediate stressing, and the anchorage for stressing at the slab edge. The intermediate stressing can also take place with specialty couplers. The intermediate anchorage allows the tendon to be stressed at the construction joint after the concrete for the first slab segment has gained sufficient strength. Long tendons are often stressed at intermediate anchorages to avoid excessive prestress losses due to friction during stressing. Tendons longer than a specific length[10] must be stressed at both ends unless they are stressed at an intermediate anchorage. When tendons are stressed at both ends, each end will have a stressing-end anchor (also referred to as a live- or active-end anchor).

Figure 2.4-6 shows a slab being constructed with bonded tendons. The two box-like structures near the center of the picture, referred to as stressing pans, will create block-outs to provide access to the strands for stressing. The tendons terminating at these block-outs are partial-length tendons that do not extend all the way to the slab edge.

[10] See section 4.8.7

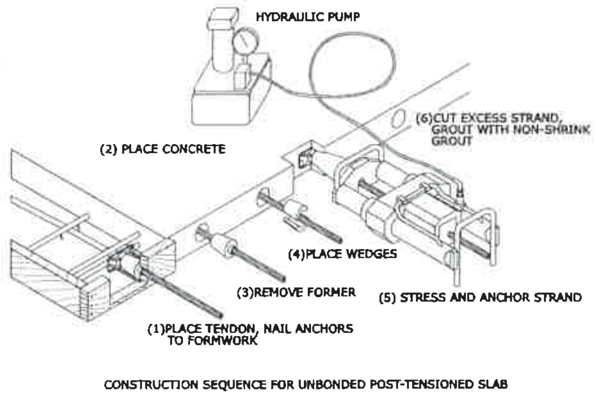


FIGURE 2.4-4 Components of an Unbonded Post-Tensioning System (P144)

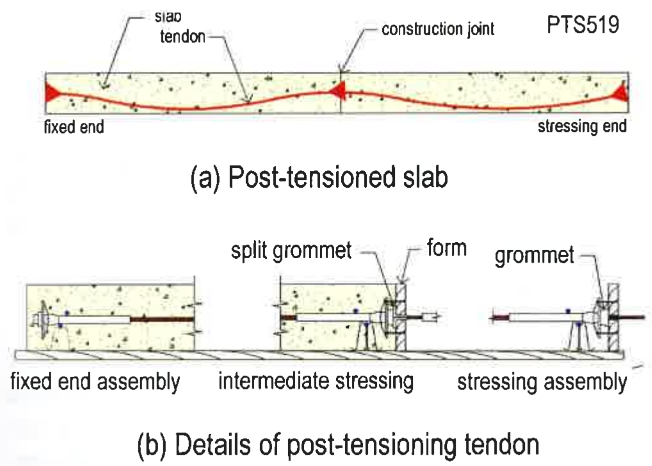


FIGURE 2.4-5 Assembly of Unbonded Mono-Strand Post-Tensioning System

FIGURE 2.4-6 Grouted System Construction in Progress (KSA; P473)

Figure 2.4-7 shows examples of typical anchorage devices used in slab system construction.

2.4.1 Prestressing Steel

Virtually all prestressing steel currently used in building construction is in the form of seven-wire strand (Fig. 2.4.1-1). Strand is available in a range of nominal diameters and several strength grades. The two most common strand diameters are 0.5 in. (12.7 mm) and 0.6 in. (15.2 mm). Half-inch (12.7 mm) strand is preferred by most installers for building construction; it is lighter and more flexible, and thus easier to place. It is also more economical for thin slabs (4 to 5 in; 100 to 130 mm slab thickness), where the design is governed by the minimum spac-

(a) Live end anchorage of a flat duct at slab edge (P146a)

(b) Dead end anchorage of a flat duct (P146b)

FIGURE 2.4-7 Stressing and Dead End Examples of a Flat Duct Grouted System ing of tendons stipulated in ACI 318.[11] Although 0.6 in. (15.2 mm) strand is used less frequently in the U.S., some installers consider it more efficient—because fewer strands are required the installation cost is lower. Strands smaller than 0.5 in (12.7 mm) are typically only used to repair unbonded tendons; instead of replacing the tendon, it is sometimes possible to extract the damaged strand from the sheathing and insert a small diameter strand.

The sizes and properties of common strands are listed in Table 2.4.1-1. Prestressing strand used in the US must conform to ASTM A 416/416M "Standard Specification for Uncoated Seven-Wire Strand for Prestressed Concrete." To qualify for use, the strands must pass specified static strength, elongation, and fatigue tests. ASTM A 416/416M allows strand with either 250 (1720 MPa) or 270 ksi (1860 MPa) Guaranteed Ultimate Strength; 250 ksi strand was common at one time but currently almost all strand used for post-tensioning is 270 ksi (1860 MPa). Higher strength strand is available but considerably more expensive than 270 ksi (1860 MPa) strand and thus not often used.

Over time, a stressed strand will "relax" slightly; this reduces the tension in the strand and thus the precompression imparted to the concrete surrounding a post-tensioning tendon. Prestressing strand is specially treated during its manufacturing process to create "low-relaxation" strand. This minimizes the relaxation that will take place while the strand is in service. The long-term loss of stress in a strand in place is about 4%.

The mechanical properties of the strand are shown on the strand's mill certificate. For design purposes,

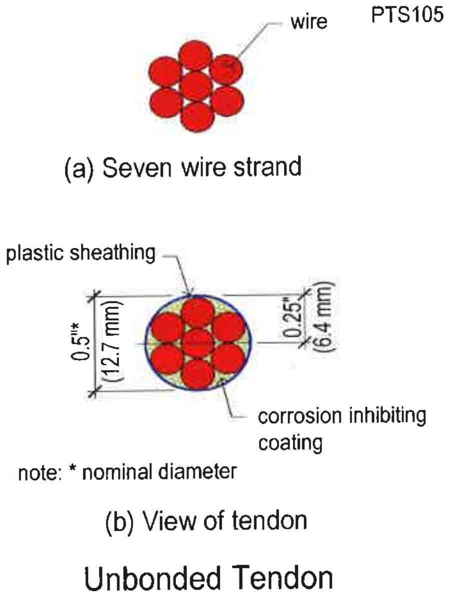


FIGURE 2.4.1-1 Section of a Seven Wire Strand and Tendon

the stress-strain relationship and modulus of elasticity recommended by the Canadian CPCI [CPCI, 1987] and shown in Fig. 2.4.1-2 are the values commonly used in the US.

2.4.2 Tendons

A. Tendon Sheathing and Ducts: Figure 2.4.2A-1 shows cross-sectional views of typical unbonded and grouted tendons. The sheathing used for an unbonded tendon is required to provide watertight encasement for the coating. The sheathing is typically extruded over the strand; this is the most economical way of producing a watertight encasement. Most sheathing is 0.050 in. (1.3 mm) thick high-density polyethylene (HDPE).

TABLE 2.4.1-1 Material Properties of Common Prestressing Strands (T110)

Seven wire strand, f_{pu} = 270 ksi (1860 MPa)		
Size designation US (SI)	0.5 (13)	0.6 (15)
Nominal diameter in (mm)	0.5 (12.7)	0.6 (15.24)
Nominal linear mass lb/ft (kg/m)	0.53 (0.775)	0.74 (1.109)
Nominal area A_{ps} in^2 (mm^2)	0.153 (98.7)	0.217 (140.0)
$0.7f_{pu}A_{ps}$ k (kN)	28.92 (128.5)	41.00 (182.3)
$0.8f_{pu}A_{ps}$ k (kN)	33.05 (146.9)	46.90 (208.3)
$f_{pu}A_{ps}$ k (kN)	41.31 (183.6)	58.6 (260.4)

[11] ACI 318-11 Section 18.12.4; no maximum spacing is stipulated in EC2

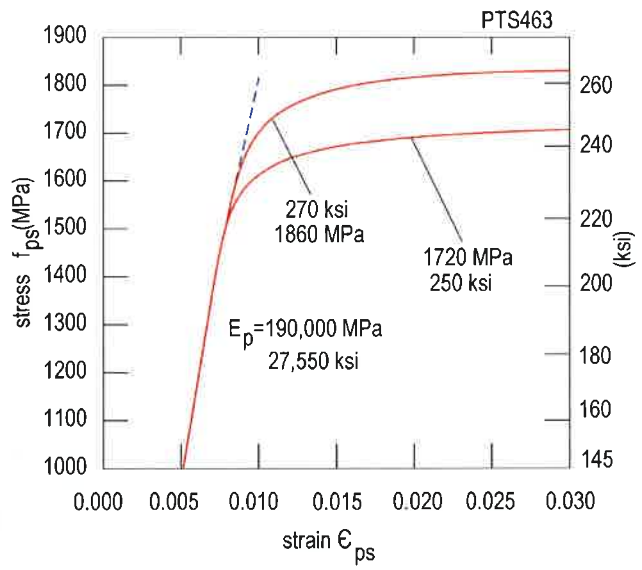


FIGURE 2.4.1-2 Typical Stress-Strain Curves for 7-Wire Prestressing Strands

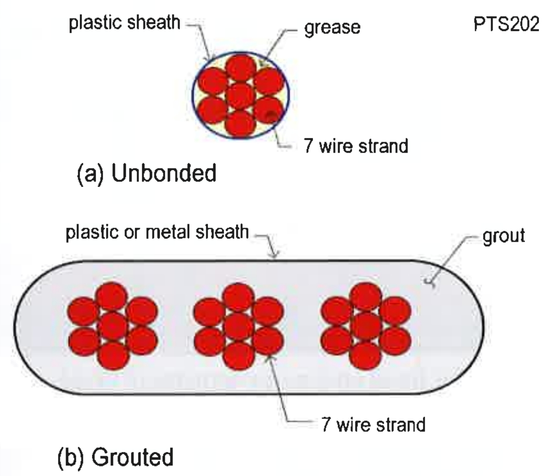


FIGURE 2.4.2A-1 Cross-Sectional Views of Typical Slab Tendons

The ducts for grouted tendons can be either metal or plastic. Metal ducts are typically made from 0.3 to 0.6 mm (0.01 to 0.02 in.) thick galvanized sheet steel, with larger size ducts requiring thicker walls.

The duct used for ordinary slab applications is typically flat and sized to hold between two and five strands, placed side by side as shown in part (b) of Fig. 2.4.2A-1. Flat steel ducts are generally 6 m (approx. 20 ft) long; the round steel ducts used in beams and applications such as transfer slabs are longer and more flexible (see Section 2.5.2).

FIGURE 2.4.2A-2 View of Plastic Ducts for Slab Construction (P150)

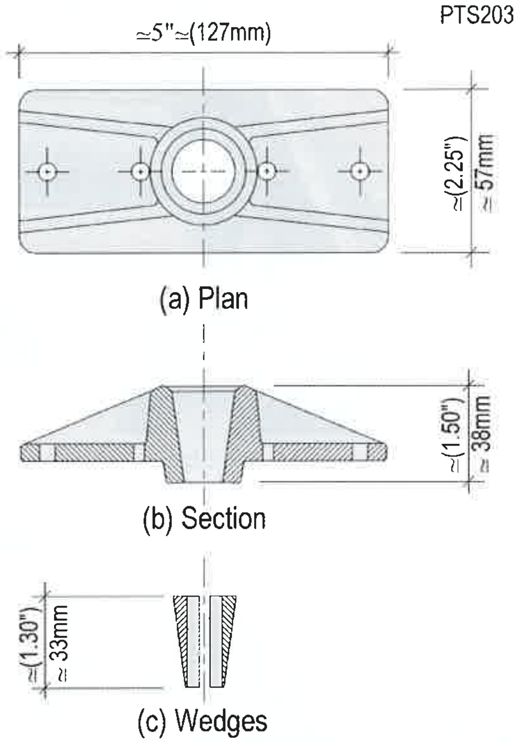


FIGURE 2.4.2B-1 Components of the Basic Unbonded Single Strand 0.5" (13 mm) Anchorage Device

Plastic ducts (Fig. 2.4.2A-2) are more recent. Plastic ducts are expected to replace metal ducts where durability is of prime concern.

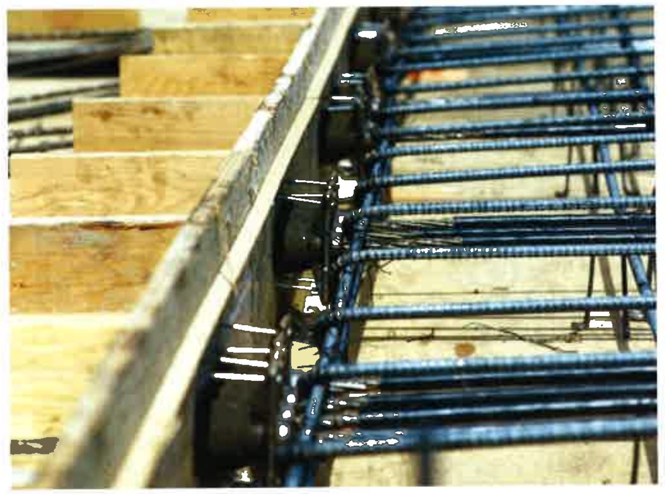
B. Anchorage Devices: At least one end of a tendon must exit at the edge or surface of the concrete to provide access for stressing. The other end of the tendon can either be a dead end (fixed end), or it is configured to be a second stressing end.

Figure 2.4.2B-1 shows the basic components of the anchorage device commonly used for unbonded systems—an anchorage plate and a set of wedges. Although the same anchorage assembly is used for both the dead end and the stressing end of the tendon, the dead end anchorage is typically attached to the strand in the shop, when the tendon is fabricated; the stressing-end anchorage is not attached until the tendon is stressed.

Anchors are typically sand castings made of ductile (cast) iron. Ductile iron is specified in terms of its minimum mechanical properties. Although it is brittle compared to steel, it is considerably stronger than other types of cast iron.

The standard wedge configuration is a two-piece truncated cone, machined with annular ridges ("teeth") on the inner surface. After machining, the wedges are case hardened (heat-treated) to create a hard surface layer. This allows the teeth to grip the strand effectively, but leaves the interior soft enough that the wedges can conform to the hole in the anchor casting.

Anchorage devices intended for use in aggressive environments are required to be protected against corrosion. The anchorage assembly is typically encapsulated in a plastic coating that is designed to provide watertight cover of prestressing steel. Connections between the components of the encapsulation system must remain watertight when subject to a hydrostatic pressure of 1.25 psi (9 kPa), which is approximately equal to a hydrostatic head of 3 ft.



(a) Live end, non-corrosive (P153a)

(b) Dead end, non-corrosive (P153b)

(c) Live end, corrosive (P153c)

(d) Dead end, corrosive (P153d)

FIGURE 2.4.2B-2 Examples of Anchorage Devices of Unbonded System in Non-Corrosive and Corrosive Environments

FIGURE 2.4.2B-3 Example of an Anchorage Device for a Flat Duct Bonded Tendon
(Courtesy PBL, Thailand P691)

(0.9m). The components are required to have a positive locking connection; systems relying on a friction connection between components are not allowed.

Figures 2.4.2B-2a,b show standard anchorage devices for a non-aggressive environment. Figures 2.4.2B-2c,d show encapsulated anchors for an aggressive environment. Structures are considered to be in an aggressive environment if they are exposed to deicing chemicals, seawater, or salt-laden air.

The anchorage plates for grouted slab tendons are typically made from the same material that is used for unbonded tendons—ductile (cast) iron—but they vary in shape and size according to the number of strands in the tendon. The wedges are the same as those used for unbonded tendons. Each strand in a grouted slab tendon is stressed and anchored individually.

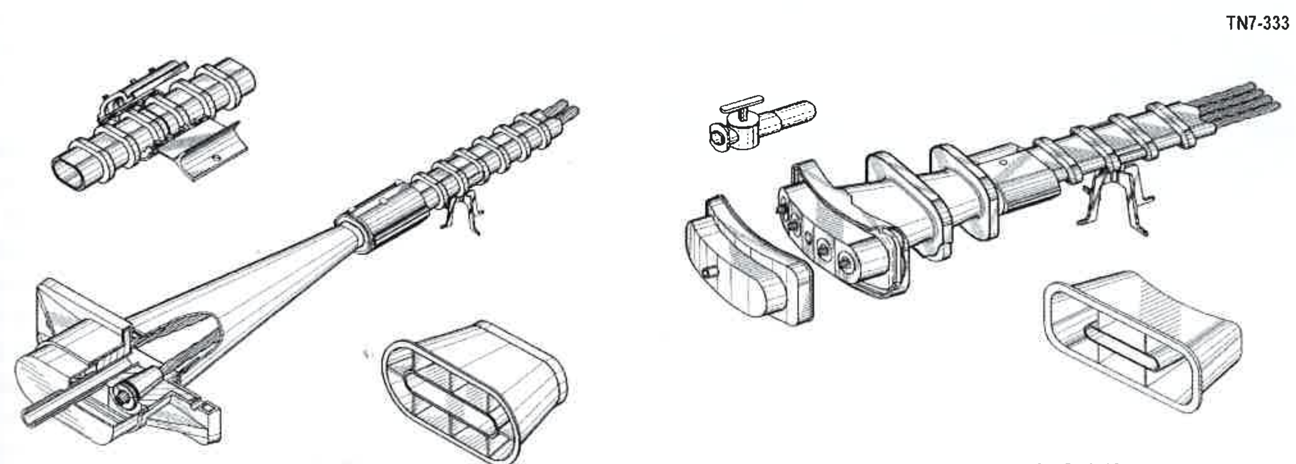
Figures 2.4.2B-3 and 4 show examples of typical anchorage devices for bonded tendons used in slabs.

C. Mono and Multi Strands: A mono-strand construction is one, in which each strand is individually stressed and seated. In multi-strand construction, the strands of an entire tendon are stressed at the same time and seated together. Multi-strand tendons are primarily used in bridge construction. Their application in building construction is typically limited to special conditions, such as transfer plates, transfer beams, and slab bands, where loads are much higher than is common for residential or commercial occupancies.

Figure 2.4.2C-1 compares mono-strand and multi-strand tendons; Fig. 2.4.2C-2 shows an example of multi-strand tendons being in building construction. The tendon shown in part (b) of Fig. 2.4.2A-1 is a mono-strand system, even though it houses more than one strand, since each of the strands is pulled and anchored individually.

2.4.3 Stressing Equipment

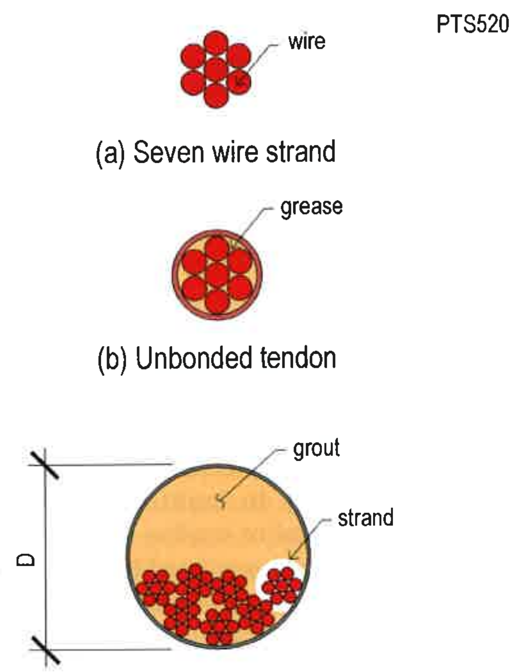
The basic components of the equipment used for stressing are:



(a) TWO-STRAND SYSTEM

(b) FOUR-STRAND SYSTEM

FIGURE 2.4.2B-4 Examples of Anchorage Devices for Bonded Tendons Used in Slab Construction
(GTI, USA;P493)



(a) Seven wire strand

(b) Unbonded tendon

(c) Multistrand grouted tendon

note: D = tendon diameter

FIGURE 2.4.2C-1 Example of a Multi- and Mono-Strand Tendon

FIGURE 2.4.2C-2 Example of Application of Multi-Strand Post-Tensioning in Building Construction (P674)

- ❖ a ram (also called a jack) to apply the jacking force;
- ❖ a hydraulic pressure pump;
- ❖ a pressure gauge;
- ❖ a calibration chart that converts the pressure reading on the gauge to the force applied to the tendon.

FIGURE 2.4.3A-1 Equipment for Mono-Strand Stressing (P158)

A. Mono-Strand Stressing Equipment: Figure 2.4.3-A1 shows a mono-strand stressing jack. Mono-strand jacks are light and can be handled by one person. The stressing operation is automatic; once the jack has engaged the strand; the operator can move a safe distance away while the tendon is stressed and the wedges are seated.

Most jacks have hydraulic power-seating capability; when the pump is switched to "retract," the seating plunger in the nosepiece of the jack extends forward until a preset pressure is reached. This seats the wedges and reduces the amount they are pulled in when the jack is disengaged.

The elongation of the strand is measured after the stressing equipment is disengaged. This provides a protection measure against rupture of concrete and dislocation of the jack during the stressing operation.

Another item to note is that the mono-strand jacks come in two configurations. The sample shown in Fig. 2.4.3A-2 rides on the strand in the case of intermediate stressing, or rides on the strand extension for end stressing. In the alternative, the strand is threaded through the jack. The former offers the advantage that it can be used to stress a strand at any point along the tendon's length, thus enabling "intermediate" stressing.

B. Multi-Strand Stressing Equipment: In multi-strand construction, all of the strands in a tendon are stressed at the same time. Jacks with different

(a) Jack rides over strand (P159a) (b) Strand threads through jack (P159b)

FIGURE 2.4.3A-2 Jacks for Mono-Strand Stressing

stressing capacities are required, depending on the number of strands in the tendon. Most multi-strand jacks are relatively heavy and must be held in position by a crane during stressing (Fig. 2.4.3B-1).

2.4.4 Grouting Equipment

When grouted tendons are used, grouting equipment is required. The grouting equipment consists of the:

- ❖ grout mixer; and
- ❖ grout pump.

Grouting equipment come in different capacities, depending on the volume of grout that is required. Figure 2.4.4-1 shows a grouting machine.

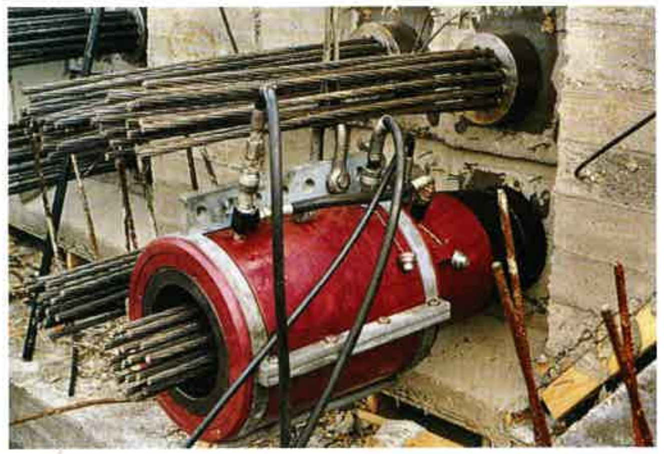


FIGURE 2.4.3B-1 Example of a Multi-Strand Stressing Jack Held in Position by a Crane (P160)

2.5 POST-TENSIONING CONSTRUCTION

Construction practices for post-tensioned buildings vary greatly among different contractors and different parts of the world. In what follows, the emphasis is placed on practices that are fairly common. For construction with unbonded tendons, the focus is the practice in the US. Construction practices in the US have matured over more than half a century, with a track record of thousands of post-tensioned buildings in satisfactory service. For grouted tendons, the bulk of the information relates to the practice in the Middle East and Asia, where in recent years a boom in building construction using grouted tendons has taken place.

2.5.1 Construction with Unbonded Tendons

Figure 2.5.1-1 shows the reinforcement layout of a

FIGURE 2.4.4-1 View of Grouting Equipment (P162)

floor slab constructed with unbonded tendons. As illustrated in the figure, the common practice is to orient the tendons in orthogonal directions. In one direction, the tendons are banded together and placed along the column lines. In the other direction, the tendons are distributed parallel to one another and at an equal spacing, as much as practical.

The reinforcement consists of prestressing tendons and nonprestressed reinforcement (rebar); there are also support bars that are used to position the tendons at the required profile. The sequence of construction is detailed next.

A. Fabrication and Delivery of Tendons: On most projects, the tendons are prepared and placed using installation (shop) drawings that are prepared by the post-tensioning supplier. The greased and plastic-sheathed single strand tendons are cut to length in the shop and fitted with a fixed-end anchor unless they will be stressed at both ends. Each tendon is

FIGURE 2.5.1-1 View of a Floor System Reinforced with Unbonded Tendons (P408)

identified with a color code painted on the anchorage or sheathing (Fig. 2.5.1A-3). The color codes are referenced on the installation drawings to show where the tendon should be placed. The tendons are then bundled and delivered to the job site (Fig. 2.5.1A-1). On large projects, a coil of greased and sheathed strand may also be provided; the strand can be cut to the length required if there have been changes to the construction or stressing locations such that a fabricated tendon is not long enough (Fig. 2.5.1A-2).

Tendons that will be stressed at construction joints as well as the slab edge are typically delivered with

FIGURE 2.5.1A-1 Tendons are Cut to Size; Provided with Dead End Anchor; Grouped; Tagged, and Delivered to Job Site (P402)

the intermediate anchorage device already in place (Fig. 2.5.1A-4). Intermediate stressing is used in the U.S. for long tendons, to avoid excessive friction losses. This is explained further in Chapter 4, Section 4.8.

B. Placement of Non-prestressed Reinforcement: Non-prestressed reinforcement whose placement will not interfere with the installation of the tendons is placed first. The objective is to avoid having to thread tendons through rebar. Work starts with placing the column drop reinforcement, where required (Fig. 2.5.1B-1). In addition one heavy bar (#7; 22 mm) is placed on either side of the drop, or column support, where there is no column drop. These

FIGURE 2.5.1A-2 Strand Delivered to Site in Coil (P403)

FIGURE 2.5.1A-3 Shop Fabricated Tendons are Color-Coded and Tagged (P404)

bars provide support for the banded tendons (part b of the figure), and the top reinforcement over the support.

Figure 2.5.1B-2 shows the installation of the first group of bars—the top bars over the supports. The top bars in the direction of the distributed tendons (orthogonal to the banded tendons) are placed first (parts a and b). They are thus the bottom layer of top bars at the support. The bars in the orthogonal direction will be placed last.

The short bars along the column line in part (a) will be raised on chairs to create the tendon profile required for the banded tendons over the column line.

Punching shear reinforcement comes in different forms, including stirrups, shear bands, and stud rails

FIGURE 2.5.1A-4 Anchors for Intermediate Stressing are installed in the Shop (P405)

(see Chapter 4, Section 11). If stud rails are used, they must be placed prior to other reinforcement (Fig. 2.5.1B-3) Stirrups and shear bands do not necessarily need to be placed before other reinforcement.

Next, the support bars for the distributed tendons are placed (Fig. 2.5.1B-4). Unlike the short bars used for the tendons in the banded direction, the support bars for the distributed tendons are generally continuous. The #4 (12 mm) bars are placed at 3 to 4 ft (900–1200 mm) intervals and are supported by chairs that raise the tendons to the heights shown on the installation drawings.

At midspan, a "slab bolster" is used instead of support bars. A slab bolster is a plastic strip with support legs that provides the minimum 0.75 in. (20 mm) cover to the tendon required at the low point.

(a) Column drop reinforcement (P406)

(b) Heavy bars on two sides of a support (P407)

FIGURE 2.5.1B-1 Installation of Reinforcement in Column Drops

(a) Top bars over support with column drop (P409)

(b) Top bars over supports without drops (P410)

FIGURE 2.5.1B-2 Bottom Layer of Top Reinforcement over Supports is Placed in Direction of Distributed Tendons

(a) Stud rails reinforcement (P411)

(b) Installation of shear reinforcement (P412)

FIGURE 2.5.1B-3 Delivery and Installation of Punching Shear Reinforcement

For ease of installation, the tendon low point is typically set at midspan, even if the loading is such that another location would optimize the design.

If bottom bars (positive moment reinforcement) are required, they are placed next. Bottom bars in the distributed tendon direction should be distributed uniformly and staggered by 12" (300 mm). (Fig. 2.5.1B-5)

At interior spans, each third bar is extended to the supports. This addresses a provision of ACI 318-11[12] that requires a percentage of bars to be extended to the supports, if they are necessary for the strength requirements of the code. Otherwise, bars do not need to extend to the supports. However, where bottom bars are required, for convenience of construction, the bar extension to the support is practiced by most engineers, irrespective whether this is necessitated for strength requirements.

FIGURE 2.5.1B-4 Support System for Distributed Tendons (P413)

12 ACI 318-11 , Section 18.9.4.3

(a) Bottom bars at interior spans (P414)

(b) Bottom bars at exterior spans (P415)

FIGURE 2.5.1B-5 Positioning of Bottom Bars. Every Fourth Bar Extends to Support at Interior Spans and Every Third at Exterior Spans

FIGURE 2.5.1C-1 Stressing Ends at the Slab Edges (P416)

FIGURE 2.5.1C-2 Tendons are Unrolled and Placed (P417)

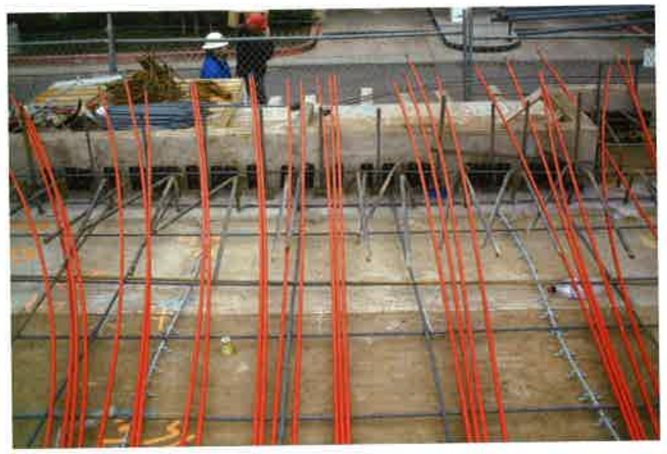
Bottom bars in the banded direction can either be distributed uniformly or grouped along the column lines below the banded tendons.

C. Tendon Installation: The bulkheads at the slab edges are marked to show where the anchorages should be placed and holes are drilled for the tendon tails. The stressing anchors and pocket formers that will create the stressing pockets are then positioned and secured to the bulkhead (Fig. 2.5.1C-1).

The tendon bundles are brought to position, unrolled and placed according to the installation drawings. The dead-end anchors are placed about 2" (50mm) in from the bulkhead and the stressing-ends of the tendons are placed over the opposite bulkhead and cut to length, if necessary. Next the sheathing is re-

FIGURE 2.5.1C-3 Dead Ends are Positioned 2" (50 mm) from the Bulkhead (P418)

(a) Live ends are secured to the bulkhead and provided with anti-bursting reinforcement (P419)



(b) Tendons are lined up with the anchorage devices, ready to be cut and threaded through (P421)

FIGURE 2.5.1C-4

FIGURE 2.5.1C-5 Tendons are secured in Position and Height to Support Bars (P420)

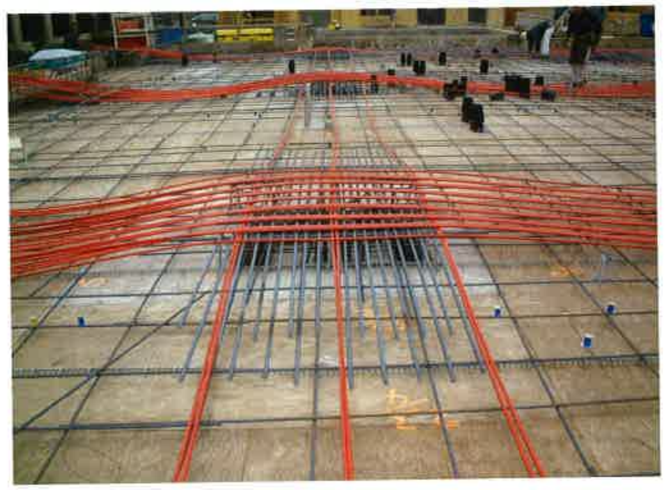


FIGURE 2.5.1C-6 Positioning of Grouped Tendons over the Supports (P422)

(a) Minimum of two tendons over support (P423)

(b) Distributed tendons placed parallel (P424)

FIGURE 2.5.1C-7 Distributed Tendons are Placed, with Minimum of Two Strands over Each Support.

moved from the stressing end of each tendon, and the strand is threaded through the stressing-end anchorage device. Figures 2.5.1C-4 through 7 illustrate the remainder of operations in tendon installation.

D. Inserts and Conduits: Conduit and duct for the electrical, data and inserts for plumbing, and other fixtures will need to be accommodated within the reinforcement layout. Tendons and nonprestressed reinforcement are moved to clear the position of inserts, where necessary. Figure 2.5.1D-1 is an example of inserts, and Fig. 2.5.1D-2 shows electrical and data conduits. In the US it is not common to provide large and oversized openings in a slab for each utility, such as bathroom plumbing. Openings are generally created to the size required. Occasionally, conduits for electrical work or plumbing become excessive (Fig. 2.5.1D-3).

E. Inspection Prior to Placing Concrete: Once everything is in place, and the floor is ready for the concrete, members of the construction team will walk the project to make sure that their work is properly positioned and secured (Fig. 2.5.1E-1).

Depending on the location of the project, additional inspections may be required before a permit to place concrete will be issued. As an example, the following reflects the practice in City of Palo Alto (California):

(i) The structural engineer responsible for the design of the slab must walk the slab, "observe" the layout and details of the reinforcement, and issue a letter stating that based on his or her opinion, the layout meets the general concept of design, recognizing that there can be deviations from the design drawings due to specific site conditions

FIGURE 2.5.1D-1 Inserts for Plumbing and Other Utilities (P525)

FIGURE 2.5.1D-2 Electrical and Data Conduits in Slab (P426)

FIGURE 2.5.1D-3 Crowded Placement of Electrical and Data Conduits in a Thin Slab (P427)

FIGURE 2.5.1E-1 Inspection by Trades Involved Prior to Placing of Concrete (P428)

(ii) A trained and certified technician from an independent "Inspection Agency," hired and paid by the owner, must inspect the reinforcement layout in detail, compare it to the structural documents, report each deviation to the structural engineer and have the deviation either corrected by the contractor, or obtain the structural engineer's written approval of the variance; and issue a letter of passing the inspection.

(iii) Once these two letters are issued, the City Inspector will walk the project, review the letters and note any corrections he or she deems necessary. If no corrections need to be made, the City Inspector will sign off on the permit to place concrete.

F. Final Cleanup: Using a strong magnet, the form is swept to collect loose nail, wires, and other pieces of metal (Fig. 2.5.1F-1). High-pressure air is then used to blow off any dust or debris that has gathered on the forms and around the reinforcement (Fig. 2.5.1F-2). In hot weather, the form may be dampened with a light spray of water before the concrete is placed.

G. Placing of Concrete: Concrete is pumped or delivered by bucket or buggy, consolidated, and finished to the required tolerance, ready to receive the floor covering (Fig. 2.5.1G-1).

H. Exposing and Preparing Stressing Pockets: Cylinders taken from the delivered concrete are tested to ensure that the concrete has gained the strength required for stressing. The standard post-tensioning anchor is sized such that tendons can be fully stressed when the concrete reaches strength of 2000 psi (13.5 MPa). However, as a precaution, most engineers require a minimum cylinder strength of 3000 psi (20 MPa) for stressing.

Typically, stressing is done two to three days after the concrete is placed. Ideally, the concrete will be

FIGURE 2.5.1F-1 Form is Swept with a Strong Magnet to Collect Loose Nails and Wire (P429)

FIGURE 2.5.1F-2 Air Pressure is used to Blow out Loose Debris (P430)

(a) Concrete is pumped and consolidated (P431)

(b) Concrete is finished with a flat surface (P432)

(c) Rotary trawler is used for a smooth surface (P433)

FIGURE 2.5.1G-1 Placing and Finishing of Concrete

(a) Form is removed from slab edge (P434)

(b) Excess concrete is chiseled off (P435)

(c) Pocket formers are removed (P436)

FIGURE 2.5.1H-1 Preparation of Tendon Tail for Stressing

(a) Wedges are used to anchor strands (P437)

(b) Wedges are inserted in anchorage casting (P438)

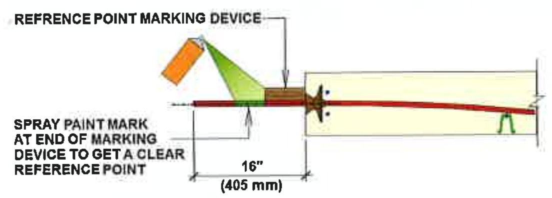
(c) Implement used to hammer the wedges into position (P439)

FIGURE 2.5.1I-1 Insertion of Wedges for Tendon Anchorage

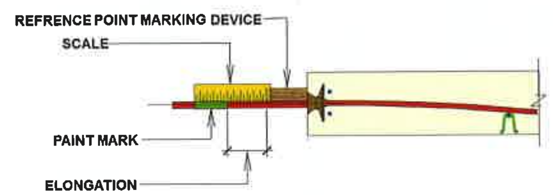
cast on a Friday so that it will gain enough strength to allow stressing on the following Monday.

The bulkheads are removed to expose the edge of the concrete, excess concrete around the stressing pockets is chipped off, and the pocket formers are removed (Fig. 2.5.1H-1) to expose the strand tails for marking and stressing.

I. Preparation for Tendon Stressing: Taking a pair of wedges from a supply box (Fig. 2.5.1I-1a), a technician positions the wedges inside the cavity of the anchorage casting (Fig. 2.5.1I-1b). Using an implement that fits over the strand (Fig. 2.5.1I-1c), the technician hammers the wedges into the cavity, making sure that they are pushed to the same depth.
The stressing operation involves (i) gripping the exposed tendon tail, (ii) applying a pre-determined tensile force to the strand; (iii) anchoring the strand; and (iv) measuring the elongation of the strand. If the elongation correlates correctly with the jack-



(a) ELONGATION MEASURMENT INITIAL MARKING



(b) MEASURING TENDON ELONGATION

FIGURE 2.5.1I-2 A Datum Mark is Sprayed on the Tail End of a Strand. The Mark is Used for Measurement of Elongation. (P496)

(a) Removal of grease from tendon end (P440)

(b) Spraying a datum mark on tendon end (P441)

(c) View of datum marks on tendon tails (P442)

FIGURE 2.5.1I-3 Preparation of Tendon Tail for Stressing

(a) Stressing equipment and crew (P443)

(b) Elongation measurement by inspector (P444)

FIGURE 2.5.1J-1 Stressing Equipment, Crew, and Elongation Measurement

ing force, the stressing is considered satisfactory. To measure the elongation, the strand tail is marked at a fixed distance from the face of the member. As indicated in Figs. 2.5.1I-2 and 3, typically a wood block combined with spray paint is used to make this reference line. The wood block reduces error due the local unevenness on the slab edge.

The strand tail is wiped clean of excess grease before it is marked so that the mark can be made clearly; this also minimizes the likelihood of slippage during stressing (Fig. 2.5.1I-3).

J. Stressing Operation: Strands are stressed to 80% of their guaranteed ultimate strength (0.8fpu), regardless of whether or not this results in stresses along a tendon that may not conform to the limits set in the code. Prior to 2011, f ACI 318 specified values for maximum allowable stress in strand at anchorage, where tendon is seated; and the maximum allowable stress anywhere along the length of a strand. As part of the installation documents, the stresses along each tendon are calculated and reported

In the U.S., tendons are generally only stressed once, to the maximum specified force, particularly in building construction.[13] However in many other countries, it is common to stress the tendons to 20% of the specified value the day after the concrete is cast, followed by the balance of the force at a later date.

The stressing crew commonly consists of three individuals (Fig. 2.5.1J-1). One technician operates the jack while a second technician operates the pump and observes the gauge to monitor the applied force. The

[13] Stage stressing may be required in transfer plates and beams, due to absence of significant design load at stressing

third individual is an independent inspector who is charged with observing the stressing operation, measuring and recording the elongations, and noting any unusual events that occur during stressing.

The inspector enters the measured elongations of each tendon on a form and notes whether any measured elongation deviates from its calculated value by more than a specified percentage. The specified percentage for U.S. projects is generally 7% but is up to 10% in several other countries. Figure 2.5.1J-2 illustrates the measurement of elongation for a strand that exits the slab edge at an angle. The elongation should be measured to an accuracy of 1/8" (3 mm).

The wedges may crack during stressing (Fig. 2.5.1J-3). This is acceptable, if the misalignment between the two wedge halves does not exceed 1/8" (3 mm).

FIGURE 2.5.1J-3 Cracking of Wedges at Stressing (P446)

FIGURE 2.5.1J-2 Measurement of Elongation of a Strand that Exits the slab at an Angle (P445)

K. Approval of Stressing Records and Finishing of Tendons: The measured elongations are sent to the engineer of record for his or her review and approval. If the deviation between the measured and calculated elongation for a particular tendon does not exceed the specified tolerance, the stressing for that tendon is considered to be satisfactory. If measured elongations deviate more than the specified tolerance, the engineer must uses his or her judgment in deciding whether to accept the stressing. The decision is based on the percentage of measurements that deviate from the calculated elongations, and the importance of the affected tendons with respect to their location in the structure. The elongations for tendons shorter than about 30 ft (10 m) tend to be out of tolerance more than the elongations for longer tendons; because the elongation itself is fairly small, the tolerance for acceptance is extremely small.

If the record in its entirety does not appear satisfactory, the engineer of record will return the report to the post-tensioning installer for explanation and/or correction. ACI 318[14] allows 2% of tendons to be lost in stressing before requiring an investigation regarding the cause and possible remedy.[15] Engineers typically equate the "2% total loss of force" of the code to under-elongation of a number of strands in each "design member," since it has the equivalent impact of force loss on the member. The literal interpretation of the wording of the code is not practical for real structures; engineering judgment is used to interpret the wording.

L. Stripping of Forms and Re-Shoring: After all of the tendons are stressed, the forms are stripped and the slab is re-shored to allow work to start on the next floor. Re-shoring generally involves placing supports at each one-third or one-fourth point along each span (Figs. 2.5.1L-1 and 2); given the time lag involved in getting the stressing records approved, this is generally done before the tendon tails are cut. Re-shoring is only necessary if the slab will need to support the weight of construction above it. Otherwise re-shoring is not necessary for a post-tensioned floor system once the tendons have been stressed—unlike conventionally reinforced concrete.

[14] ACI 318-11, Section 18.20.1

[15] ACI-318-11, Section 18.20.4

FIGURE 2.5.1L-1 Re-shoring of Flat Slab Construction (P447)

FIGURE 2.5.1L-2 Re-shoring of Beam and Slab Construction (P448)

M. Cutting the Strand Tails and Finishing off Tendons: Once the stressing records are approved by the engineer of record, the tendon tails can be cut and the stressing pockets can be filled. For protection against corrosion and fire, strand tails are cut to provide a minimum of 1" (25 mm) cover from the face of the concrete. There are four methods of cutting strand tails: (i) flame cutting; (ii) rotary abrasive saw; (iii) hydraulic shears; and (iv) plasma cutting.

Flame cutting with oxyacetylene gas is the most commonly used method. The flame is directed at the strand tail about ¾" (20 mm) behind the wedges (Fig. 2.5.1M-1). The heat generated has not been observed to impair the performance of the wedges and anchorage casting. However, as a precaution against setting fire to the formwork, flame cutting is not allowed in certain parts.

Rotary abrasive saws typically cannot be used for monostrand construction because they cut the strand tail flush with the face of the concrete as shown in Fig. 2.5.1M-2. This does not provide the 1 in. (25 mm) minimum clear distance between the strand end and the face of the concrete required for fire and corrosion protection.

Hydraulic shears such as the pocket shear (Fig.2.5.1M-3) have a cutting nose that fits into the stressing pocket; the blades in the cutting nose shear the tendon tail just behind the wedges. The process is fast and provides a clean cut of the strand tail. However, due to the cost of the equipment, hydraulic shears are typically only used if flame cutting is prohibited.

(a) Flame to cut strand tail (P449) (b) Flame-cut fused strand end (P450)

FIGURE 2.5.1M-1 Cutting of Strand Tail, Using a Flame

FIGURE 2.5.1M-2 Cutting of Strand Tail, Using Abrasive Saw (P451)

Plasma cutting through electric current is another option for cutting strand tails. If plasma cutting will be used, oversized pocket formers are required—the stressing pockets need be larger than is typical to allow the equipment's grips to extend into the stressing pocket (Fig. 2.5.1M-4).

N. Securing Stressing Pockets: Excess material from the stressing pocket is removed (Fig. 2.51N-1a) and the strand tail is protected with an anti-corrosive spray (Fig.2.51N-1b). If the tendon is an encapsulated system for aggressive environments, a cap filled with protective grease is fitted over the end of the strand (Figs. 2.51N-1c and d). Depending on the hardware used, there are several options for capping the anchorage device for added protection of the wedges against corrosion.

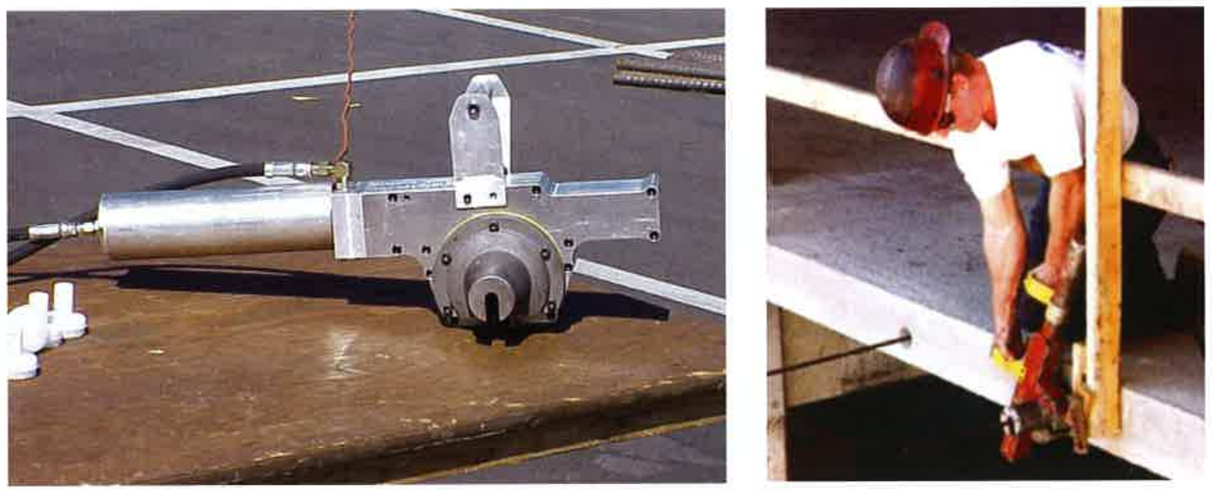
The stressing pocket is filled with a stiff non-shrink grout and finished as illustrated in Fig. 2.5.1N-2.

There is more in construction of unbonded post-tensioning than detailed in the description covered herein. Other items, such as care in tendon layout to follow the design-specified profile and avoiding sharp changes in curvatures; proper positing of the stressing anchors; and cleaning of the anchor cavities to make sure the wedges will seat correctly are among the critical steps.

2.5.2 Construction with Grouted Tendons

Many of the steps in construction with grouted tendons, such as stressing and recording elongations, are similar to those described in Section 2.5.1 for unbonded tendons. Therefore, the focus of the following will be the steps that are unique to grouted tendons.

Figures 2.5.2-1 through -4 show slabs that are reinforced with grouted tendons. The first is a project in the Middle East. Apart from the grouted tendons, two things are notable. First, the tendon spacing is essentially the same in both directions. When using metal ducts, this practice is encouraged, since flat metal ducts are not as flexible as unbonded tendons. Flat metal ducts cannot readily swerve to follow columns that are not aligned, or avoid openings or other obstructions. The maximum change of angle on plan



(a) View of pocket shear (P452) (b) Pocket shear in operation (P453)

FIGURE 2.5.1M-3 Cutting of Strand Tail, Using Pocket Shear

(a) Plasma cutter (P454)

(b) Cutting strand tail using plasma cutter (P455)

FIGURE 2.5.1M-4 Cutting of Strand Tail, Using Plasma Cutter

(a) Excess material is removed from recess block (P456)

(b) A corrosion inhibiting spray, such as Zinc-based galvanizer is used for added protection (P457)

(c) A grease cap is used to cover the exposed metal (P458)

(d) The cap is hammered (screwed) in position (P459)

FIGURE 2.5.1N-1 Preparing Recess Block to be Sealed Off

(a) View of capped stressing pocket (P260)

(b) Water is splashed into the recess block using a brush (P261)

(c) Shrinkage compensating, stiff grout is pounded onto the recess block (P262)

(d) The recess is finished off with a smooth surface in line with concrete slab (P263)

FIGURE 2.5.1N-2 Finishing and Sealing of Stressing Pockets

is typically limited to 10o, in order to avoid buckling or collapse of the duct. The other item of note in the first two figures is the bottom mesh. Often, the mesh is not required by design, but it is nevertheless customary to use it in many Middle East countries. In most cases, in particular at the upper levels of multi-story buildings, the benefits of mesh reinforcement do not justify its cost, and the environmental impact on excessive use of material. It is also noteworthy that bottom mesh increases the downward deflection due to shrinkage

Not all tendon layouts using grouted tendons with flat metal ducts follow the bi-directional equal spacing shown in Fig. 2.5.2-1. However, it more common than the banded/distributed layout shown in Fig. 2.5.2-2.

Figure 2.5.2-3 shows a similar project in the US, where it is more common to use plastic ducts for grouted tendons. In addition, bottom mesh reinforcement is seldom used in the U.S. The figure shows the slab ready for concrete.

The typical construction sequence with grouted tendons is as follows:

A. Material Delivery and Tendon Preparation
B. Installation of Non-Prestressed Reinforcement
C. Tendon Installation
D. Inspection Prior to Placing of Concrete
E. Placing and Finishing of Concrete
F. Preparation for Tendon Stressing
G. Partial Tendon Stressing
H. Full Stressing and Validation of Stressing Records
I. Stripping of Forms and Re-Shoring

FIGURE 2.5.2-1 Slab Reinforced with Grouted Tendon Using Metal Ducts (P164)

FIGURE 2.5.2-3 Slab Reinforced with Grouted Tendons Using Plastic Ducts (Courtesy VSL; P700)

FIGURE 2.5.2-2 Grouped/Distributed Tendon Layout (P165a)

FIGURE 2.5.2-4 Tendon Layout of a Heavily Loaded Slab Using Grouted Tendons
The close spacing of the tendons in the distributed direction reflects the heavy design load of slab (Courtesy Freyssinet Gulf; P693)

J. Cutting of Tendon Tails and Finishing of Stressing Pockets
K. Grouting

A. Material Delivery and Tendon Preparation: Practices vary with respect to tendon fabrication. Some contractors prefer to have the strand cut to length and the dead ends formed or attached in the shop. The strand is then delivered to the job site, bundled according to where it will be placed. Alternatively, the strand can be delivered as a coil that is cut to length on site. The practice also depends on whether rigid sheet metal or somewhat flexible corrugated ducts are used

Figures 2.5.2A-1 and 2 show duct and coils of strand are delivered to the job site. The strands can be pushed through the ducts before the tendons are installed (Fig. 2.5.2A-3). However, the more common practice with flat ducts is to place the ducts in position before pushing the strand through them (Figs. 2.5.2A-4). In particular, when using corrugated ducts, it is easier to place the duct first and then push the strand through. (Figs. 2.5.2A-5 and 6).

The strand at the fixed end of the tendon will extend out of the duct; the bond between the exposed strand and the concrete provides some anchorage for stressing but the strand must also be mechanically anchored. There are several ways to provide mechanical anchorage. The most common is the onion shaped configuration, shown in Fig. 2.5.2A-9 . Figure 2.5.2A-7 shows the device that is used to create the onion shaped anchorage. Other types of fixed-end anchorages are shown in Fig. 2.5.2A-8.

FIGURE 2.5.2A-1 Bundle of Flat Ducts Delivered to Site (P168)

FIGURE 2.5.2A-4 Strand is Pushed into an Installed Duct (P171)

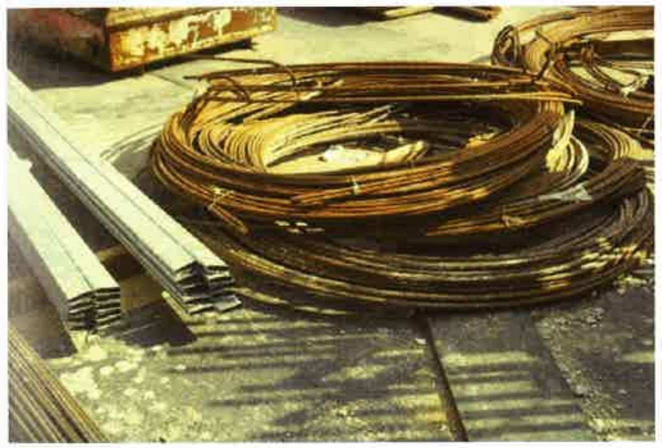


FIGURE 2.5.2A-2 Bare Strands and Ducts Are Delivered to the Site for Tendon Preparation (P169)

2.5.2A-5 Bundle of corrugated Ducts Ready for Installation (P172a)

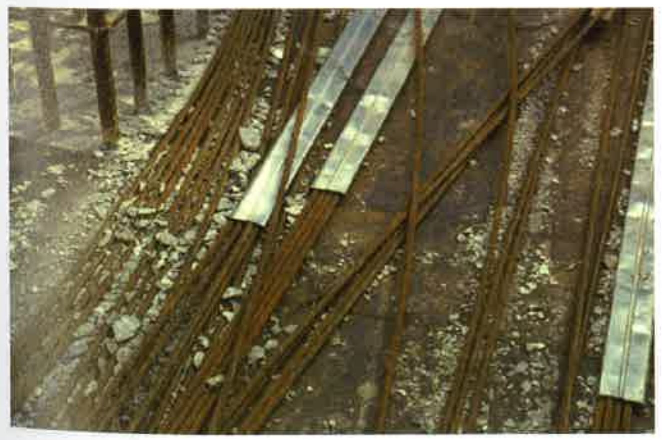


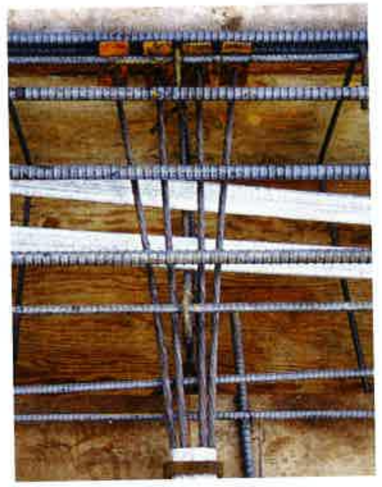
FIGURE 2.5.2A-3 Strands are pushed through the Ducts, Preparing the Tendon for Installation (P170)

Figure 2.5.2A-6a Empty Flexible Ducts are Placed in Position (P172b)

FIGURE 2.5.2A-6b Strands are Pushed into Positioned Ducts (P173)

Figure 2.5.2A-7 Device Used to Form Onion-Shaped Dead Ends (P511)

(a) Dead end using metal strip (P175)



(b) Single anchorage castings used as anchorage device (P513)

FIGURE 2.5.2A-8 Examples of Alternative Dead Ends

B. Installation of Nonprestressed Reinforcement: Unlike the US, where the non-prestressed and prestressing reinforcement are typically placed by the same crew, allowing coordination between the work, in many other countries the two are placed by different trades. The non-prestressed reinforcement is placed first as much as practical. Often the work starts with placing a bottom mesh, even though it is generally not required by design, when using ACI 318/IBC. The mesh may be as light as 6 mm bars at 150 mm spacing (W4.5 @ 6" o.c.) but the common option is 12 mm bars spaced 250 mm on center (#4@12" o.c.).

In the US, it is neither required by the building codes, nor is it common practice to place a bottom mesh in post-tensioned slabs, whether the tendons are bonded or unbonded. One reason a mesh is not necessary is the low limit imposed by ACI 318 for the hypothetical extreme fiber tensile stresses, coupled with the requirement of a minimum precompression.

In the European Code, EC2,[16] there is neither an absolute limit on allowable tensile stresses, nor a requirement for minimum pre-compression (see Chapter 4, Section 4.10.3 for details). European Code designs are based on probable crack widths, for which a mesh is beneficial

C. Tendon Installation: Once most of the nonprestressed reinforcement is in place, the tendons are threaded through. Often, due to congestion of non-prestressed reinforcement the threading of tendon becomes an arduous task. Figure 2.5.2C-1 shows a slab with non-prestressed reinforcement in place, where the post-tensioning crew is starting to install the tendons.

[16] EC2 EN 1992-1-1:2004

FIGURE 2.5.2A-9 Most Common Dead End Configuration (P174a)

FIGURE 2.5.2C-1 Tendon are Threaded through Installed Non-prestressed Reinforcement
(Dar Al-Handasah Beirut; P178)

Where reinforcement congestion at the slab edge makes it difficult to place the anchorages, the tendons can be stressed from the surface of the slab. Figures 2.5.2C-2 and 3 show stressing pans used to provide access to the tendons from the top of slab. The pans must be positioned so that there is at least 900 mm (3 ft) clearance behind the anchorages for the stressing jack.

There must be bursting reinforcement behind both the stressing-end anchor and the fixed-end anchor (Fig. 2.5.2C-2 and 4).

Duct splices should be provided with a sleeve and taped to prevent concrete paste entering the duct (Fig. 2.5.2C-5).

Live end anchorage devices were installed first and secured in position. The stressing recess at the face of concrete is often created by Styrofoam shaped to fit the anchorage face (Fig. 2.5.2C-6). The joint between the Styrofoam and the anchorage casting must be taped to prevent cement paste from getting into the casting.

Individual chairs are used to raise the tendons to their design profile. Once tendons are placed according to the installation plans, a technician writes the required chair height along the length of the ducts. Chairs are typically placed at 1m (40") spacing along the tendon (Fig. 2.5.2C-7 and 8). The legs of the chairs are pulled apart to provide the right chair height. Once adjusted, the chairs' feet are nailed to the form.

The tendon ducts are pierced at the high points and provided with vent tubes to let the air escape as the duct is injected with grout. The opening around the tube is taped to prevent concrete paste from getting into the duct and causing problems during stressing. Vents and grout inlet tubes are provided at the stressing and dead ends. (Figs. 2.5.2C9 and 10).

D. Inspection Prior to Placing of Concrete: In most jurisdictions, inspections of both the tendons and non-prestressed reinforcement before concrete is placed are mandatory. However, based on author's experience, the practice is not always as detailed and rigorous as it is in California. Often, the construction documents vest the responsibility for quality control in the post-tensioning supplier, who is charged with ensuring that the tendons are placed according the installation drawings and stressed properly. From

FIGURE 2.5.2C-2 Close up of a Stressing Pan at Top of Member (P180)

FIGURE 2.5.2C-3 Row of Stressing Pans Next to Wall (P181)

FIGURE 2.5.2C-4 Anti-Bursting Reinforcement behind Anchorage Device (P183)

the author's experience, in most countries it is not common to have an independent agency for quality control. A visit by a representative of the building department prior to placing concrete may be required, but it is often a formality rather than a detailed checking and approving of the reinforcement layout. Inadequate quality control means that problems

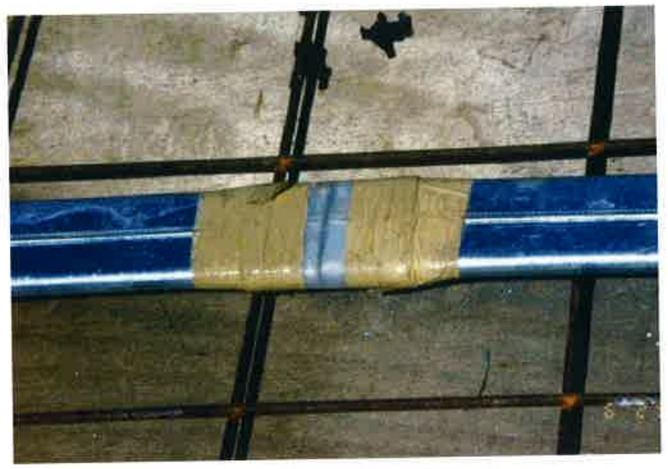


FIGURE 2.5.2C-5 Taped Duct Splice (P184)

FIGURE 2.5.2C-6 Pocket Former for Stressing using Tape Over Styrofoam (MEPS; P185c)

FIGURE 2.5.2C-7 Required Chair Heights are Marked along the Length of Each Tendon (P186)

FIGURE 2.5.2C-8 Individual Chairs Create the Design Profile (P189)

FIGURE 2.5.2C-9 Air Vent Tubes are Provided at Tendon Ends (KSA; P190)

FIGURE 2.5.2C-10 Provision of Air Vents at High Points (Thailand; P191)

FIGURE 2.5.2D-1 Rough Handling of Ducts in Place Results in Duct Collapse and Damage (P475a)

FIGURE 2.5.2D-2 Local Collapse of Duct from Rough Handling (P476)

such as damaged ducts may not be detected. Buckled ducts can result in a large friction loss and incomplete grouting of the tendon. Figures. 2.5.2D-1 and 2 show examples of damage caused by rough handling of the ducts.

The steps in clearing the formwork of debris and preparing it for the concrete placement are similar to those described for unbonded tendons in Section 2.5.1.

E. Placing and Finishing of Concrete: Concrete is pumped to location and consolidated. Where a floor is to be covered by stone or tiles, in anticipation of application of a layer of mortar prior to placing the floor cover, the surface is not finished flat and smooth (Figs. 2.5.2E-1 and 2.5.2 E-2).

FIGURE 2.5.2E-1 Placing of Concrete (P192)

FIGURE 2.5.2E-2 Surface is Rough Finished in Anticipation of Mortar and Tile/Stone Topping (P193)

FIGURE 2.5.2F-1 Anchorage Device Ready to be Stressed (P194)

F. Preparation for Tendon Stressing: Once the concrete reaches the strength required for stressing, the material used to create the stressing recesses is removed and the debris is cleared from the recess block to expose and position the strand ends for stressing Where pans at the top surface the concrete were used, the pans and the exposed surface of anchorage casting are cleared of concrete paste. Figure 2.5.2F-1 shows an anchorage device and strands ready to be stressed.

G. Tendon Stressing: The strands are stressed one at a time, regardless of the number of strands in the duct. Stressing of strands within a duct is sequenced to minimize the likelihood of fretting (abrasion) between the strands. Some contractors use jacks that require the strand to be threaded through the device (Fig. 2.5.2G-1). Where a tendon has to be stressed at the top of a member, a jack with a bent nose is use (Fig. 2.5.2G-2). A justification for the use of this jack style, compared to jacks that ride on top of strand is that bonded tendons are not stressed at intermediate anchorages the way unbonded tendons are. Where continuity of force is required, either couplers or overlapping tendons as shown in Fig. 2.5.2G-3 are used.

Since the strands are loose within the duct, they are first stressed to between 20% to 25% of their final stressing force to eliminate the slack. The strand is then stressed to full force and the elongation is measured. The elongation associated with the second stressing must correlate to the difference in the stressing force between the initial and final stressing.

In many applications, tendons are stressed to between 20% and 25% of their design force the day after the concrete is cast, to reduce shrinkage cracks. The balance of the design force is applied after the concrete has gained the strength specified for stressing. The benefit of two-stage stressing in the reduction of shrinkage cracks is debatable. The practice is not customary in the US, despite the fact that US projects generally feature far less non-prestressed reinforcement. The practice possibly developed because of the need to do an initial stressing to eliminate the slack in the strand.

The strand tails are marked for elongation measurements using the same procedures that are used for unbonded tendons. Elongations are usually measured after the strand is anchored

FIGURE 2.5.2G-1 Stressing Operation at Member Edge (P195)

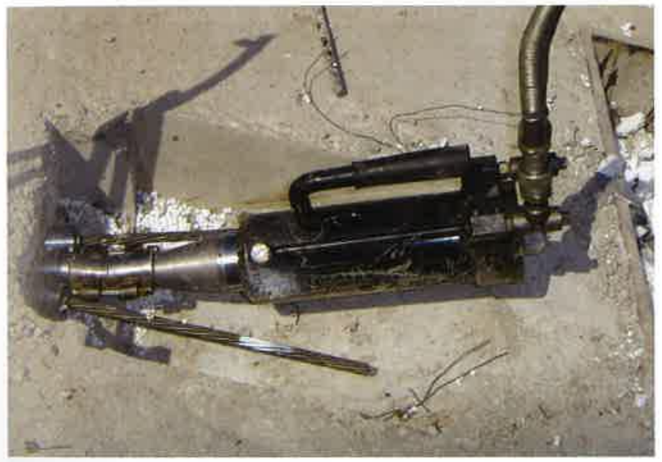


FIGURE 2.5.2G-2 Stressing Operation at Member top Surface (P196)

FIGURE 2.5.2G-3 Intermediate Stressing at a Construction Joint through Overlapping Tendons (P512)

FIGURE 2.5.2G-4 Stressed and Anchored Strands Using Donut Barrel (P199b)

Figure 2.5.2G-4 shows a three-strand tendon that has just been stressed; donut (barrel) anchors were used to anchor the strands.

H. Validation of Stressing Records: The stressing records must be approved before the tendons are grouted. The responsibility for observation of the stressing process, record keeping, and approval of the records varies from region to region. Often, the post-tensioning supplier is charged with the task of record keeping. In many countries, variations between the calculated and measured elongations up to 10% are deemed acceptable.[17]

I. Stripping of Forms and Re-Shoring: Once the stressing records are approved, the forms can be stripped, and the floor re-shored. Re-shoring is generally necessary if the slab is going to be used to support the weight of construction above it.

J. Cutting of Tendon Tails: The tendon tails are typically cut with abrasive rotary saws. Unlike unbonded tendons, where the stressing pockets are small and shallow, the stressing recess for a bonded tendon is generally large enough that the saw can cut the tail just behind the wedges. This provides the cover required for fire and corrosion protection.

K. Tendon Finishing and Grouting: Next the stressing recesses are sealed off with a stiff grout to avoid leakage of the liquid grout that will be pressure injected into the ducts (Fig. 2.5.2K-1).

[17] ACI 318 recommends 7%

A grout mixer and pump are used to prepare and inject the grout (Fig. 2.5.2K-2). For better quality control, bags of pre-mixed grout are typically used. Samples of the mixed grout are taken for quality control. The grout is pumped into the duct through an insertion tube at one end of tendon. The vents at the high points are used to monitor the progress of the grouting. Once grout with acceptable consistency comes out of a vent (Fig. 2.5.2K-3), the vent is tied to force the grout down the duct. The vents are tied off in sequence down the tendon when grout with acceptable consistency comes out. When grout comes out the vent at the far end of the tendon, the grouting is concluded.

2.5.3 Marking and Recording of Tendon Positions

It is a good idea to record the as-installed location of the tendons, in case it becomes necessary to modify the structure in the future. Marking the tendon locations helps to identify where repairs or drilling will require special precautions.

A. Marking of Tendons on Finished Floor: One way to mark the tendon locations is to spray paint the locations on the formwork before the concrete is cast; the paint marks will be transferred to the slab soffit (Figure 2.5.3A-1.) Another option shown in Fig. 2.5.3A-2 is to paint the tendon locations on the slab soffit after the forms are removed.

B. Photo/Video Recording of Reinforcement: An alternative to marking the slab soffit is to record the position of reinforcement through photographs and or videos and file the records as part of the as-built documents of the construction. Recording the tendon positions is an investment at the time of construction that may prove extremely valuable if it subsequently becomes necessary.

2.6 ECONOMICS AND MATERIAL QUANTITIES

The benefits of post-tensioning and the economics of its construction vary greatly in different parts of the world. In Northern California, some concrete contractors prefer a post-tensioned alternative to the corresponding reinforced concrete design, regardless of the span and configuration; the cost per ton of post-tensioning in place is almost the same as non-prestressed reinforcement, and the post-tensioned alternative will use less material. In contrast, in South Korea, the cost of post-tensioning in place is several times that of non-prestressed reinforcement; in part because the labor costs are considerably lower than in California and the strand cost is higher. However a post-tensioned alternative may be selected for other reasons, for example the architectural layout or the fact that post-tensioned construction will take less time.

FIGURE 2.5.2K-1 Stressing Pockets are Sealed Off to Avoid Spilling at Grouting (P201)

FIGURE 2.5.2K-2 Grout Mixer Using Pre-mixed Bags (P202)

FIGURE 2.5.2K-3 Grout is Allowed to Spill Out of a Vent to Check its Consistency Prior to Locking it Off (P204e)

(a) Spray paint on form at low point region of tendon (P205a)

(b) Impression of tendon location at slab soffit (P205b)

FIGURE 2.5.3A-1 Identification of Tendon Location through Spray Paint on Soffit

The following is a general overview of estimated reinforcement quantities for common residential and commercial buildings. Labor requirements and construction practice in the U.S. are also briefly covered.

Cost comparisons often refer to graphs such as the one in Fig. 2.6-1, where the economy of a post-tensioned slab versus a conventionally reinforced slab is shown as a function of span length. A span of about 7 m (23 ft) is viewed as the cross-over point between the two options. While a relationship such as shown in the figure is valid, compliance with different building codes and local practice can override the general case. As an example, when ACI 318 is used for the design of a column-supported floor system, a minimum amount of prestressing is required, based on the cross-sectional geometry of construction.[18] In addition, ACI 318 limits the spacing of tendons.[19] The first requirement means that there is a minimum span length for which a post-tensioning tendon can be fully utilized; the second means that there is a minimum slab thickness for full utilization of post-tensioning. These requirements do not exist in the corresponding European code, EC2.[20] These and other requirements, such as allowable stresses, mean that design quantities are often a function of the building code. In addition to the design requirements governed by the applicable building code and engineering principles, local perception of "good practice," can play a significant role in the quantities commonly used. For example, a relatively heavy bottom mesh is used for post-tensioned slabs in the KSA, (Kingdom of Saudi Arabia) but a mesh is generally not

FIGURE 2.5.3A-2 Soffit of Slab is Painted to Show the Location of Tendons (Mumbai; P206b)

18 ACI 318-11 minimum P/A; Section 18.12.4

19 ACI 318-11 tendon spacing; Section 18.12.4

20 EC2 (EN 1992-1-1;2004(E)

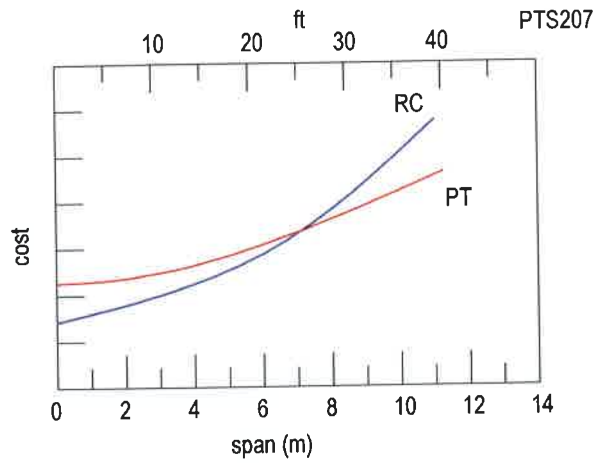


FIGURE 2.6-1 Relationship between Construction Cost and Span for Post-Tensioned and Conventionally Reinforced Floor Systems

used in the U.S. This greatly influences the quantity of non-prestressed reinforcement in a post-tensioning alternative, and hence its competitiveness versus the comparable reinforced concrete design.

There are several features in post-tensioned floor systems that lead to their selection as construction of choice, regardless of the cost differences between a post-tensioned slab and its conventionally reinforced alternative (see Section 2.2). In addition to the material, labor, and other inherent advantages discussed earlier, the increased speed of construction and simpler forming means there is faster turnover of forms; this reduces the amount of formwork required and leads to a more cost effective construction. Likewise, the reduction or even elimination of beams, and simplifications in the geometry of the floor system through selection of a flat soffit lead to a reduction in the cost of forming and overall construction. Although these considerations are central to overall cost, they are not covered in this section; the focus of this section is "material quantities."

2.6.1 Material Quantities

A. Practice and Project Examples: The quantities covered consist of concrete, prestressing strands, and non-prestressed reinforcement (rebar). The values quoted are the minimum amounts required by efficient design using the governing building codes. The reinforcement used for engineering detailing is reported separately.

In the U.S., the long tradition of post-tensioning construction has resulted in a greater awareness and familiarity of design engineers and contractors with post-tensioning; this, along with a relatively clear and explicit building code, means that design and construction practices are well matured. Reinforcement is generally specified and placed "where needed," to meet the requirements. If a design does not follow this criterion, the post-tensioning contractor will generally have the work re-designed by an experienced post-tensioning engineer. As a result, engineers are aware they should take full advantage of the material specified for construction.

Elsewhere, where post-tensioning is emerging as a preferred choice, the construction is sometimes a mixture of conventionally reinforced concrete and post-tensioning practices. Floors are reinforced as if they were conventionally reinforced, and then post-tensioning is added. Thus, a large amount of non-prestressed reinforcement is used in addition to post-tensioning tendons. The base reinforcement is mostly in form of a heavy bottom mesh—and at times, both a top and bottom mesh (Fig. 2.6.1A-1).

As an example, the quantities for a project value engineered and constructed by Suncoast Post-Tension[21] are given in Table 2.6.1A-1. The project is a typical multi-story hotel constructed in the US with 26 ft (8 m) spans, supported on columns and shear walls, and designed according to ACI 318. The table lists

FIGURE 2.6.1A-1 View of Reinforcement in a Floor System Ready to Receive Concrete (Beirut; P208)

21 Suncoast Post-Tension Houston www.suncoast-pt.com

TABLE 2.6.1A-1 Reinforcement Quantities for Floors of a Multistory Hotel (T111)

Concrete	7 in (180 mm)	8.5 in (220 mm)
Rebar	1.3 psf (6.34 Kg/m2)	5.5 psf (26.85 Kg/m2)
Post-Tensioning	0.85 psf (4.15 Kg/m2)	0 0

the quantities used for the post-tensioning alternative, along with the values of the original reinforced concrete slab.

The total weight of post-tensioned and non-prestressed reinforcement in the post-tensioned alternative is 2.15 psf (10.49 kg/m2) compared to 5.5 psf (26.84 kg/m2) in the reinforced concrete slab.[22] Thus the conversion to post-tensioning reduced the total weight of reinforcement (combined PT and rebar) by over 2.5 times. Due to high labor cost, the in-place unit price of PT and rebar are fairly similar, so the savings were significant, even without including the reduction in volume of the concrete.

For the multistory building of Fig. 2.6.1A-2, constructed in Hawaii in 2009, typical quantities for the floor would be:

- Slab thickness: 6 in (150 mm)
- Post Tensioning 0.70 psf (3.4 kg/m2)
- Rebar 1.0 psf (4.88 kg/m2)

B. Base Quantities for Code Compliance: This Section covers the requirements of the International Building Code (IBC 2009) and the European Code EC2 (EN1992-1-1:2004). Since the IBC concrete requirements are derived from ACI 318, the results compare EC2 to ACI 318.

The quantities used in each construction derive from two requirements. First, it is the reinforcement to comply with the in-service (SLS) and safety (ULS) requirements of the applicable code. This reinforcement is by computation. Second, it is the reinforcement for "structural detailing." This is the reinforcement commonly used for crack control at discontinuities; along the perimeter of a slab and similar conditions, where reinforcement is added for improved performance.

22 The reinforcement totals refer to typical floors. They do not include trim bars, such as around openings

The quantities required for code compliance are deterministic. These are shown in the graphs of following figures for the codes covered. The quantities for structural detailing depend on the complexity of the project, the judgment of the designer, and local practice regarding what constitutes "good structural detailing." The practice varies widely in different countries. The quantities given in Table 2.6.1D-1

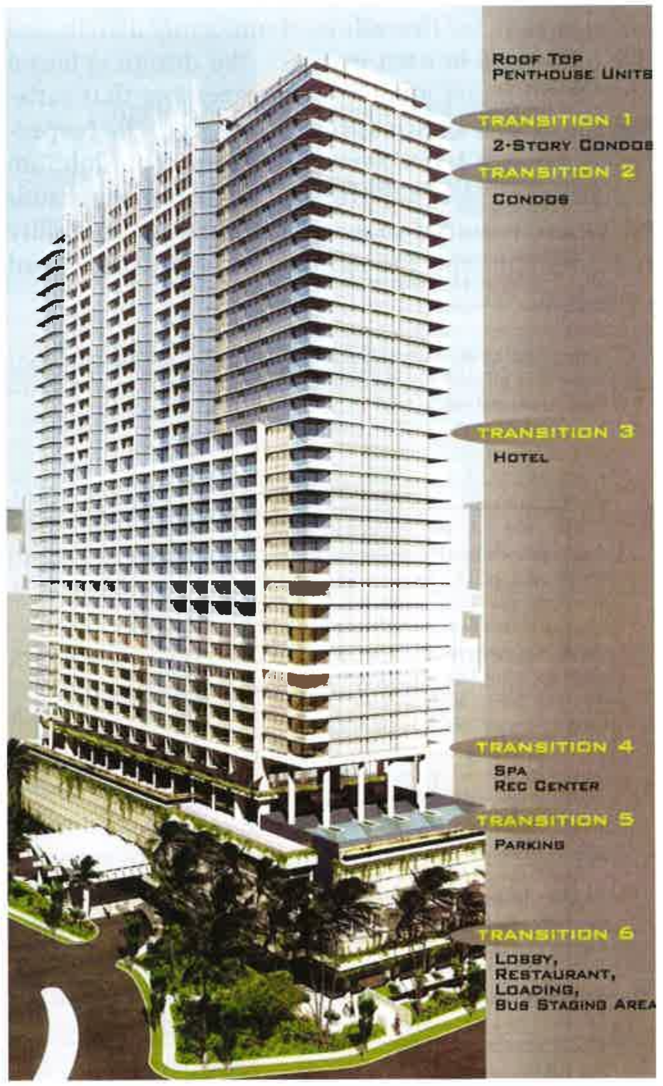


FIGURE 2.6.1A-2 Multi-Story Post-Tensioned Building in Hawaii (P209)

reflect the US practice for common residential and commercial buildings.

For comparison among the different options, a reference floor is selected. The quantities are calculated for the reference floor using different load values and building codes. The results are compiled in following graphs.

The geometry and tendon layout of the reference floor is shown in Figs. 2.6.1B-1 and 2. Interior spans are 9.00 m (29'-6") long; exterior spans are 7.50 m (24.6 ft) long; slab thickness is 200 mm (8 inch); columns are 600 mm (23' 6") square. Other parameters of the reference floor are listed in Section C.

In addition to selfweight, the floor is subjected to four different values of superimposed dead load (SDL). For each value of SDL, the floor is analyzed and designed for five values of uniformly distributed live load (LL). In each instance, the design is based on the minimum amount of prestressing that satisfies the in-service stress requirements of the respective codes—ACI 318 typically governs the minimum requirement. Non-prestressed reinforcement is added, where required to meet the other serviceability requirements and the ultimate strength (ULS) load combinations.

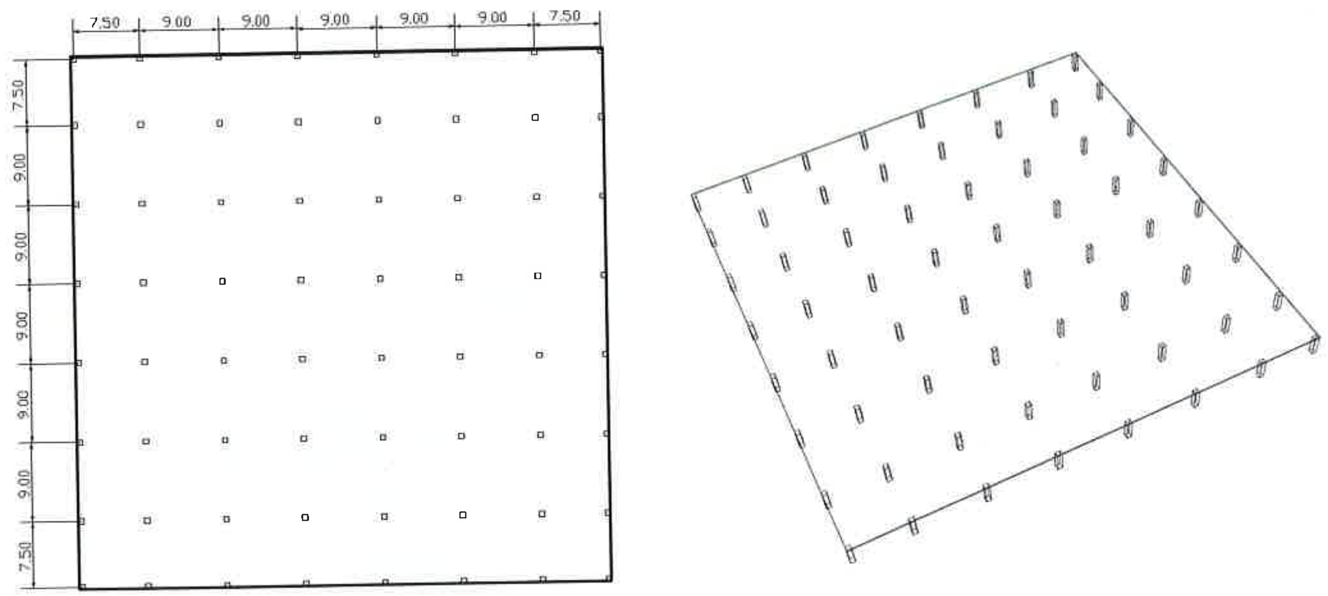
The reference floor system was designed for the following load values:

- Superimposed Dead Load (SDL) 1 to 3 kN/m2; (20 to 60 psf)
- Live Load (LL) 2 to 5 kN/m2: (40 to 100 psf)

The comparison was carried out for the commonly used bonded and unbonded post-tensioning systems. In each case, the typical construction parameters, as detailed in Section 2.6.1C, were used. The same tendon layout was used for all conditions. Because the spans were the same in both directions and the loading was uniform, the same number of tendons were required in each direction (Fig. 2.6.1B-2). In practice, tendons are typically banded over the column lines in one direction and uniformly distributed in the orthogonal direction. Whether tendon are banded in one direction and distributed in the other direction, or distributed in both directions does not impact the required material quantities. However, banding the tendons in one direction considerably simplifies their installation.

The tendon profile for the interior spans is shown in Fig. 2.6.1B-3. The tendon cover is the same for both bonded and unbonded systems, but the distance to the centroid of the strands is different because the ducts used for bonded tendons are larger than the sheathing used for unbonded tendons.

For the lower values of load, continuous tendons from one end of the structure to the other end were



(a) Plan of floor system (spans 9m; 29' - 6") (b) 3D view of floor system

FIGURE 2.6.1B-1 Floor System Selected for Reference Floor (P210a,b)

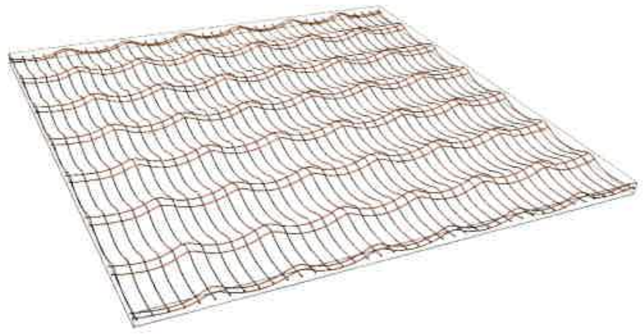
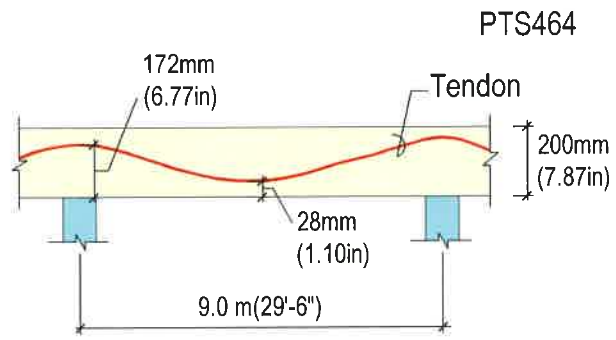
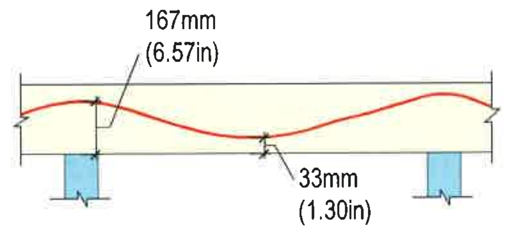


FIGURE 2.6.1B-2 View of Tendon Layout in Slab (P211)



(a) Unbonded tendon geometry



(b) Grouted tendon geometry

FIGURE2.6.1B-3 Tendon Profile of Reference Floor

adequate. For the higher load values, tendons had to be added in the exterior spans, to meet the allowable stress limits.

An average precompression of 125 psi (0.84 MPa) was specified for all cases, based on the requirements of ACI 318.[23] EC2 does not require a minimum precompression but, the same tendon layout and precompression was used for both cases to allow comparison of quantities.

A notable feature in the comparative quantities is a specific ACI requirement[24] for grouted systems, namely that the nominal design moment (M_n) of a section reinforced with grouted tendons must be greater than or equal to 1.2 times its cracking moment (M_{cr}).

$\emptyset M_n \geq 1.2 M_{cr}$ (Exp 2.6.1B-1)

Depending on the slab geometry and loading, non-prestressed reinforcement may need to be added to increase the moment capacity ("cracking moment rebar"). Interestingly, EC2 has a similar criterion for beams reinforced with unbonded systems. EC2 requires the nominal design capacity (M_n) of a beam reinforced with unbonded tendons to be greater than or equal to 1.15 times its cracking moment (M_{cr}).

For bonded systems with low values of loading, the ACI requirements of (i) cracking moment rebar combined with the low limit on allowable tensile stresses and (ii) the minimum level of average precompression result in more non-prestressed reinforcement than the European Code, EC2. At higher load values, however, the fact that the allowable stress in the tendons at the ULS is greater under ACI 318 results in less non-prestressed reinforcement than EC2.

Figures 2.61B-4 and 5 illustrate the amount of reinforcement in relation to the applied load for the reference slab.

The generally higher amount of rebar required for unbonded systems compared to the rebar required for a comparable grouted system is due to the ACI 318 minimum reinforcement requirements for unbonded systems. ACI 318 does not require a minimum area of non-prestressed reinforcement for bonded systems. The initially higher amount of reinforcement for bonded systems at low values of load stems from the requirements for cracking moment rebar discussed above; the cracking moment requirement applies to grouted systems only. At higher values of loads, the larger amount of post-tensioning available provides the necessary margin of safety against cracking moment. Hence, there is less need for supplemental non-stressed reinforcement. The requirements of the two codes are discussed in greater detail in Sections 4.10 and 4.11.

Figures 2.6.1B-6 and B-7 show the quantities of post-tensioning and non-prestressed reinforcement used

23 ACI 318-11; Section 18.12.4
24 ACI 318-11; Section 18.8.2

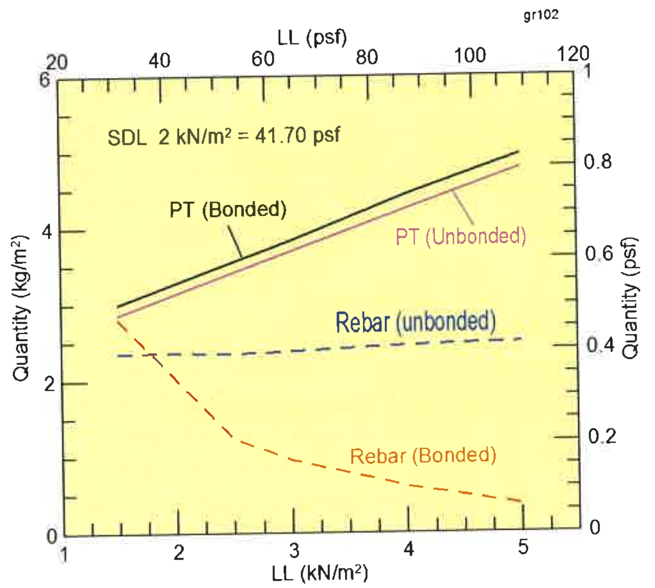


FIGURE 2.6.1B-4 Quantities Based on ACI 318 for Reference Floor Reinforced with Unbonded or Grouted Systems. For detailing add 1.5 kg/m2 (0.30 psf) (P643)

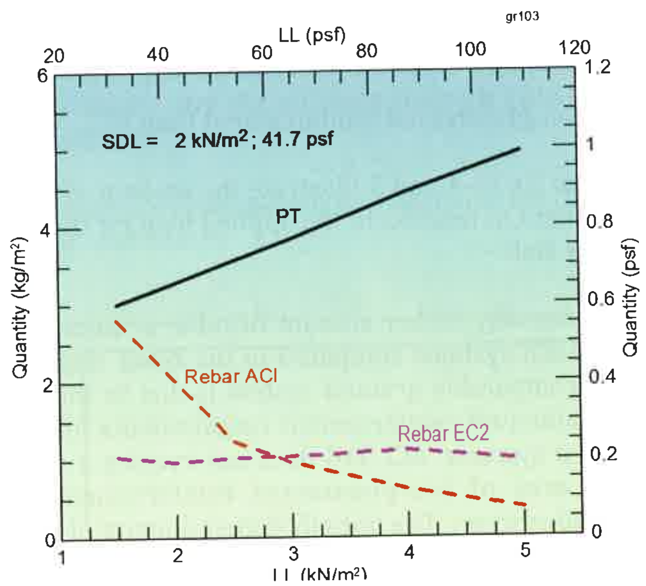


FIGURE 2.6.1B-5 Grouted Post-Tensioning Quantities for the Reference Floor (P642)

for the grouted and unbonded systems for the range of superimposed and live load values analyzed

C. Data and Parameters of Reference Slab: The material properties, post-tensioning systems, and other input data for the analysis of the reference floor system are listed below:

Concrete:
Weight = 2500 kg/m³ (150 pcf)
Specified strength at 28 days = 34 MPa (5000 psi)
Modulus of Elasticity = 29,958 MPa (4,030 ksi)
Poisson's ratio = 0.2

Post-Tensioning:
MATERIAL
Low relaxation, seven wire strand
Strand nominal diameter = 13 mm (0.5 in)
Strand Area = 99 mm² (0.152 in²)
Modulus of Elasticity = 200000 MPa (29,000 ksi)
Guaranteed ultimate strength (f_{pu}) = 1860 MPa (270 ksi)
Jacking stress = 80% ultimate
Seating loss = 6 mm (0.25 in)
Long-term stress loss = 75 MPa (10.88 ksi)

UNBONDED SYSTEM
Tendon diameter (greased and sheathed) = 13 mm (0.5 in)

Stressing
Angular friction coefficient, μ = 0.07
Wobble Friction = 0.0046 rad/m (0.0014 rad/ft)

GROUTED (BONDED) SYSTEM
Duct depth = 20 mm (0.79 in)
Distance of duct centroid to centroid of strand (z) = 3 mm (0.12 in)

Stressing
Angular friction coefficient, μ = 0.20
Wobble friction coefficient, K = 0.025 rad/m (0.0076 rad/ft)

Cover
Nonprestressed Reinforcement
Interior span top and bottom cover = 20 mm (0.79 in)
Exterior span bottom cover = 40 mm (1.57 in)

Prestressing Tendons
Interior span top and bottom cover = 20 mm (0.79 in)
Exterior span bottom cover = 40 mm (1.57 in)
Tendon Profiles
Interior spans = reversed parabola with inflection points on either end at 0.1 times the span length

Exterior spans = exterior end—simple parabola; interior end—reversed parabola with the inflection point at 0.1 times the span length; low point at midspan.

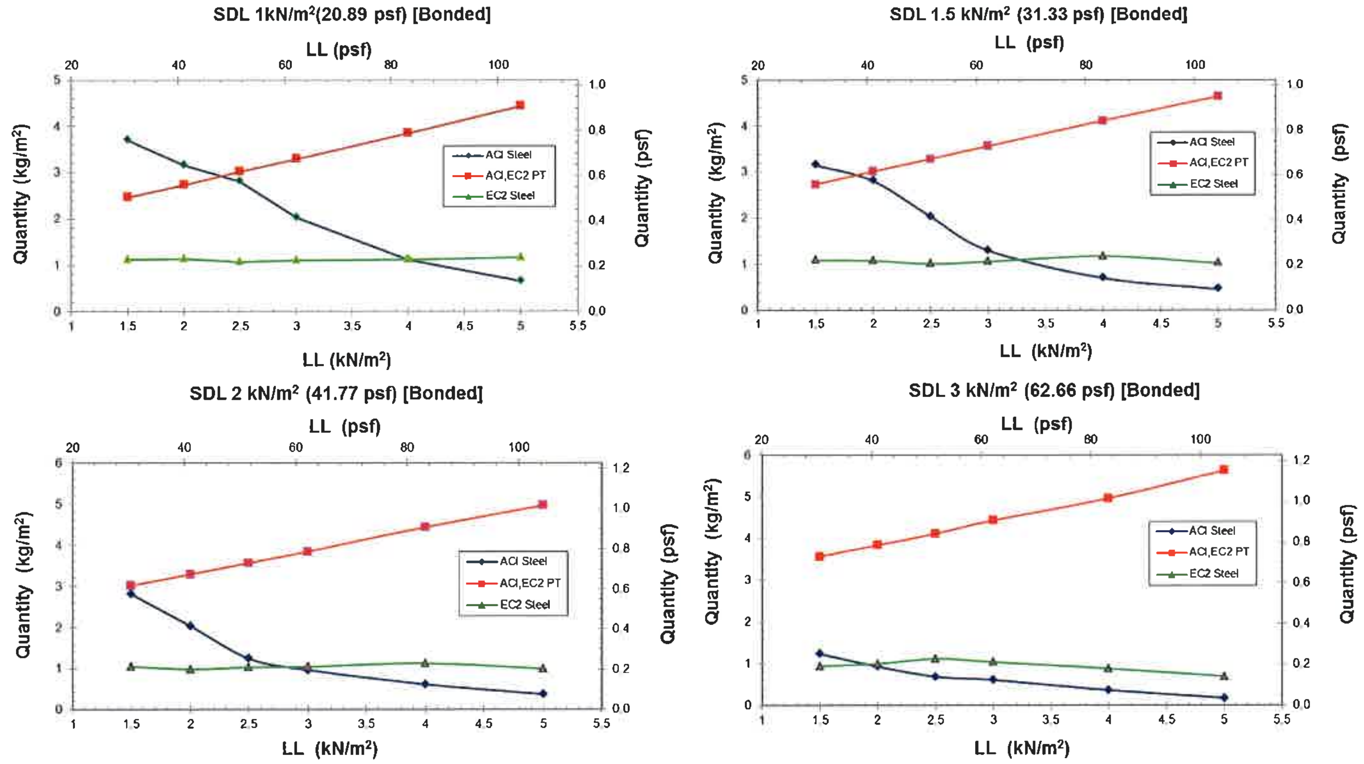


FIGURE 2.6.1B-6 Quantities of Prestressing (PT) and Non-prestressed Reinforcement Using Bonded (Grouted) Tendons (PTS280)

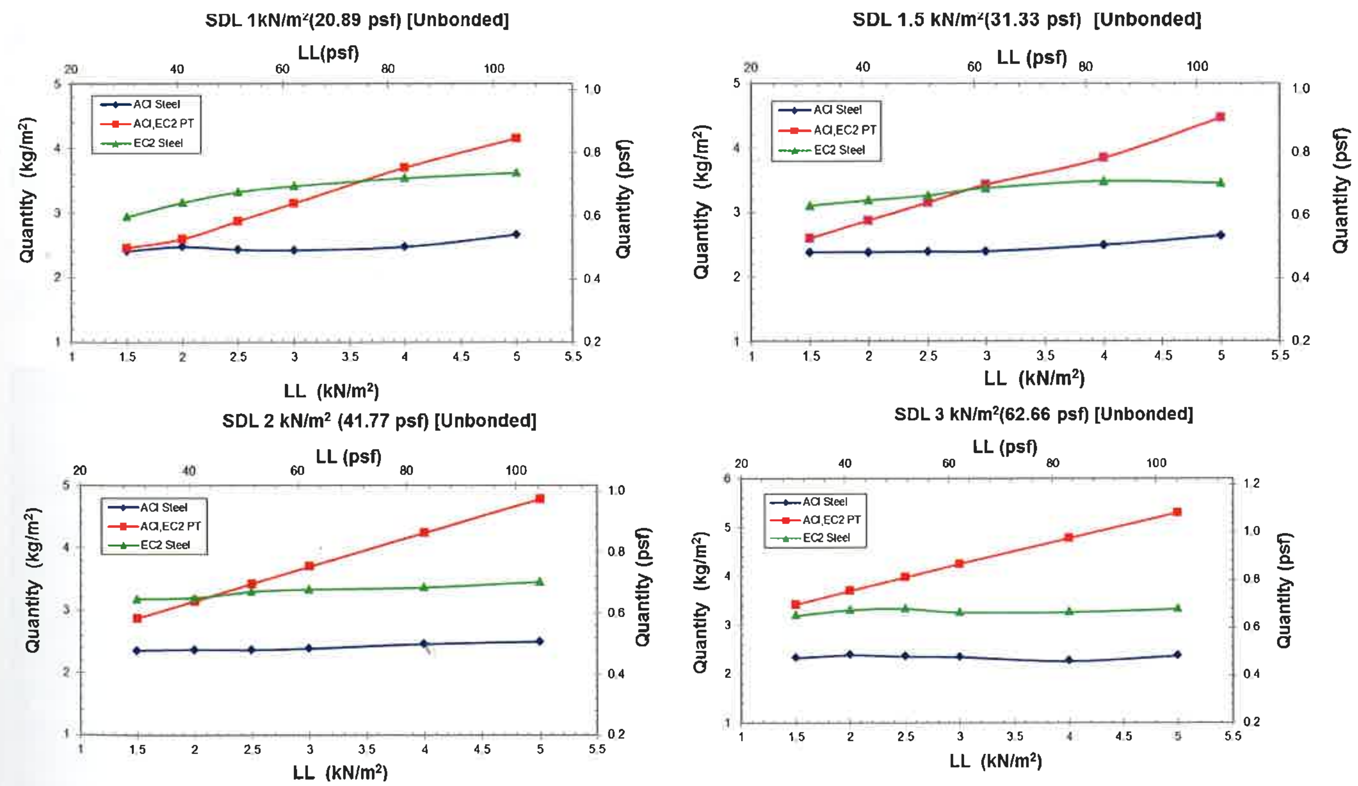


FIGURE 2.6.1B-7 Quantities of Prestressing (PT) and Non-prestressed Reinforcement Using Unbonded Tendons (TN280)

TABLE 2.6.1D-1 Non-stressed Reinforcement for Detailing (T112)

	Description	kg/m²		lb/ft²	
		One-way	Two-way	One-way	Two-way
1	Reinforcing behind anchorage devices (back-up bars, hair pins)	0.19	0.39	0.04	0.08
2	Shrinkage and crack control (rebar due to restraining elements)	1.45	1.45	0.30	0.30
3	Trim steel (openings, penetrations, corners, discontinuities)	0.48	0.19	0.10	0.04
4	Support steel for tendons*	0.97	1.7	0.20	0.35
5	Column capitals	---	0.68	---	0.14
6	Completing beam rebar cage**	1.45	---	0.30	---
Total		4.54	4.41	0.94	0.91

Notes: Not included are reinforcement which may be necessary to resist wind/seismic loading (drag and chord bars), closure strips, and construction joints.
* 12 mm bars at 1.20m (#4 @ 42") spacing, continuous with lap for distributed strands; for banded tendons
12 mm bars at 1m spacing (#4 x 6'-3" @ 3'-0" o.c).
** 2 x 26 mm (2#8) continuous bars, top and bottom of beam cage, plus one 16 mm bar (#5) at mid-height of stem, where no rebar reported by analysis.

Analysis and Design Tool
ADAPT-PT[25] version 2012 was used to optimize the design

D. Reinforcement for Detailing: In addition to the reinforcement calculated for code compliance, a certain amount of reinforcement is added at (i) discontinuities such as re-entrant corners, steps, and openings to control cracking, and (ii) where reinforcement is needed to distribute or direct the applied loads to the load path envisaged by the design engineer. The amount of reinforcement needed for detailing depends on the complexity of the structure. Table 2.6.1D-1 provides a rough estimate for common residential and commercial buildings; the values apply to both unbonded and grouted construction systems.[26]

2.6.2 Construction Cost

The bulk of the construction cost of a post-tensioned floor consists of material, field labor, and equipment; management, engineering, and administrative costs are essentially the same as for a conventionally reinforced slab. The unit cost of each of the primary components of material, field labor and equipment varies greatly from country to country, and within the same country from region to region. The following list provides costs for a typical post-tensioned floor system constructed in Northern California in 2013:

A. Unit Cost of Material: The approximate cost of material delivered and placed for a typical mid-rise building construction project in Northern California is as follows:[27]

Cost of rebar, purchased, bent, and placed (60 ksi; 420 MPa) \$1.05/lb; \$2.31/kg,

Cost of prestressing material, including hardware, placing and stressing: \$2.70/lb; \$5.94/kg.

Total cost of construction of concrete floor, including forming, labor and concrete, but excluding rebar and post-tensioning: \$582/CY; 762/m3.

For normal projects, contractors generally carry 20,000 sf (1,858 m2) of forming and break a project into pours of this size, using a crew that places both non-prestressed reinforcement and prestressing. For most buildings, the construction cycle is typically one week. This includes stripping and re-shoring the previously cast concrete; moving the forms to next pour and installing them; placing rebar and prestressing tendons, performing quality control and any required inspections, placing the concrete, and stressing the tendons. Ideally, the concrete is placed on Thursday afternoon or Friday morning so that it has developed enough strength to allow the tendons to be stressed on Monday

25 ADAPT-PT, 2012, www.adaptsoft.com
26 ADAPT- Floor Pro, 2012, www.adaptsoft.com

27 Values are for Fall of 2013 www.largoconcrete.com

B. Unit Cost of Labor: Labor costs vary more than material and equipment costs from country to country and region to region. In regions of high labor costs, the benefits of post-tensioned structures include:

Labor
- ❖ Less reinforcement, which results in lower labor costs for handling and placing
- ❖ Simplification in construction, which reduces the cost of labor for forming
- ❖ Shorter construction cycle.

As an example of productivity and labor costs, consider the following concrete floor slab that was constructed in San Francisco (Fig. 2.6.2B-1). The slab area for each level was 17,672 sf (1,642 m²). Once the forms were set up, the reinforcement and post tensioning for each floor were placed in four days by eight technicians. On average, each technician placed 552 sq (51.5 m²) of reinforcement (PT and non-stressed) each day.

2.7 REPAIR; RETROFIT; MAINTENANCE; AND LIFE CYCLE

Questions often arise regarding the ease of repairing or remodeling a post-tensioned floor system. This includes whether it is even possible to drill holes or make new openings in a post-tensioned slab. The short answer is "yes", it is both possible and practical.

FIGURE 2.6.2B-1 Slab ready to Receive Concrete
(Courtesy Nishkian Menninger; Portero; San Francisco; P213)

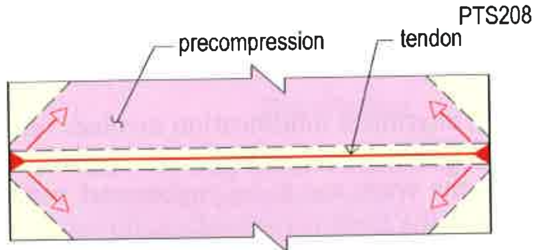
However, anytime a modification involves removing a portion of a slab and cutting rebar or prestressing tendons, the work has to be engineered, regardless of whether the floor is conventionally reinforced or post-tensioned. The loss of load-bearing capacity resulting from the removal of concrete and reinforcing needs to be evaluated, and if necessary compensated for. When the slab is post-tensioned, there are additional considerations, particularly if the slab is reinforced with unbonded tendons.

2.7.1 Floors Reinforced with Grouted Tendons

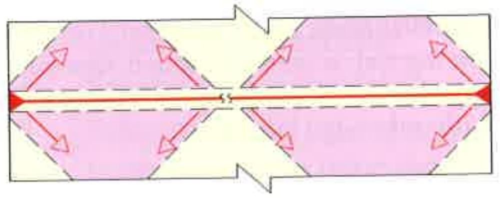
When a slab is reinforced with grouted tendons, the procedures for repair or remodeling are similar to those for a conventionally reinforced slab and the same precautions apply. The prestressing steel is bonded to the concrete surrounding it in the same way that non-prestressed reinforcement is; once in service, the prestressing steel functions much like non-prestressed reinforcement. Nevertheless, there are some important functional differences.

In particular, at the strength limit state (ULS), prestressing strand is capable of developing three to more than four times the force of non-prestressed reinforcement of the same cross-sectional area. Thus cutting a prestressing strand can be more detrimental than cutting a rebar of comparable size if the design relies on the full strength of the cut strand. Because the strand carries more force, it requires a longer length than a rebar of comparable size to develop its full strength. As a result, the loss of effectiveness of the strand extends over a longer distance from the location of the cut.

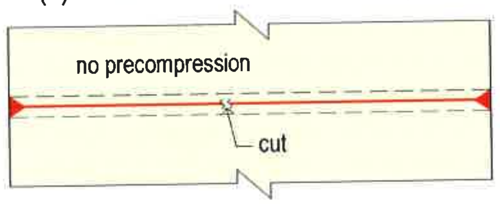
In addition, bonded prestressing strand provides "local" precompression that is beneficial to crack mitigation; this precompression is lost if a section of the strand is removed. However, cutting a bonded strand does not entirely eliminate its original contribution to the precompression in the floor system. The precompression from each strand is dispersed into the slab when the strand is stressed; once the tendon is grouted, the precompression is locked into the floor system. When a tendon is cut, there will be a local reduction in precompression, as indicated in Fig 2.7.1-1, but the precompression imparted from the remainder of the strand will remain. Furthermore, since there is generally more than one prestressing



(a) Plan : slab with continuous tendon



(b) Bonded tendon severed at a point



(c) Unbonded tendon severed at a point

FIGURE 2.7.1-1 Loss of Precompression from a Severed Tendon

strand in a slab, the loss of precompression from one strand does not eliminate the precompression in the floor, even adjacent to the location of the cut.

2.7.2 Floors Reinforced with Unbonded Tendons

When dealing with grouted tendons, it is not necessary to re-stress and re-anchor a strand that has been cut. The grout that is injected into the duct, once hardened, locks the force into the tendon. In contrast, when an unbonded strand is cut, it loses it force along its entire length. Thus its contribution to both the serviceability and safety of the structure is completely lost. In addition, unlike with a grouted tendon, the cut tendon's contribution to the precompression in the floor system will be lost over the entire length of the tendon (Fig. 2.7.1-1c).

The loss of force and effectiveness of a tendon over its entire length require that the contribution of each tendon be fully evaluated and, where necessary, compensated for in the repair or retrofit. The numerous buildings constructed with unbonded post-tensioning in the US over the last sixty years include many examples of post-tensioned floor remodeling projects. These range from the creation of small openings to the removal of critically large sections of the floor.

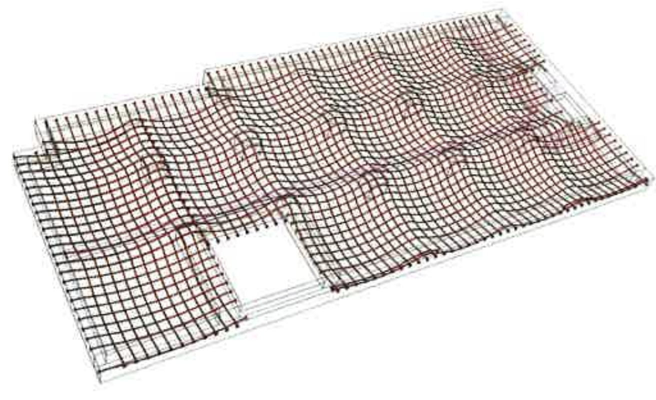
The repair/retrofit procedure generally consists of exposing the affected tendons; carefully cutting them; re-stressing them and re-anchoring them at the face of the cut. The sequencing of the work may vary, depending on the number of tendons that need to be cut and the location of the new opening or the repairs. As with the repair and rehabilitation of any other type of floor system, the work should be done by a crew that specializes in similar structures.

As an example of extensive remodeling of a floor system reinforced with unbonded tendons, consider the remodeling at the Hult Business School in San Francisco, engineered by Simpson Gumpertz & Heger.[28] The closely-spaced tendons in both directions posed a special challenge when it was necessary to cut a large 14.5x11.5 ft (4.4x3.5 m) opening for a new stairwell in the middle of a central panel 30x27 ft (9.14x8.23 m). Nevertheless, the work was done successfully.

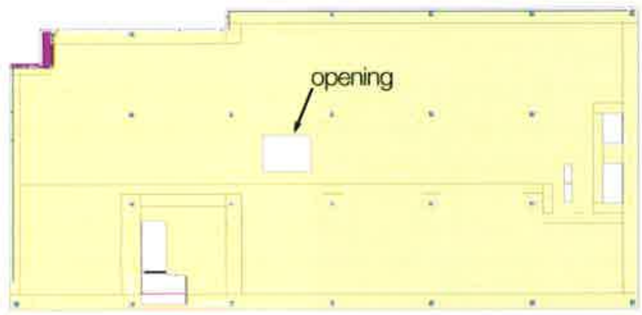
(a) View of floor system and stairwell opening (P478)

(b) View of opening showing anchored tendons (P479)

FIGURE 2.7.2-1 Stairwell Opening Created at Center of a Post-Tensioned Panel (San Francisco, courtesy SGH)



(a) Densely spaced tendon layout of the original floor system (P214)



(b) Plan of the floor system showing the position of stairwell opening created in the middle of a central panel (P215)

FIGURE 2.7.2-2 Creation of an Opening (14.5x11.5 ft ; 4.4x3.5 m) in the Middle of a Central Panel

Figure 2.7.2-1 shows the new opening. The floor system was a waffle slab (Fig. 2.7.2-2a), reinforced in both directions with one tendon in each waffle stem. Part (b) of the figure shows the position of the opening in the central panel. To create the opening, the tendon locations were identified and each tendon was exposed, cut, re-stressed and re-anchored at the face of the new opening by a firm specializing in post-tensioning repair.[29]

Figures 2.7.2-3 and 4 illustrate other examples of major retrofit of a post-tensioned floor. Figure 2.7.2-3 shows the removal of concrete to expose the tendons for an opening. Figure 2.7.2-4 shows the cut strands re-positioned at the edges of the new opening ready to be re-stressed and re-anchored once the concrete for the strip around the new opening is cast.

From the standpoint of a structure's serviceability and safety, the loss of a single strand either inadvertently or by design, is usually not critical. Structural analysis for the effects of a lost tendon often demonstrates that the structure has enough redundancy that it can lose one or more tendons without compromising its intended performance. If it is necessary to replace a lost strand, one option is to extract the damaged portion of the strand and re-thread the sheathing with a smaller diameter, but higher strength strand. The new strand is coupled to the existing strand and the tendon is re-stressed. If only a short length of new strand is required, it is typically

FIGURE 2.7.2-3 Removal of Concrete around Tendons (P480)

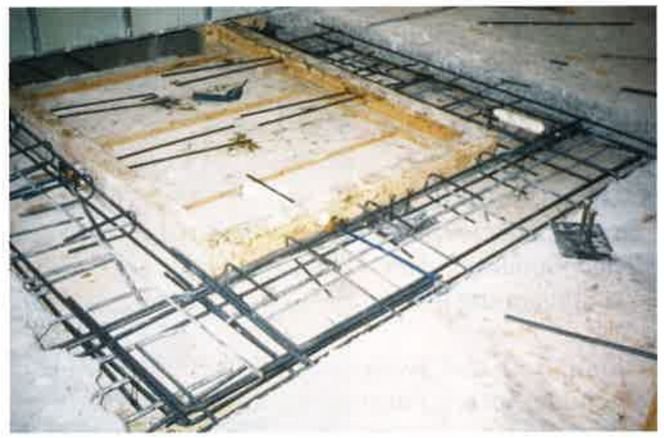


FIGURE 2.7.2-4 Tendons are De-stressed and Re-anchored at the Edge of an Opening (P481)

[28] SIMPSON GUMPERTZ & HEGER, San Francisco
[29] Guido Schwager; www.schwagerdavis.com

possible to use a strand of the same diameter as the original strand rather than smaller strand, as long as the sheathing that the strand will be threaded through is not damaged.

There continues to be considerable discussion that floor slabs constructed with unbonded tendons do not lend themselves to extensive re-modeling which requires new openings to be made in the slab. However, experience accumulated in the US from numerous floors reinforced with unbonded tendons and successful major repairs are testimony of the ability of this type of construction to handle the challenges of extensive re-modeling.

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CHAPTER 3

DESIGN OF CONCRETE FLOORS

Post-Tensioned Tower; Ocean Heights

(Courtesy CCL, P689)

3.1 GENERAL REQUIREMENTS

The primary concerns of a structural engineer are the safety, serviceability, and economy of the structures he or she designs. Safety is understood as the structure's ability to withstand code required loads without excessive damage or injury. Serviceability is achieved if the structure performs as intended throughout its expected life span. Economy is taken to mean the structure's owners feel that both its short and long-term costs are reasonable.

Legality of the design procedure, defined as compliance with applicable building codes, is also important. It is not always easy to establish however, particularly for post-tensioned structures. Building codes tend to follow, rather than lead, practice with respect to post-tensioning. Much of what is currently considered appropriate practice for post-tensioned design is not incorporated into the codes yet.

3.2 REQUIREMENTS OF DESIGN PROCEDURE

An important issue which is often overlooked by code developers is that, unless design procedures are fairly simple and expedient, they will not be adopted by design professionals. This can be important when a design professional is asked to evaluate and select from different structural alternatives.

An example within the field of concrete design is the use of post-tensioning in building construction. A number of consulting engineers are reluctant to select a post-tensioned alternate, because the design may require more time and effort than a conventional concrete structure, although a post-tensioned alternative may be more economical and have superior performance.

The challenging part of automating the design of buildings constructed with concrete has been the flat slab floor systems. In most cases, skeletal members such as beams, columns, and frames made from them can readily be automated for analysis and design. Concrete buildings are rarely limited to skeletal members; however, the floor slabs are typically a significant portion of the building and its design.

This Chapter reviews concrete slab design concepts and presents the method for automating the design of both reinforced and post-tensioned concrete buildings. The method presented eliminates many of the problems associated with the integrated design of concrete buildings. A procedure for selecting load paths and guidelines for the layout of reinforcement are also presented.

3.3 CONCRETE DESIGN IN RELATION TO OTHER MATERIALS

The following example highlights the principal features that distinguish concrete design from other types of design. To better highlight the differences, the example considers two materials: concrete, and glass, each of which has a distinctive feature in terms of design. Although the example is hypothetical, it illustrates how material properties affect design requirements and procedures [Aalami et al, 2005, 2001].

The focus of the following is on:
- ❖ Determination of "load path" in resisting applied forces
- ❖ Determination of "design values," for which resistance need be provided; and
- ❖ Differentiating feature of "analysis process" for concrete floors

Figure 3.3-1 shows a partial plan of a plate or slab under uniform loading. We review the design of the area surrounded by supports A, B and C, marked

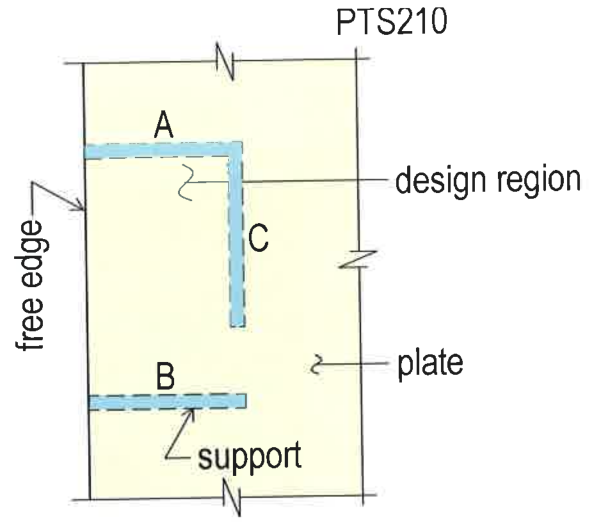


FIGURE 3.3-1 Partial Plan of Roya Structure

"Design Region." The objective is to satisfy the serviceability and safety (strength) requirements of this region.

A. Glass: Consider first a glass plate. The serviceability of the glass plate is determined by acceptable deflection; its safety is measured by the load that causes it to crack. Cracking occurs when the tensile stress at the surface reaches a value that is a material property of the glass. Glass is an extremely brittle material; once cracking is initiated, it will spread and cause failure. Hence, the design procedure consists of:

- ❖ Estimating the deflection under service load; and
- ❖ Determining the load at which the maximum tensile stress reaches the cracking strength of glass.

For serviceability design, deflections can be estimated using approximate methods based on the plate's geometry, support conditions, material properties and service loading. As noted above, however, failure occurs when the stress at any point on the plate reaches the cracking strength of the glass. In order to get a reliable estimate of the glass plate's safety, both the location of the maximum stress and the stress value in relation to the applied loading need to be determined accurately.

The geometry and supports of the glass plate must be modeled accurately since they directly affect the magnitude of maximum stress. In most cases, the actual load path must be determined either analytically or experimentally. Approximate methods based on assumed load paths will not produce accurate results. The need to accurately determine the stress at a point in order to ensure the safety of glass plate is what differentiates the response of glass under increasing load from that of the other materials. Local stresses calculated by finite element analysis are sensitive to the number of mesh divisions and the accuracy of the finite element formulation. In order to determine the value of stress at point, a very fine finite element mesh and an appropriate formulation must be used.

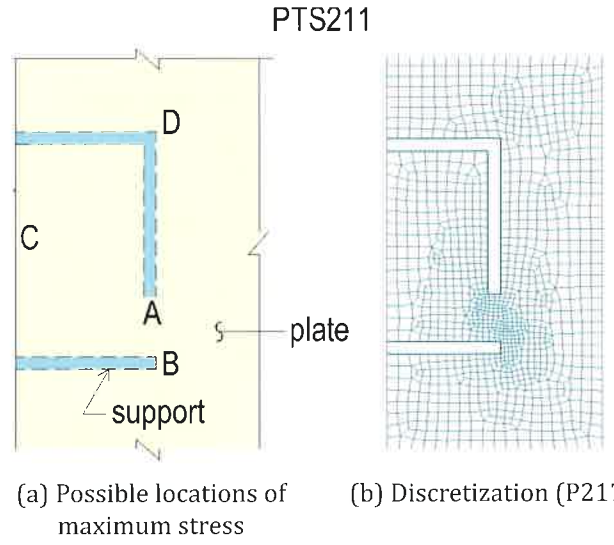
To arrive at a safe design, the following information is required

- ❖ Location governing the design: This is the location, where maximum stress occurs. The location is not known a-priori. It is determined by the elastic response of the plate under the applied loads, its material properties and support conditions. The manner in which the applied load is transferred to the support is referred to as "load path." In this case, the load path is determined by the response of the structure to the applied load. The location and magnitude of the maximum stress are the characteristics of the structure, as opposed to the choice of the designer. To determine the location, designers have to analyze the structure. Typically a finite element plate analysis is performed. The structure is subdivided into a fine mesh, such as illustrated in Fig. 3.3A-1b and analyzed to determine the location of maximum stress.

- ❖ Magnitude governing the design: From a first finite element analysis the probable location of maximum stress can be estimated. Let the maximum location be point A shown in Fig. 3.3A-1a. Next, it is necessary to validate the reliability of the solution value obtained from the analysis. Since local stresses are subject to details of discretization of the finite element cells at the location, the solution has to be validated.

The validation of the solution takes place by selecting a finer finite element mesh at the location of interest, and obtaining new solutions. This iterative process of multi-solution has to be continued, until the difference in value of the stress between successive iterations at the location of interest diminishes. The solution is then converged. In short, the solution obtained for the "design" value has to be validated through multiple analyses, with progressively finer mesh.

In summary, in design of glass plate (i) the "load path" is determined by the response of the structure; (ii) the "critical location" that governs the design is generally not known a-priori by the design engineer. It is determined through the response of the structure. And, (iii) the solution has to be validated through multiple analyses, when using finite element method (FEM).

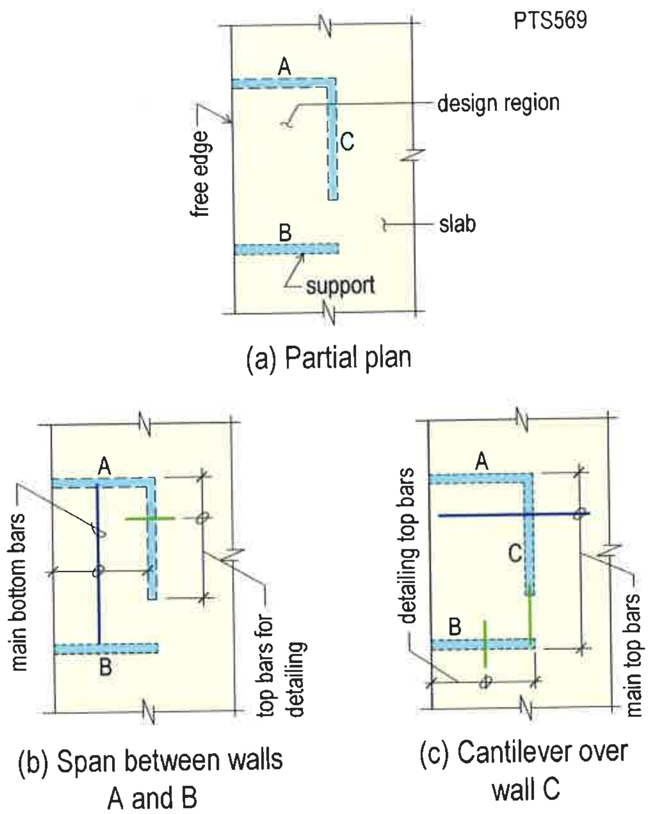


(a) Possible locations of maximum stress (b) Discretization (P217)

FIGURE 3.3A-1 Glass Plate and its Discretization for Local Stress Analysis

B. Concrete: Consider a concrete slab, such as the region representing Roya balcony in Fig. 3.3B-1. The design engineer's first step is likely to decide, (i) whether the required reinforcement will be placed at the bottom of the slab spanning between supporting walls A and B (Fig. 3.3B-1b), (ii) or to place the reinforcement over wall C (Fig. 3.3B-1c) and design the balcony as a cantilever. The process of envisaging a reinforcement layout by the engineer is the assignment of a "load path" for the slab to resist applied forces. Unlike the previous example of glass, in this case the load path is assigned by the design engineer. Different engineers may select different load paths. But each can conclude with a safe structure.

Next, the force demand on the slab resulting from the selected load path must be determined. At this stage, the design engineer is likely to select a "finite width" of the slab, for which the design moment, and shear will be determined. For the balcony example, using the option of placing reinforcement at bottom, the selected width is likely to be the width of the balcony. The balcony will be viewed as a "design strip" extending between the walls A and B, for which the design moment will be determined at mid-length between A and B for a "design section," with the finite width" equal to the width of the strip.



Modeling Options of a Plate Region

FIGURE 3.3B-1

The underlying reason behind the flexibility in selection of a load path for concrete slabs is the ability of a concrete section to crack and undergo a finite amount of rotation prior to failure. This is a reflection of the section's ductility. Cracking results in redistribution of demand values to locations in slab, where resistance is available. In order to mobilize the assumed load paths and redistribute the load resistance in a floor, the slab must possess a minimum ductility. All major building codes ensure a minimum ductility by controlling the amount of reinforcement in a section, such as to effectively limit the depth of the neutral axis in bending.

Figure 3.3B-2 is a schematic for response of a typical region of a post-tensioned beam designed for bending according to ACI 318. Part (a) of the figure shows the rotation (Ψ) of the beam section with increasing moment. It illustrates that the beam continues to bend past the elastic limit of its reinforcement. The ability of a section to deform beyond its elastic limit, without failure defines its ductility. The ratio of the ultimate curvature, Ψu , to the curvature at first yield, Ψy, gives a measure of the ductility of the section. The ductility cab be expressed by $m = \Psi_u / \Psi_y$

Through restrictions imposed on the depth of neutral axis (c) (Fig. 3.3B-2b), major building codes ensure that sections designed in bending possess adequate ductility. Part (b) of the figure illustrates the ductility ratio (m) of a typical design based on ACI-318, with the value of the neutral axis. Most major building codes restrict the depth of neutral axis (c) to values that do not exceed $0.5d_t$.

In the example of Roya balcony, the computation of the design moment is straightforward. For a complex geometry, with irregular support layout, it is likely that similar to the glass plate example, FEM will be the analysis processor to determine the design values. This is the trend with increased reliance on BIM[1] technology, where seamless integration among architects' and other building trades' drawings and data are becoming the norm. Unlike the glass example, recent developments in FEM technology enable design values for a concrete floor to be determined with a good degree of accuracy, (i) using a relatively coarse mesh, and (ii) one analysis only, as opposed to multiple analyses necessary for validations of results in the case of glass plate.

Figure 3.3B-3 illustrates in part (a) a fine mesh coupled with multiple analysis runs for the glass option

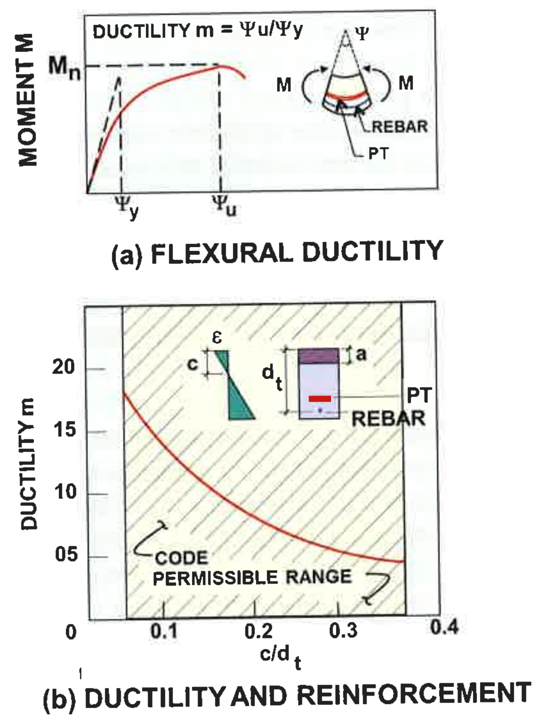


FIGURE 3.3B-2 Ductile Response and Design Requirements of Member in Bending (P647)

[1] Building Information Modeling

and part (b) a suitable mesh for a single analysis for a concrete slab. The underlying reason is that for glass plate, the design is based on the stress value at a "point," whereas for concrete, it is based on the "total value" of an action (moment, etc) over a design section having a finite width. The "total value" is the integral of the "forces," as opposed to local stress at a "point" for glass.

It is important to note that the coarse discretization suggested for concrete design is only valid, if the FEM used for analysis is explicitly formulated to extract "integral" values of force solutions over finite lengths of design sections. This requirement is further expounded in Chapter 4, Section 4.9.5.

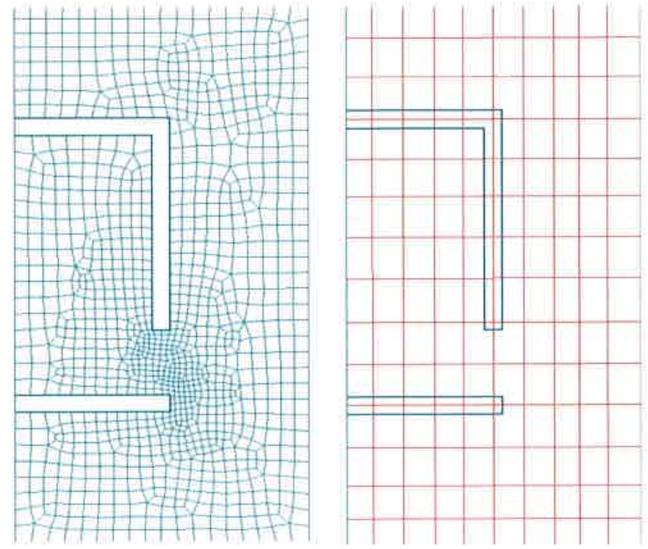
In summary, the primary design criteria for the concrete plate are: (i) deflections and crack widths must be within acceptable limits under service conditions, and (ii) slab must not collapse under code stipulated factored loads.

Using the load path designation, followed by analysis and design of design sections concludes with the amount and location of the primary reinforcement. The determination of primary reinforcement, however, is followed by "structural detailing." The "structural detailing" fulfills the design procedure by ensuring that:

- The load path envisaged by the engineer can develop at loads equal to, or greater than, code stipulated values; and
- Local cracking at discontinuities is controlled.

Referring back to Roya balcony, in Fig. 3.3B-1c, the slab is modeled as a cantilever supported by wall C. This load path requires "primary" top bars over wall C as shown. As part of "Structural Detailing," top bars will be placed over walls A and B for crack control. The figure shows detailing bars over wall B only.

Figure 3.3B-4a shows an example of "Structural Detailing" for development of the load path. In this figure, a concentrated loading is distributed over the width of the assumed load path by distribution steel placed below the load. The added reinforcement ensures that the load path between walls A and B will materialize as envisaged by the designer. Note, however, that although this type of reinforcement is required for both safety and serviceability, it might not be reflected in the outcome of many automated design methods, since a number of such schemes



(a) Discretization for glass plate (P217)

(b) Discretization for concrete slab (P218)

FIGURE 3.3B-3 Discretization Options for Glass Plate and Concrete Slab

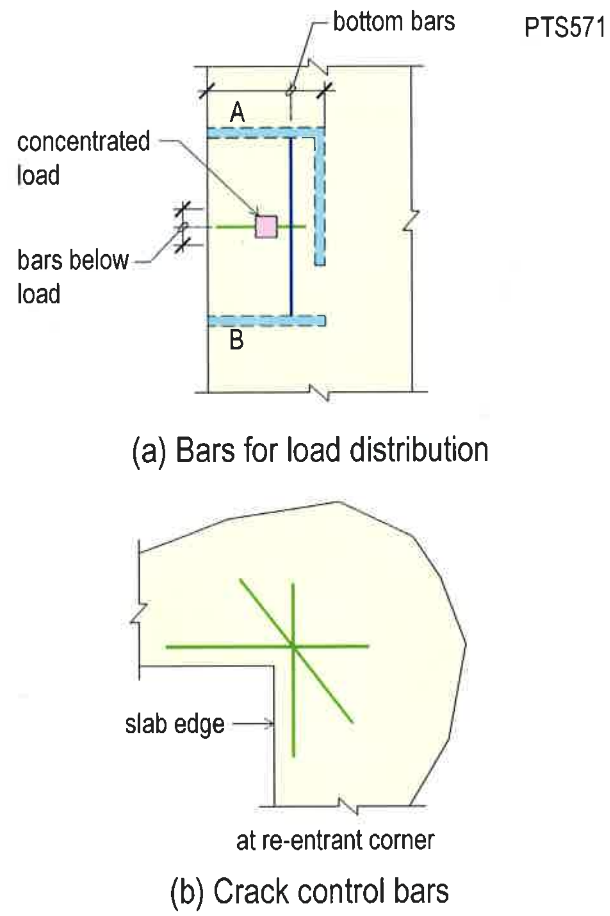


FIGURE 3.3B-4 Two Examples of "Structural Detailing"

report "total" amount of reinforcement for a design section and leave the distribution to the designer.

The reinforcement shown in part (b) of the figure is an example of "structural detailing" for crack control.

The detailing that represents the bottom bars shown in Fig. 3.3B-5 with the bar size, number, length and location of individual bars to be placed in the slab as shown in (a-ii) is referred to as "Construction Detailing". Another example of "Construction Detailing" is the selection of the correct lap splices, hooks, and bar bending details as illustrated in part (b) of the figure.

In North America, "Construction Detailing" is shown on fabrication (shop) drawings generated by the materials suppliers. "Structural Detailing," on the other hand, is done by the design engineer and is shown on the structural drawings. In many other parts of the world, however, there is no distinction between Structural Detailing and Construction Detailing. Unlike the practice in North America, the drawings generated by the design engineers also reflect the Construction Detailing.

The following general conclusions can be drawn about the design of concrete floor systems:

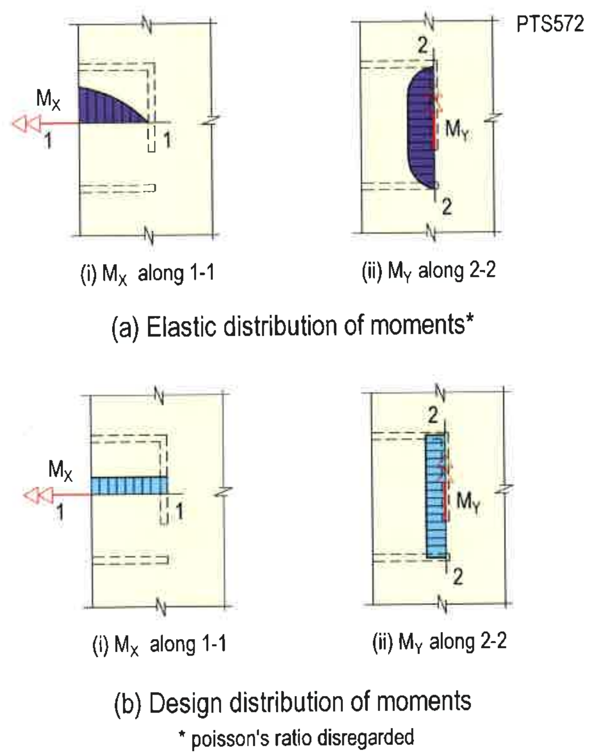


FIGURE 3.3B-6 Elastic Distribution and Design Moments

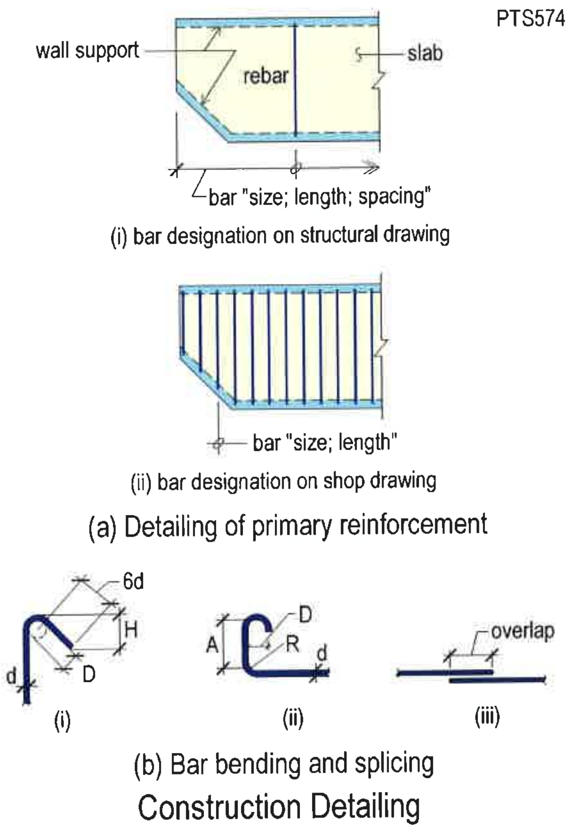


FIGURE 3.3B-5 Examples of Construction Detailing

❖ For concrete design, the engineer must designate a load path in order to determine the reinforcement. This is unlike the glass alternative, where the load path must be determined by analysis. The load path designation leads to the magnitude and layout of the primary reinforcing bars that define the resistance developed by the slab. Often, there is more than one possible load path. The load paths that are selected make up the skeleton of the "structural system" of the building.

❖ Safety of concrete design is not sensitive to local stresses. As an example, the distribution of moments determined from elastic theory will be similar to the schematics of Fig. 3.3B-6a. The simplified, equivalent moment shown in Fig. 3.3B-6b is generally used for reinforcement calculations, however. The reinforcement necessary for strength in each direction is that required to resist the total moment, i.e. the integral below the moment curve. For safety of the structure, the layout of the reinforcement is typically not critical, as long as the bars are within the region corresponding to the moment they are designed to resist. This is based on the premise that failure follows the formation of a hinge line and the hinge line will mobilize all of the reinforcement that crosses it.

(a) Slab reinforced with unbonded tendons (b) Slab reinforced with grouted tendons (Tehran)

FIGURE 3.4-1 Views of Post-Tensioned Floors (P219) (P551)

This highlights another feature of concrete design, namely the total moment is used for design. The distribution of the moment and local values of the moment are not critical. This feature places concrete at a great computational advantage since total (integral) values of the actions are not as sensitive to finite element discretization as local values. Finite element software is generally formulated to satisfy static equilibrium, regardless of the density of the mesh used to discretize the structure. A coarse mesh gives essentially the same "total" moment over a design section as a fine mesh. This observation is based on the premise that the finite element formulation used is specifically developed to properly determine the "total" values as is commonly used for design of concrete members (see Chapter 4, Section 4.9.5).

3.4 DESIGN CHARACTERISTICS OF POST-TENSIONING

The need to designate a "load path" was identified as one of the characteristics that differentiate concrete design from glass plate design. Post-tensioned concrete design (Fig. 3.4-1) adds another layer of complexity that requires additional engineering judgment and input. Consider the example of Fig. 3.3B-1a, in which the slab region is assumed to span between walls A and B. A post-tensioned design alternative of this region is shown in Fig.3.4-2. The region is reinforced with post-tensioning tendons between walls A and B; the post-tensioning is supplemented by mild steel as shown.

In addition to selection of load path, which in this case it is designated as a one-way slab spanning between walls A and B, two additional assumptions are required to complete the analysis. These are generally grouped into (i) tendon profile, and (ii) amount of prestressing. Tendon profile defines the shape of tendon along its path. Tendon profile includes the distance between the center of tendon and centroid of the slab along the tendon length. The amount of prestressing can be expressed in different forms, such as the number of strands, or the force provided by prestressing. Typically, the amount of nonpre-

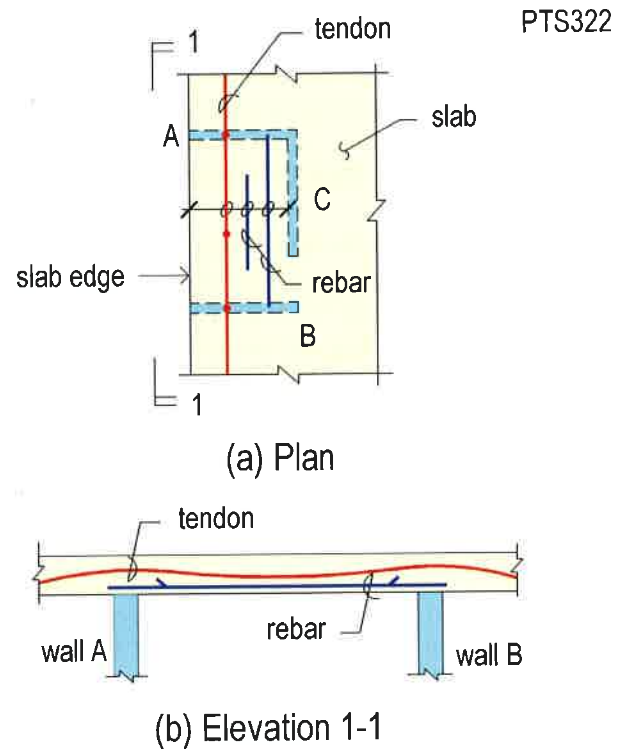


FIGURE 3.4-2 Display of a Tendon Layout Option

stressed reinforcement required in addition to prestressing tendons will depend on what is selected for cited parameters.

In a nonprestressed floor, determination of the required reinforcement is fairly routine, once the load path is determined. The results in terms of required reinforcement will be essentially the same, regardless of the designer's experience or inclination. In a post-tensioned floor, however, the engineer has considerable latitude when selecting the amount of prestressing and the tendon profiles. Depending on the engineer's assumptions and what he/she uses as design criteria, different designs will result.

A corollary observation is that for a given geometry, material properties and loading, there will be a single design outcome for a conventionally reinforced member, such as the one shown in Fig. 3.4-3a. However, for the post-tensioned alternative (part b of the figure), depending on the design engineer's entry value assumptions for (i) prestressing force and (ii) profile/shape, there results a multitude of designs. While all designs can meet the minimum requirements of the governing code, they can have different material quantities. Engineers with greater experience in post-tensioning design are likely to arrive at more economical outcomes. This leads to the recognition that there is an "optimization" aspect in design of post-tensioned members that is absent in the conventionally reinforced counterpart. Computer programs with optimization features become beneficial in design of post-tensioned members.

3.5 ANALYSIS AND DESIGN PROCESS

3.5.1 Analysis and Design Steps

The design process for concrete floors is summarized in the flow chart (Fig. 3.5.1-1). There are four steps: (i) structural modeling; (ii) analysis; (iii) design, and (iv) structural detailing. The structural modeling step covers the designation of load paths. The analysis step determines the actions (moments and shears) that each load path must resist. The design step gives the area of reinforcement required for each designated load path to resist the computed actions. And, finally, at structural detailing step the layout of the reinforcement is defined. Structural detailing also specifies the additional steel that is not determined by computation, but considered necessary for crack control or distribution of applied loads.

3.5.2 Structural Modeling

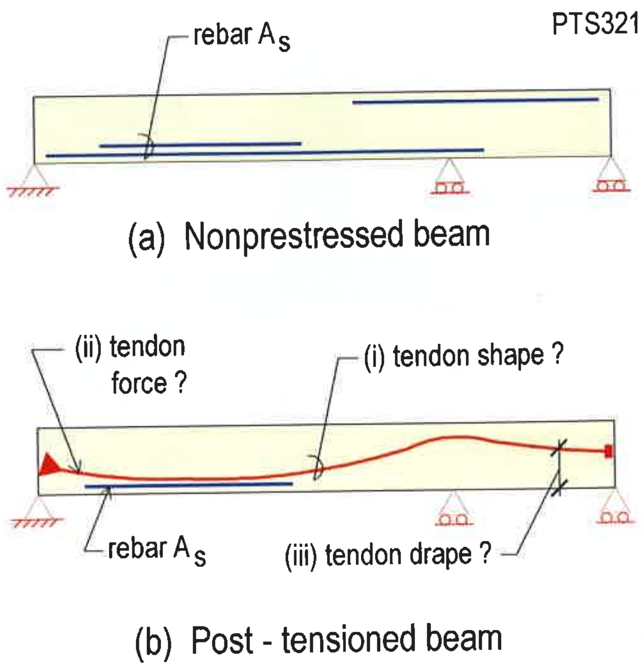


FIGURE 3.4-3 Design Information Required for a Post-Tensioned Member and Conventionally Reinforced Alternative

Structural modeling includes the designation of load paths. This defines the structural system to be analyzed. The structural modeling also includes the definition of how design values for reinforcement and stress check are to be determined. The structural modeling is followed by "analysis," at which stage the numerical values associated with the "structural modeling" are computed. The following explains the structural modeling.

Structural modeling starts with the designation of the load paths. The load path designation is complete, if the self-weight and applied loading at every location is assigned an explicit load path that leads to a support.

The strip method of load path assignment requires dividing the floor into support lines generally in two orthogonal directions. Each support line has its own tributary area. The support lines indicate the assumed load paths; a support line, together with its tributary area, is referred to as a "design strip."

For most structures, selection of the load paths is essentially independent of the analysis method. Consider a typical floor (Nahid floor) from a multi-story building with columns and walls above and below as shown in Fig. 3.5.2-1. The following describes the structural modeling of the floor and illustrates the procedure for

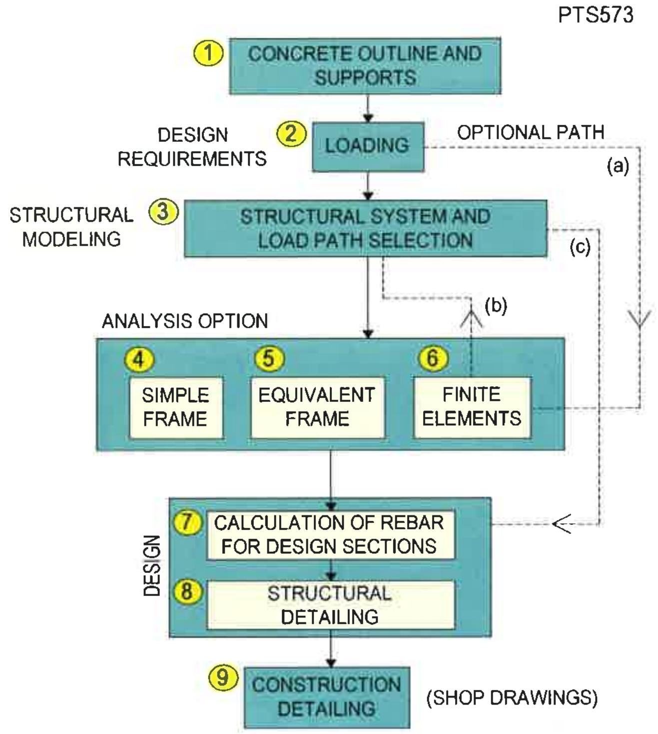


FIGURE 3.5.1-1 Overview of Design Process

selecting load paths and design sections.

A. Define outline of floor slab and supports: As a first step, the engineer defines the slab edge and any openings, steps or other discontinuities. Next, he/she identifies the location and dimensions of the walls and columns supporting the floor. The supports for this example are shown in Fig. 3.5.2B-1. Note that, beams are considered as part of the floor system rather than the support system. They are therefore modeled and designed contiguous and in conjunction with the floor slab.

B. Define Support lines: The engineer then determines a series of support lines in each of the two principal directions. Typically, these are lines joining adjacent supports along which an experienced engineer will intuitively place reinforcement. Fig. 3.5.2B-2 shows the support lines, labeled A through G, in the X-direction (F is not a designated line of support). Fig. 3.5.2B-3 shows the five support lines, labeled 1 through 5, in the Y-direction.

If a floor system is highly irregular, i.e. the columns are significantly offset from one another; the support lines may be less obvious. The criteria for selection are the same as in a regular slab, however. The support lines are the lines along which an experienced structural engineer is likely to place the primary reinforcement for resisting the gravity load.

The creation of support lines joining the physical location of supports is critical. Many provisions of the building codes such as minimum reinforcement, relate to checks specific to "'support" or "span." Detailing of reinforcement also depends on the position of support, and support width. Selection of a support line along the line of supports that also includes each physical support affords the identification of "span" and "support" for code compliance.

C. Define Tributary Areas and Design Strips: Typically, the midpoints between support lines are used to designate the tributary areas for each support line. The midpoints are joined to identify the boundaries of the tributary. Fig. 3.5.2B-1b shows the support lines in the X-direction. Points 8 and 9 would be used to determine the boundaries for the tributary of support line B, for example. The tributaries for the design strips in the X-direction are hatched in Fig. 3.5.2C-1. Figure 3.5.2C-2 shows the support lines in the Y-direction with their associated tributary areas.

While the selection of support lines along the line of supports is central to the identification of design strips for code compliance, the tributary associated with each support line need not necessarily extend to half the distance between adjacent support lines. The delineation of design strip widths can be adjusted to suit the design based on engineering judgment, or other construction restrictions. As an example, for the same floor an alternative design strip selection based on straight lines is discussed in 3.5.2H. Note that the modeling retains the support lines, but simplifies the allocated tributary delineations.

D. Design Sections: Design sections are typically drawn across each design strip at the locations

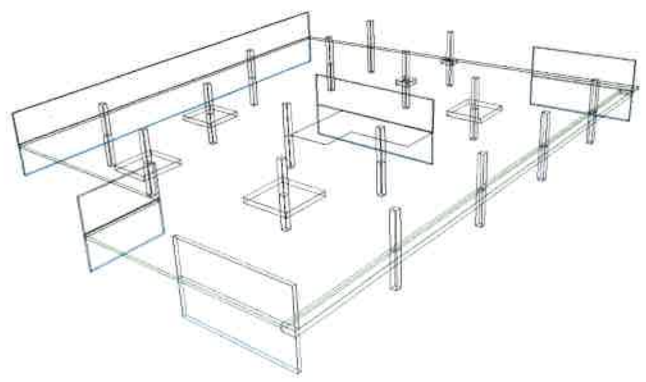
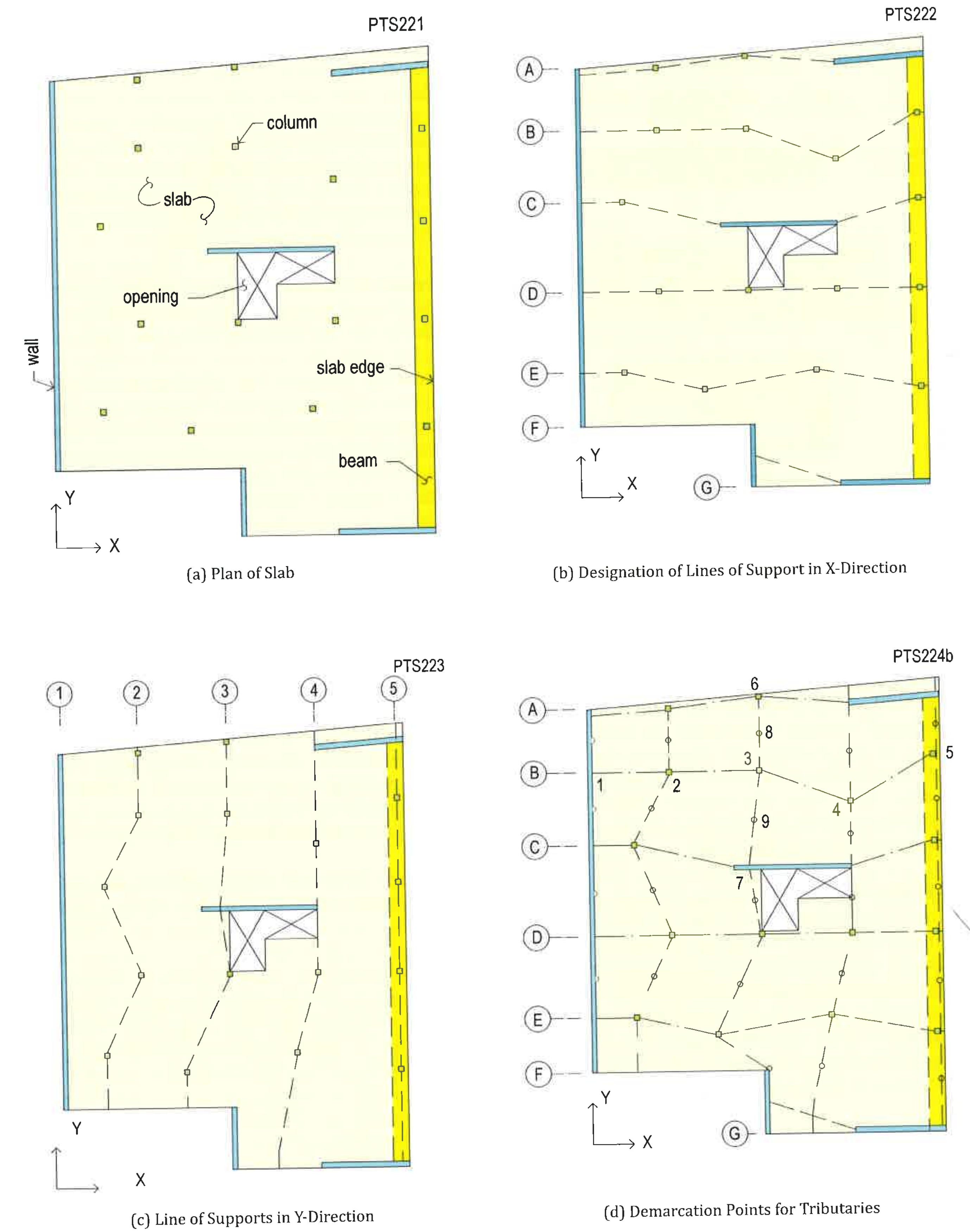


FIGURE 3.5.2-1 3D View of Nahid Floor (P472)



(a) Plan of Slab

(b) Designation of Lines of Support in X-Direction

(c) Line of Supports in Y-Direction

(d) Demarcation Points for Tributaries

FIGURE 3.5.2B-1 Nahid Floor and the Initial Steps in Structural Modeling

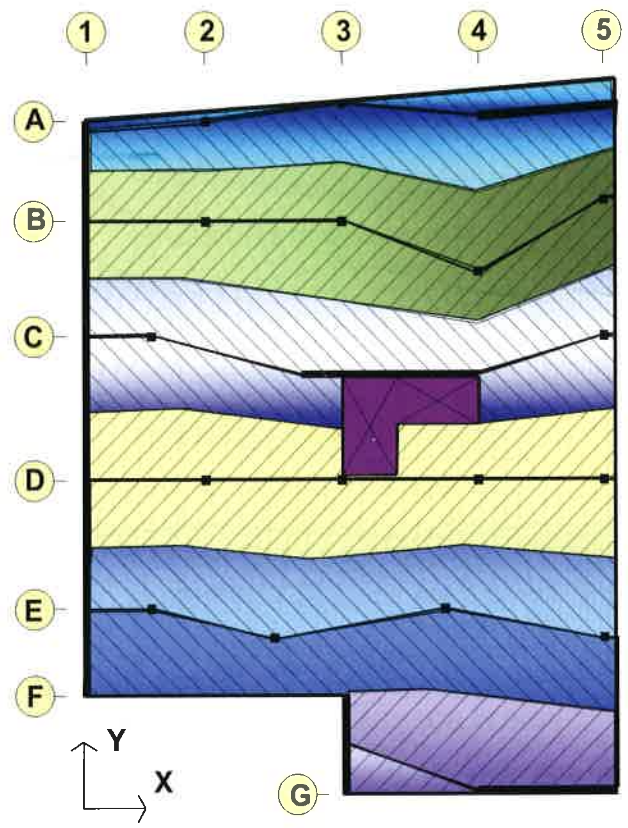


FIGURE 3.5.2C-1 Design Strips in X-Direction
(P653)

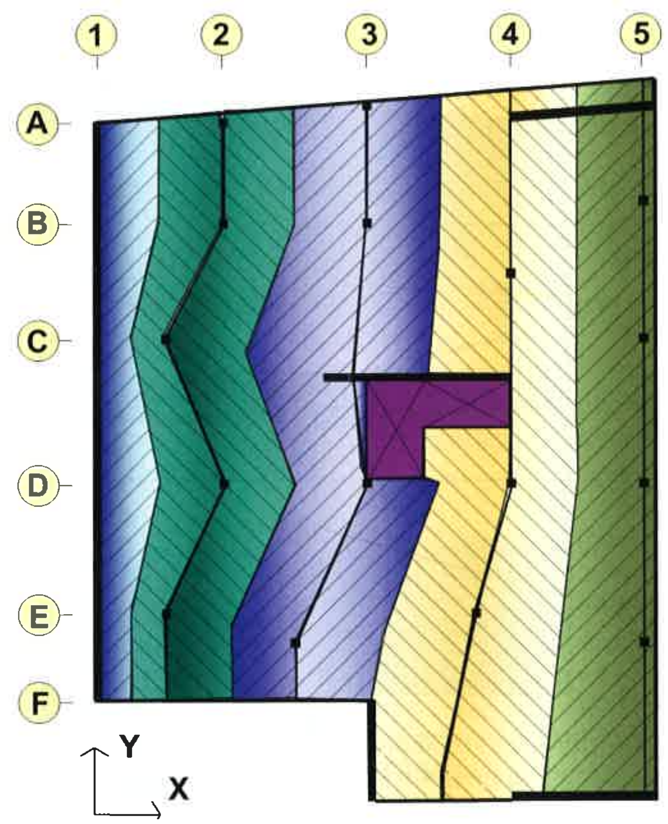


FIGURE 3.5.2C-2 Design Strips in Y-Direction
(P654)

where the integrated actions on the design strip are greatest. There is no limit to the number of design sections that can be specified. Note that, the maximum design actions in the field may not be at the midpoint of the spans. In addition, peak design actions for the strength and serviceability checks may not occur at the same location. Figure 3.5.2D-1 shows selected design sections for two of the design strips in the X-direction. For hand calculations, where design sections will be limited in number and possibly based on engineering judgment, design sections are typically limited to those at the face-of-support and midspan as shown in Fig. 3.5.2D-1.

When using an automated design, a larger number of design sections are made along each span, in order to capture the maximum value of design actions, thus providing a more appropriate layout for the required reinforcement (Fig. 3.5.2D-2)

E. Analysis Methods: In the analysis step, the response of the floor to the applied load is determined. The floor's response is used to arrive at the actions that each of the design strips need to resist. There are two basic methods of analysis, namely "Frame Analysis," and "Finite Element Analysis." The frame analysis is carried out either using "Simple Frame," or "Equivalent Frame."

E1. Frame Analysis: In both the Simple and Equivalent frame options, each design strip is extracted from the floor and re-constructed with appropriate support conditions and loading to create an approximated frame model [ACI 318-11]. Each design strip is analyzed as an independent structural system, isolated from the adjacent design strips.

Consider design strip B, shown as a separate entity in plan (Fig. 3.5.2E-1) and in elevation (Fig. 3.5.2E-2). For plane frame analysis, the strip is straightened along its line of support as illustrated in Fig. 3.5.2E-1 (b). The span length thus corresponds to the slant distance between adjacent supports. Note that the tributary widths may vary over a single span. To simplify the analysis, these varying tributary lines are typically idealized as straight boundaries (Fig. 3.5.2E-1c). Usually, the idealized tributary is chosen to be conservatively larger than the actual tributary.

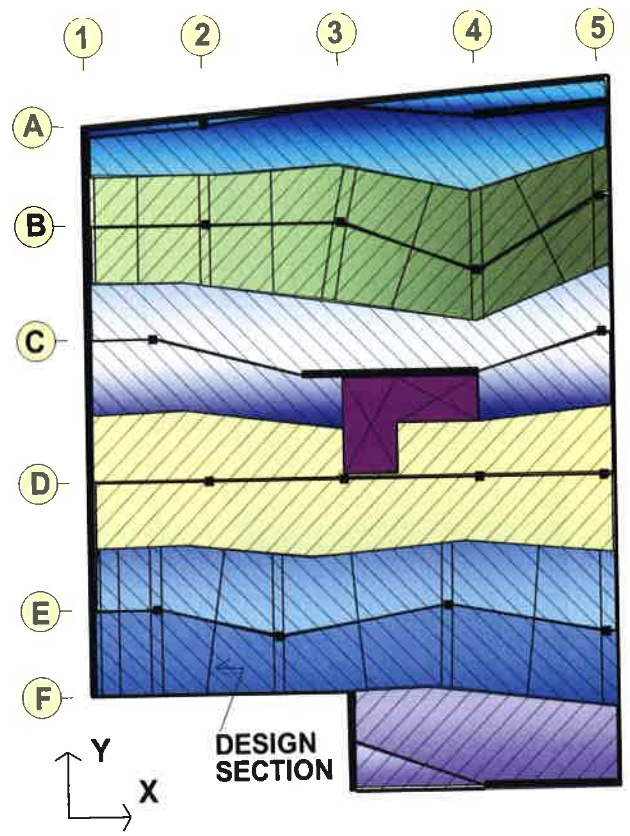


FIGURE 3.5.2D-1 Selection of Limited Design Sections (P655)

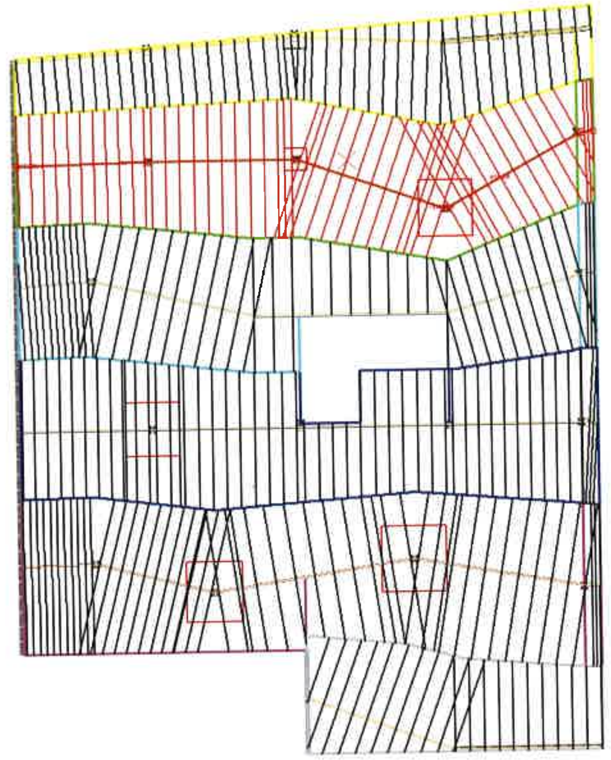


FIGURE 3.5.2D-2 Software-Generated Multiple Design Sections (P222)

If the change in tributary width in any given span varies by more than 20%, it may be worthwhile to model the tributary as a series of steps to reduce the reinforcement required. Additional approximations may be necessary for other non-standard conditions.

In the Simple Frame Method (SFM) the stiffness of the isolated frame is based strictly on the cross-sectional geometry of its parts as illustrated in Fig. 3.5.2E-1c and Fig. 3.5.2E-2. The Equivalent Frame Method (EFM) features a modification in the stiffness of the members that is intended to better approximate the biaxial nature of the response of the entire floor system that is lost in the unidirectional modeling of the design strip. The differentiating details of the two schemes are expounded in Chapter 4, Section 4.9.2.

E2. Finite Element Analysis. As noted above, if either the Simple Frame Method or the Equivalent Frame Method is used, each design strip must be extracted from the floor system and analyzed as a plane frame. With the Finite Element Method (FEM), the entire floor can be analyzed at one time.

The Finite Element Method (FEM) is based on the division of the structure into small pieces (elements) whose behavior is formulated to capture the local response of the structure (Fig. 3.5.2E-3). Each element's definition is based on its material properties, geometry, location in the structure, and relationship with surrounding elements. The mathematical assemblage of these elements into the complete structure allows for automated computation of the response of the entire structure. FEM inherently incorporates the biaxial behavior of the floor system when determining the actions in the floor.

The information required for geometry, loading and boundary conditions is the same for the Simple Frame, the Equivalent Frame and the Finite Element methods. The first two methods are more approximate than the Finite Element Method. However, all three yield lower bound (safe) solutions. Generally, the EFM results in lower column moments.

The results of an FEM analysis must be processed as "design strips" and "design sections" for code stipulated serviceability and strength checks, however. As with the frame methods, the design strips are based on the assumed load paths. The design strips do not need to be selected before the analysis though. Figure

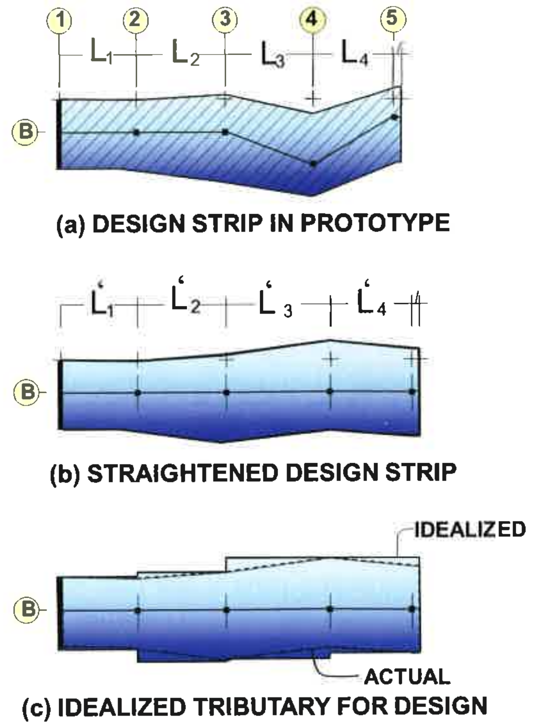


FIGURE 3.5.2E-1 Extraction and Idealization of a Design Strip (P656)

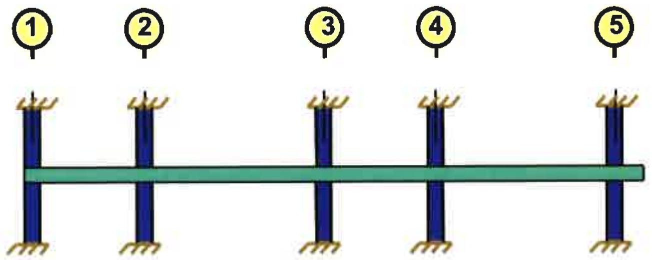


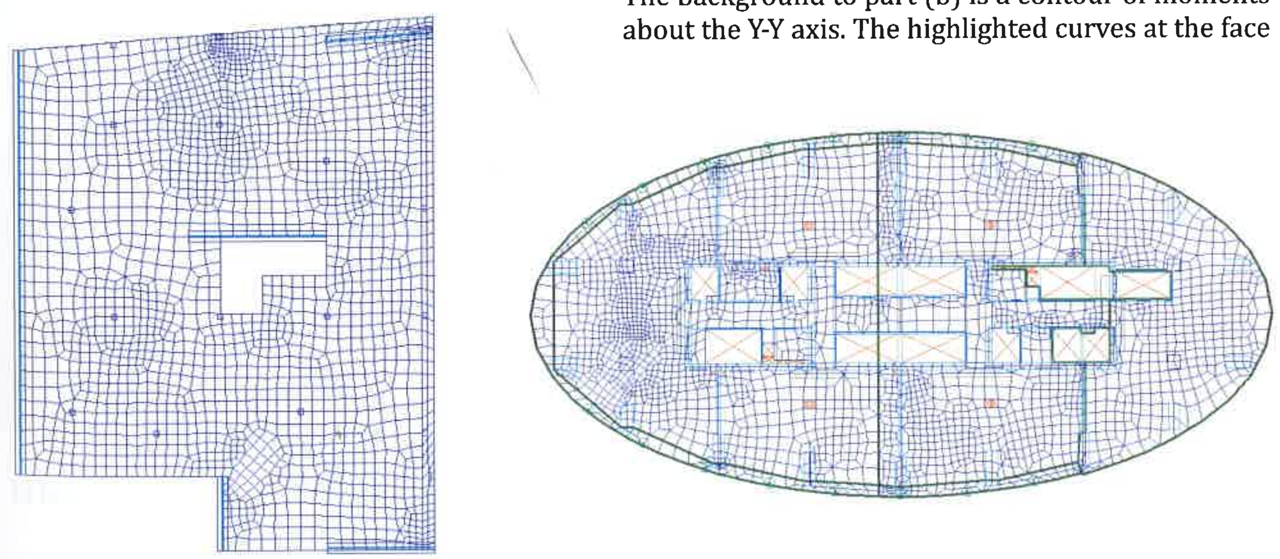
FIGURE 3.5.2E-2 Elevation of Design Strip for Analysis (P657)

3.5.2E-3 illustrates two examples of floor system discretization for finite element analysis, using adaptive quadrilateral well-proportioned finite element cells.

Not all the finite element formulations are the same and deliver the same results. Neither extraction of design values from a finite element analysis follows a unique procedure. Depending on the formulation for analysis and post processors for determination of design values, such as moment and shear, different values will be reported. Chapter 8 offers a discussion on this topic.

F. Determination of Design Actions: To evaluate the code compliance of each design section for its in-service (SLS) and safety (ULS) conditions, the design actions on each design section will be calculated. When using the frame method of analysis, the solution directly gives the total value of each of the actions that a design section has to resist. However, the raw solution from a finite element analysis gives the values of the actions at the node of the finite element cells scattered over the entire floor system. The design values for each design section have to be extracted from the solution at the nodes.

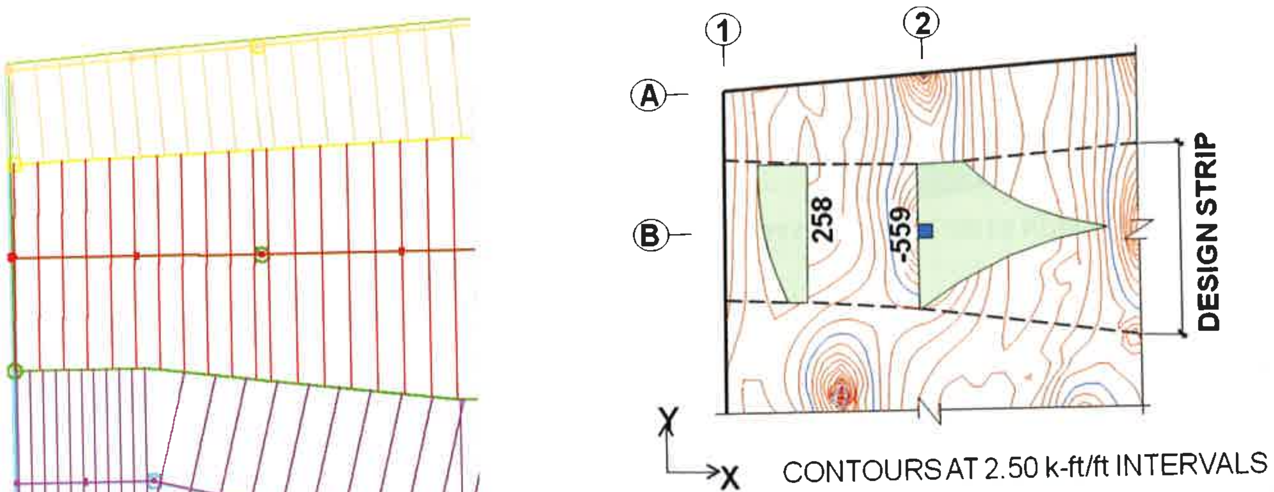
Refer to Fig. 3.5.2F-1. It illustrates the top left of Nahid floor shown in Fig, 3.5.2D-2. Part (a) of the figure shows the associated design strip and design sections. Design sections are at the face of the column support and at close interval along the span. The background to part (b) is a contour of moments about the Y-Y axis. The highlighted curves at the face



(a) Discretization of Nahid floor (P223)

(b) Discretization of Shams, (KSA; P224a)

FIGURE 3.5.2E-3 Examples of Floor System Discretization for FEM Analysis



(a) View of design sections (P497)

(b) Distribution of moment along selected design sections and their integral values (P498)

FIGURE 3.5.2F-1 Partial Top Left Plan of Nahid Floor

of the support and midspan show the distribution of the moment along the respective design sections. For design and code compliance, the distributed values of actions along each of the design sections has to each be integrated to its resultant that acts on the design section. The values (-559, 258) shown in part (b) of the figure, each is the integral (sum) of the associated distribution of the moment.

In general, the outcome of the distribution of actions from a FEM analysis on a design section can be reduced to six resultants, three forces and three moments, all expressed at the centroid of the design section. Figure 3.5.2F-2 is an example of a design section, showing its geometry and the centroid at which the resultant of the distributed actions will be expressed for design and code compliance check. Note that the resultant of the actions on a design section is a force R that acts at a point in space "A." For the purpose of design, the force R in transferred to the centroid of the design section at O and resolved into its six components.

In real structures, design section will not always be in shape of simple rectangles. Steps, cut-outs, slab bands and beams result in a more complex geometry as exemplified in the figure. However, regardless of the complexity of the geometry, the distribution of the actions on each is reduced to the six items at the respective centroid for design. Options and details for the determination of design section forces are given in Section 4.9.5.

G. Design: At design stage, the integrated values of the actions at the centroid of each design section are applied to the entire geometry of the respective section for both the serviceability (SLS) and strength (ULS) requirements of the respective building codes. This includes the determination of farthest fiber stress values for crack control, minimum code required reinforcement, as well as reinforcement for strength of the section.

For example, the moment used for the determination

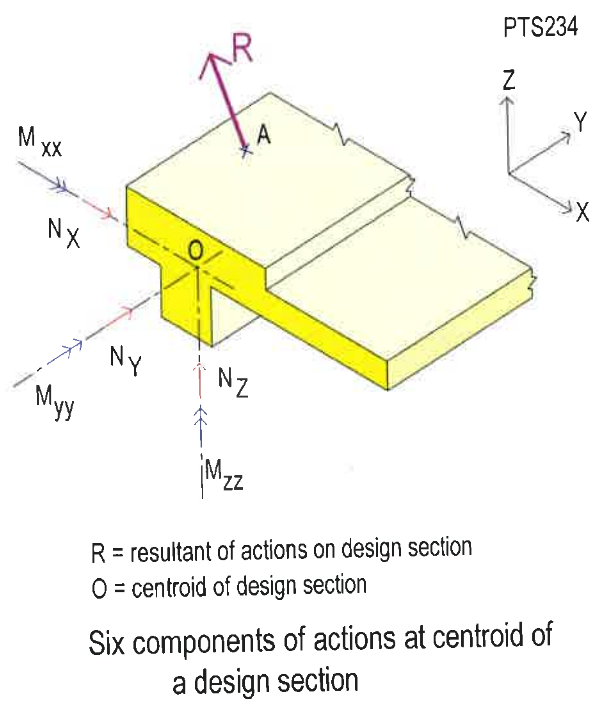
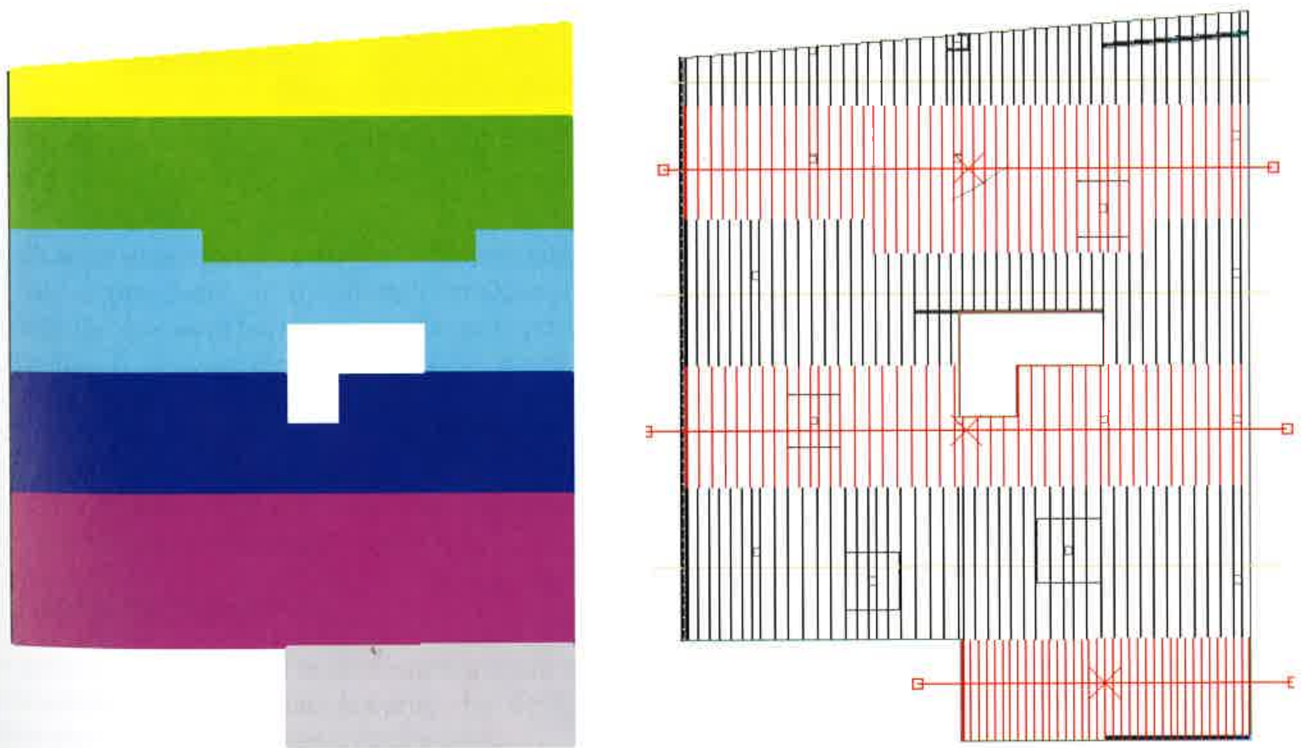


FIGURE 3.5.2F-2 View of a Design Section and Associated Forces

of the code-intended farthest fiber stresses and reinforcement at each design section is the area (integral) of the moment distributions shown in Fig. 3.5.2F-1 At the face of the interior support , the design moment is Myy = 559 k-ft (757.89 kNm). At design stage, this integral value is applied to the gross cross-sectional geometry of the design section to determine the hypothetical stresses for code compliance. Likewise, factored integral value is applied to the entire cross-section to calculate the required reinforcement. It is recognized that the computed value is "hypothetical," and when used in conjunction with the code recommended limits is deemed to result in a "serviceable," floor slab. The stresses at a point in a floor system will be different. At many points the local stresses will be higher. In Concrete Society's Technical Report [TR43, 2005], there is an option of selecting half the width of the tributary and use it with higher values of allowable stress. However, since the outcome of the code check is worked out to be the same, there is no apparent advantage in the application of a design strip different from the total width.

H. Remarks on Design Strip Configuration: The breakdown of a floor system into design strips described in the preceding is based on the requirements of building codes for serviceability limit state (SLS) and ultimate limit state (ULS) checks. For an example, the stress check for SLS is based on a hypothetical extreme fiber value that is derived from the forces in a slab applied to the slab width that is tributary to a support line; the amount of minimum reinforcement to be placed over a support is based on the cross-sectional area of a slab that is tributary to a support line. These and similar provisions lead to the necessity of using design values that are derived from "support lines," and the associated "tributaries." The combination leads to the breakdown of a floor system into "design strips." The breakdown of a floor system into design strips that do not associate with a support line and its tributary does not lead to proper check for code compliance. However, the subdivision of a floor to strips that closely match the tributary is considered acceptable, provided the modeling covers the entire floor surface (Fig. 3.5.2H-1). The latter follows the practice of design strips that are extracted from a floor system and analyzed in isolation, where the strips are idealized into straight edges. The subdivision of a floor system into design strips with parallel edges and parallel design sections is acceptable (Fig. 3.5.2H-1).



(a) Squared design strips (P499a)

(b) Parallel design sections (P499b)

FIGURE3.5.2H-1 Option for Creation of Parallel and Square Design Strips and Design Sections

3.6 REFERENCES

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CHAPTER 4

DESIGN CONCEPTS AND PROCEDURES

Post-Tensioned Building under Construction (US; P500)

This Chapter presents the concepts and procedures that form the basis of the design practice of concrete floors – with particular reference to post-tensioning. It reviews the information you need, and are likely to draw upon, when you design either a conventionally reinforced, or post-tensioned floor system. The information provided in this Chapter is applied to the longhand numerical examples offered in Chapters 6 and 7.

4.1 PRINCIPAL OBJECTIVES

The principal objectives of most designs are safety, serviceability, economy and keeping the design within the bounds of the legal requirements.

4.1.1 Safety – Ultimate Limit State (ULS)

Safety is established by illustrating that the designed floor can withstand the code stipulated overload without collapse. Clearly, a soundly designed structure may fail, or sustain disproportionate damage, under excessive and catastrophic loading, not envisioned in its code-based design. Hence, "safety" and damage are not absolute. Safety is a guarantee of a defined level of performance within the range of loading specified in the applicable code.

To establish "safety," it is sufficient to demonstrate that under all of the code stipulated loading conditions the structure can develop an uninterrupted load path - from the point of load application to the foundation - capable of sustaining the applied load and all corresponding actions generated in the structure. Towards this effort, the common design procedure for safety aims to ensure two criteria - that there exists a load path that is *adequate* and it would be *mobilized*.

The *adequacy* of a load path is achieved by ensuring that, at any point along its length, the load path can withstand the actions calculated at that point. In design practice, adequacy is determined for only one engineer-selected load path, generally referred to as the "structural system." The engineer-selected load path is a "designated" system, in the sense that the natural load path of a floor may be different from the path selected by the engineer. The designated load path may provide an acceptable design, so long as the engineer can demonstrate its adequacy.

From standpoint of savings in material, a structure's natural load path, commonly known as *natural (elastic) response*, can lead to an economical design. However, other considerations, such as selection of construction technique, or design procedure, may favor alternative load paths. While the natural load path is of academic interest, and helps to form a sound engineering judgment, it is neither required for a successful structural design, nor it is commonly practiced.

To ensure safety, the designated load path must develop on demand. Should the natural load path of a floor system-- reflected in the initial elastic response to loads proves inadequate, the ensuing cracks are deemed to re-direct the loads to the paths, where resistance is provided. The ability of a structure to crack, deform, and re-direct (re-distribute) the actions is a characteristic of the structure's *ductility*. A central part of the engineer's work, in conjunction with a designated load path, is to ensure that the structure has adequate ductility.

In concrete design, ductility is achieved by controlling the reinforcement in a design section. Using code specified limits, the design engineer ensures that the reinforcement is adequate, and that it yields well before crushing of concrete in compression. Early yielding of reinforcement results in loss of local stiffness and diversion of demand for resistance to other regions of a floor system.

The procedure for the selection of load path for floor systems is detailed in Chapter 3.

4.1.2 Functionality – Service Limit State (SLS)

The functionality of a design is generally referred to as "serviceability" in the language of building codes. The primary premise of serviceability is that, when in use, the structure meets the functionality expectations of its users, and requires reasonably low maintenance over its expected life span. Items generally checked to ensure the serviceability of a floor slab are:

A. Deflection: Deflection is controlled for (i) the comfort of occupants, (ii) the proper operation of installations, such as counter tops and doors, and (iii) to avoid damage to non-structural members that are likely to be affected due to excessive deformation. Deflection control also serves to maintain an occupant's perception that a structure is safe. An excessively sagging floor may undermine the confidence of its occupants as to the adequacy of the safety margin, even when the structure meets all safety requirements.

B . Cracking: Cracking can be unsightly if it is visible, wide and extensive. Concrete cracking may also expose the reinforcement to corrosive elements and/or lead to leakage. It may also give the impression of poor workmanship, structural weakness and inadequate safety.

Cracks do not generally compromise the safety of a floor system, provided all design requirements for strength are met.

C. Durability: Depending on the location and exposure of the floor slab, the design must meet the life expectancy of the structure. Several of the important factors affecting the durability of the structure are the selection of material, concrete cover provided for the reinforcement, the concrete's density and composition, the presence of cracks, and exposure to corrosive elements. When using post-tensioning, durability can also be affected by deterioration of improperly selected and executed post-tensioning components.

D. Vibration: Improperly designed long spans, coupled with inadequate stiffness may lead to floor vibrations that are perceived by occupants and considered undesirable. Floor vibration is controlled to be within the acceptable range of perception and objection of a floor system's occupants [ADAPT-TN290, 2013]

E. Fire resistivity: Depending on its function, a floor system must withstand a minimum, code-specified lapse of time - expressed in hours - of direct exposure to fire prior to failure. Minimum member dimensions for fire endurance, and cover to reinforcement are the primary means of controlling the fire resistivity of a concrete member.

F. Maintenance, Sustainability, Carbon Footprint: With increased awareness and emphasis on environmental concerns, and sustainability of our activities, the "carbon" footprint of the material used in construction is becoming a design consideration. In parts of the world[1], it is a requirement for a building permit to qualify for a "GreenPoint" rating that reflects the environmental impact of the construction materials used, including energy saving efforts.

4.1.3 Economy

Construction cost, maintenance and the useful life of a structure are important factors in determining a design's economic achievement. Economic decisions are also influenced by local practice, local availability of construction materials, equipment and labor, and the contractor's expertise in construction technology. Consequently, an attempt to optimize a design for minimum material usage alone, without consideration for these other factors, is usually not the most rational approach. All inclusive integrated economic solutions which consider the entire cost factors of a construction are becoming more feasible through application of BIM[2].

4.1.4 Legality

In the litigious environment of the United States, prudence demands that design efforts be in line with applicable building codes and common building practice, as much as practical. However, for post-tensioned building construction, this obvious recommendation might not be a simple task to follow.

Unlike many other examples of innovative construction technology, the current practice in post-tensioning design has not yet been fully implemented into prevailing building codes. This deficiency is most likely due to the unique environment in which the technology evolved.

A. Post-Tensioning in USA: The development of post-tensioning in building construction in the United States in the early 50's was the effort of a handful of pioneers in building construction. The technology was not based upon the results of concerted research efforts, nor regimented studies. Rather, the technology evolved from the empirical knowledge developed by innovative engineers, as construction proceeded. Many aspects of the practice were extensions from conventionally reinforced counterparts and intuition of the early designers. Confidence on validity of practice was derived from the construction seeming fulfilling its objectives, as opposed to necessarily having been based on scientific or engineering principles. Subsequent research at the universities validated many aspects of the practice. Numerous post-tensioned projects were constructed, before work on the respective code[3] and technical publications were initiated. The code and publications have mostly lagged behind industry practice ever since.

While work on the published material and the code are in progress, they do not yet cover all the critical aspects of the current practice in design of post-tensioned building structures. The innovation in design, hardware and construction has continued and are helping to keep this industry competitive. In short, when it comes to post-tensioning design, there is a notable gap between what is reflected in the building code and viewed by as "legal," and the "standard of practice," and more importantly, by what can be supported by sound "engineering principles."

B. Post-Tensioning and Euro-Code: The emphasis of the Euro-Code EC2[4] is on the general requirements and design principles, as opposed to prescriptive instructions on practical design of building structures. As such, it is more difficult than ACI-318 for typical design engineers to comfortably navigate through EC2 and conclude with a detailed design. Section 4.10.3 of this book provides a detailed flow chart for design of post-tensioned floor systems using EC2 provisions.

4.2 MATERIALS

This section covers the materials that are commonly used in construction of post-tensioned

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[1] Such as City of Palo Alto, in California
[2] BIM Building Information Modeling
[3] ACI 318 -11
[4] EC2 (EN 1992-1-1;2004)

buildings, including transfer plates and mat (raft) foundations. The objective is to list the range of commonly used materials, intended to assist engineers in their first selection of design parameters. Detailed material information is available in text books [Nawy 2000; Collins et al 2000]. In the following, where a range of values are listed, the recommended choice is shown in square brackets, thus [ ].

4.2.1 Concrete
Cylinder 28-day strength f'c; $(f_{ck})^5$ = 4000 to 6000 psi; (28 to 40 MPa)
Weight (W) = 150 pcf; (24 to 25 kN/m³)
Modulus of Elasticity

- Using ACI-318, E = $W^{1.5}33\sqrt{f'_c}$ = 3834 to 4696 ksi (26435 MPa to 32378 MPa)
- Using EC2, E = $22 \times 10^3 \times [(f_{ck}+8)/10]^{0.3}$ = 32308 to 35220 MPa

Creep Coefficient= 1.5 to 2.5 [2]
Ultimate Shrinkage value = 400 to 550 micro strain [400]
Poisson's ratio = 0 to 0.2 [0.2]

4.2.2 Prestressing Steel

A. Strand: seven wire; low relaxation
Guaranteed Ultimate Strength = 270 ksi; (1860 MPa)

- More common: nominal diameter - 0.5 in (12 to 13mm) [0.5 in; 13mm] Effective area - 0.153 in2 (99 mm2)
- Less common: nominal diameter - 0.6 in (15mm) Effective area - 0.217 in2 (140 – 150 mm2) [140 mm2]

Modulus of Elasticity = 19000 ksi (200,000 MPa)

B. Unbonded System:
Angular coefficient of friction, μ = 0.07
Wobble coefficient of friction, K = 0.0014 rad/ft (0.0046 rad/m)
Effective force after all losses[6] = 175 ksi (1200 MPa)

C. Bonded (Grouted) System: Using corrugated or flat sheet metal ducts
Angular coefficient of friction, μ = 0.25
Wobble coefficient of friction, K = 0.0014 rad/ft (0.0046 rad/m)
Effective force after all losses[7] = 163 ksi (1100 MPa)

D. Stressing:
Jacking force = 0.75 – 0.85 times the ultimate strength [0.85]
Anchor set (wedge draw-in) = 1/4 in (6mm)

4.2.3 Non-Prestressed Steel
Yield stress = 60 ksi (420 – 460 MPa) [460 MPa]
Modulus of elasticity = 29,000 ksi (200,000 MPa)

4.3 SIZING

As a general rule, to take full advantage of post-tensioning, the geometry of a structure, such as slab thickness, and support spacing is best to be selected such that the minimum prestressing requirements of the code would not govern the design. An economical design is one for which the reinforcement provided is necessary in its entirety to meet the "strength" requirements of design. In many instances, the amount of prestressing and the supplemental non-prestressed reinforcement necessary for service condition exceed the strength requirements of design. Such designs do not fully utilize the entire reinforcement for the safety check of the structure.

4.3.1 Support Spacing
In an optimum design the combination of the provided prestressed and nonprestressed reinforcement is fully utilized to meet the strength requirements of design (ULS). The best ratio of prestressed to non-prestressed reinforcement however, varies from country to country depending on the relative in-place cost per unit weight of each. Depending on the country, the unit weight cost ratio of prestressing to non-prestressed steel in place varies from 1:1 to 4:1, making it impractical to arrive at a universally applicable sizing parameters.

In general common residential and commercial building designs benefit from the following optimum dimensions:

5 Where 28 day 200 mm cube strength is used, divide the cylinder strength by 0.8
6 For stressing lengths described in Chapter 6, Section 6.1
7 For stressing lengths described in Chapter 6, Section 6.1

For solid slabs:

- Interior span length = 26 to 33 ft (8 to 10 m)
- Exterior span length = 20 to 26 ft (6 to 8 m)

The addition of overhangs leads to an economical increase of exterior span lengths.

4.3.2 Slab Thickness
For flat slab floor systems supported on columns, well established guidelines for selection of slab thickness are the PTI recommended span/depth ratios [PTI,1985] reproduced in TABLE 4.3.2-1. Where vibration is not a critical criterion, such as in parking structures thinner slabs may be selected. Excerpt from a similar table [TR43, 2005] is reproduced in Table 4.3.2-2. Other recommendations are available in the literature of post-tensioning hardware suppliers.

Where perceived vibration is not critical, such as in parking structures, a slab thickness of 4.5 in (114 mm) for a 17 ft (5.18 m) span giving a span to depth ration of just over 45 has been used successfully.

4.3.3 Beam Dimensions
TABLE 4.3.2-1 also includes recommendations for beam dimensions. Beams are generally economical, when the aspect ratio of a panel exceeds two. For panels with smaller aspect ratio flat slab construction, or flat slab in combination with slab band (Section 4.6.2) are generally more economical.

For example, in parking structures with a layout common in the United States parallel beams are typically spaced17-21 ft (5 to 6.5m) and span 60 to 65 ft (18 to 20 m). For this layout, the panel aspect ratios are typically between 3.5 to 4.

Depth = 30 to 36 in (760 to 915 mm); span/depth ratio approximately 20 to 25
Width = 14 to 16in (355 to 405 mm)

A beam depth of 30 in (762 mm) is more common.

The geometry of a typical parking structure frame in the USA using beam and one-way slab construction is illustrated in Fig. 4.3.3-1.

4.3.4 Common Sizing Examples
The following figures illustrate samples of common dimensions for buildings and parking structures using flat slab construction.

4.4 DURABILITY

4.4.1 Exposure to Corrosive Elements
Corrosion protection is achieved through a combination of one or more of the following items: (i) cover to post-tensioning tendons, (ii) width control of probable cracks, (iii) selection of appropriate post-tensioning system, (iv) reinforcement coated with protective film (epoxy coated), and (v) composition and placing of concrete. The following covers the first three items listed.

4.4.1.1 Cover to Reinforcement: Concrete cover, used as protection for reinforcement against weather and other effects, is measured from the concrete surface to the outermost surface of the steel or prestressing duct. Where stirrups enclose the longitudinal reinforcement or prestressing ducts, such as in beams, the minimum cover specified is to the outer surface of the stirrup.
The minimum specified cover depends on whether the concrete surface is "exposed to weather" or not. "Exposure to weather" means direct contact

TABLE 4.3.2-1 Recommended Span/Depth Ratios (T124)

	Continuous spans		Simple spans	
	Roof	Floor	Roof	Floor
One-way solid slabs	50	45	45	40
Two-way solid slabs (supported on columns only)	45-48	40-45		
Beams	35	30	30	26

Note: The above ratios may be increased if calculations verify that deflection, camber, and vibrations are not objectionable.

TABLE 4.3.2-2 Recommended Span/Depth Ratios [TR43, 2005] (T125)

	Total imposed load		Span/depth ratio
	kN/m^2	psf	6 m < span<13m 20' <span < 40'
Flat slab	2.5	50	40
	5	100	36
	10	200	30
Slab with drop panel*	2.5	50	44
	5	100	40
	10	200	36

* Minimum drop panel dimension span/3 on plan, extending not less than ¼ of slab thickness below soffit.

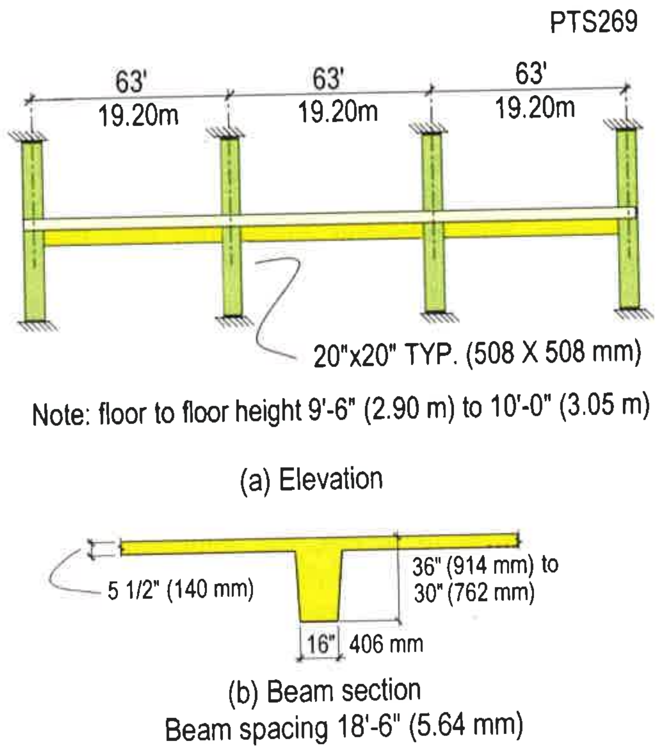
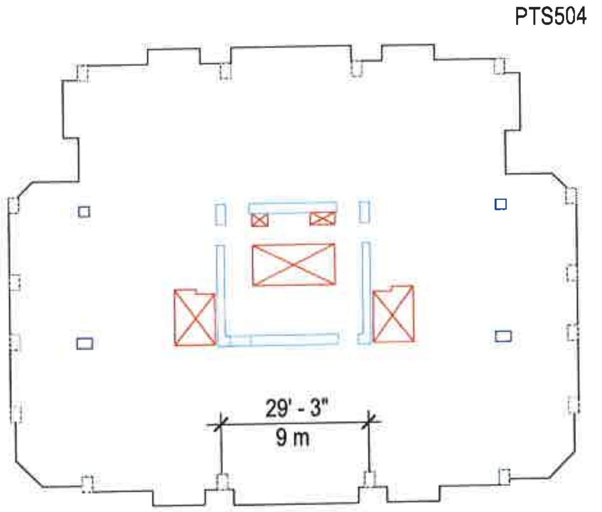


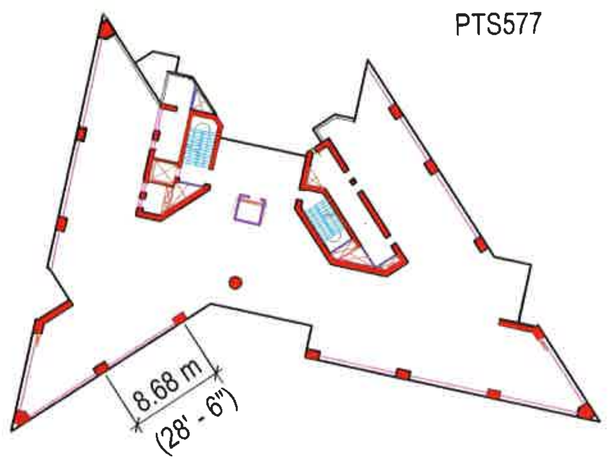
FIGURE 4.3.3-1 Geometry of a Typical Parking Structure Frame in USA

to both moisture and temperature changes; not just to temperature changes. Slab and beam soffits are not considered as "exposed," unless subjected to "wetting," including condensation. Also, if alternative methods of corrosion protection are provided, the minimum cover requirements may be waived.

Table 4.4.1.1-1 lists the minimum cover for prestressed slabs.



(a) Typical floor plan of Diesel Tower (San Francisco)



(b) Typical floor of Taazez Tower (KSA)

FIGURE 4.3.4-1 Floor Plan Examples of Contemporary Post-Tensioned Multi-Story Buildings

4.4.1.2 Crack Width Control: Several major building codes, such as EC2 specify a "design crack width" not to be exceeded, when the structure is in service. The "design crack width" is selected based on the likely exposure of the member to corrosive elements (Table 4.4.1.2-1). The following excerpt from the European Code[8] illustrates the concept.

- Cracking shall be limited to an extent that will not impair the proper functioning or durability of the structure or cause its appearance to be unacceptable.

[8] EC2 EN 1992-1-1:2004, Section 7.3.1

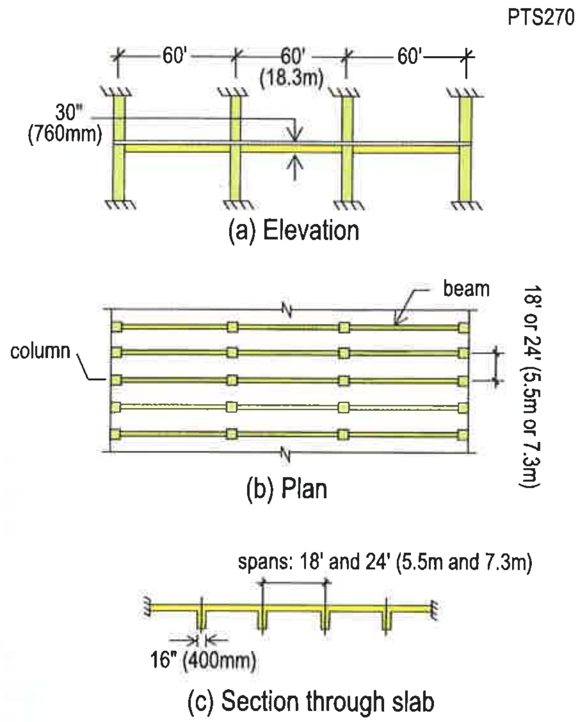


FIGURE 4.3.4-2 Typical Dimensions of a Post-Tensioned Beam and Slab Parking Structure Frame in USA

- Cracking is normal in reinforced concrete structures subject to bending and shear, torsion or tension resulting from either direct loading or restraint or imposed deformations.
- Cracks may be permitted to form without any attempt to control their width, provided they do not impair the function of the structure.

EC2 further stipulates that, for each design a limiting crack width, w_{max} (herein referred to as "design crack width") that accounts for the function and the ambient environment of the structure, as well as costs of limiting cracking, should be established.

The "design crack width" is the average width of cracks that are likely to form, as opposed to the maximum observed width. The design for a given crack width accounts for the particulars of the reinforcement, its placing, properties of the concrete material and the applied forces when the structure is in-service.

TR43[9] recommends the following crack widths, depending on the post-tensioning system selected irrespective of the structure exposure.

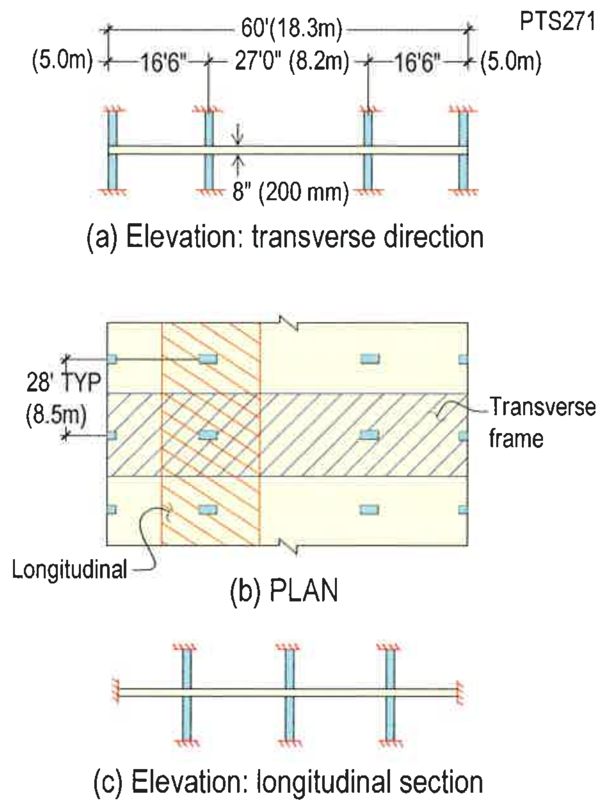


FIGURE 4.3.4-3 Typical Dimensions of a Post-Tensioned Flat Slab Parking Structure in the USA (The figure also displays the design strips)

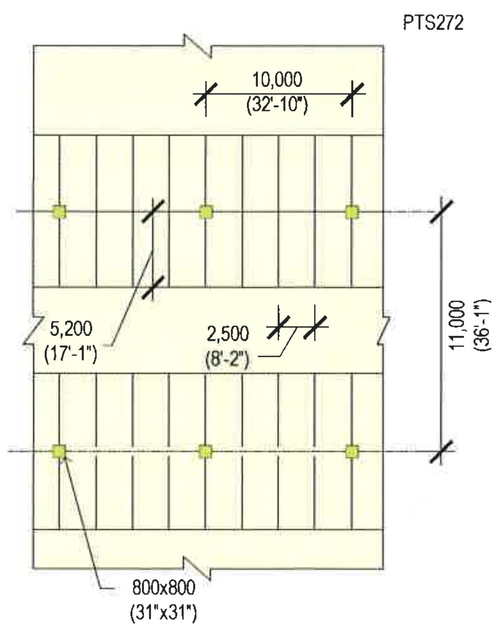


FIGURE 4.3.4-4 Plan of a Parking Structure in Croatia (Courtesy BBR, Croatia)

[9] TR43, Table 3, Section 5.8.1

- ❖ For grouted beams and one-way systems up to 0.2 mm
- ❖ No specification for unbonded tendons

4.4.1.3. Selection of Post-Tensioning System: In addition to other measures of corrosion protection, such that are intended for use in non-corrosive environments, and (ii) those that are permitted for use in corrosive environments. The system used for non-corrosive environments allows limited lengths of strand ; to be left unprotected beyond that provided by concrete cover; whereas the system intended for the corrosive environments, also referred to as "encapsulated " system, requires that the entire strand, and its wedges to be covered with a protective material. Figures 4.4.1.3-1 and 4.4.1.3-2 are examples of the two systems.

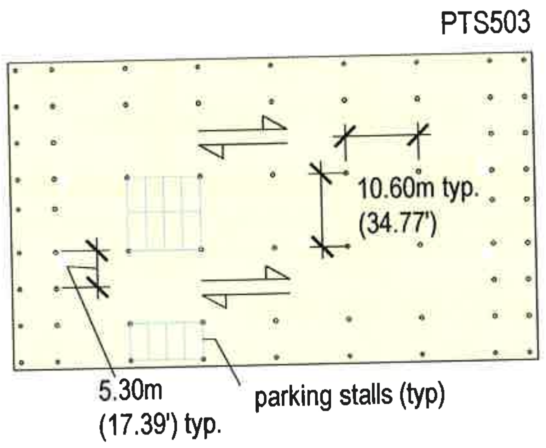


FIGURE 4.3.4-5 Layout of a Parking Structure in Kolkata

Specifically, for the United States, the geographical ambient environment is treated using the chart proposed by Carl Walkers, [Carl Walker,1998].

As an example, the map shown in Figure 4.4.1.3-3 divides the United States into five zones, determined by the presence of deicing salts, or airborne salts from oceans. Structures in Zones III and CC-II (coastal chlorides) should be built to meet stricter durability requirements than those needed for structures in Zones I, II, and CC-I. Zones CC-I and CC-II are not specific to the United State. They apply to all locations. The remainder of the zones, however, depends on the locality of a building and the environmental weather conditions not impacted by the vicinity to an ocean or sea.

4.4.2 Fire Protection

The fire resistance is defined as the period of exposure to a standard fire, before a member collapses under a predefined, sustained applied loading. The details of the standard test and criteria are given in [ASTM-E119-83].

Minimum dimensions for post-tensioned members, required to meet various fire endurance requirements, are a function of the type of steel used, the type of concrete used, the amount of cover provided to reinforcement, and - in the case of beams - the beam width. The engineer may either use the recommended covers, given in ACI-423/ IBC 2012 and reproduced herein, or demonstrate by calculation that the cover and reinforcement used are adequate for the specific design parameters

TABLE 4.4.1.1-1 Minimum Cover to Reinforcement for Prestessed Floors[10] (T126)

Member and Condition	Minimum cover mm	Minimum cover inch
Concrete cast against and permanently exposed to earth	75	3.0
Concrete slabs and joists exposed to earth or weather	25	1.0
Concrete not exposed to weather or in contact with ground:		
(i) Slabs and joists	20	0.75
(ii) Beams primary reinforcement	38	1.5
(iii) Ties and stirrups	25	1.0

[10] ACI-318-11, Section 7.7

FIGURE 4.4.1.3-1 Example of Application of a post-tensioning System in Use in a "Non-Corrosive" Environment (P501)

FIGURE 4.4.1.3-2 Example of Application of a post-tensioning System in Use in a "Corrosive" Environment (P502)

selected. The computational procedure for the verification of cover to reinforcement and its amount other than those stipulated in the building codes is referred to as the "rational approach." The rational approach for justification of adequate fire resistivity is detailed in the PTI manual [PTI Design Manual, 1981],

A. Minimum Cover: The current recommendations for rebar cover, required to achieve various fire ratings, are given in TABLE 4.4.2A-1 for slabs and TABLE 4.4.2A-2 for beams [IBC, 2006]. A concrete member is categorized as either "restrained" or "unrestrained". The background to the classification is as follows. Where fire occurs, rise in temperature of a slab soffit tends to expand the member and push against the surrounding parts. The restraint of the surrounding tends to form compressive forces on the heated portion. The thrust from the constraint of surrounding members acts near the bottom of the slab, where heat is most. The additional compressive force to the bottom layer of a slab enhances a slab's strength. At the same time, rise in temperature of reinforcement and prestressing lowers the yield stress of each and leads to loss of strength.

On this premise, the interior spans of a floor system, and exterior spans that are bound by heavy beams are considered "restrained." Exterior spans of floor slabs which are not provided with a heavy beam or wall along the outer boundary of the floor are considered "unrestrained."

TABLE 4.4.1.2-1 Exposure Classification and Suggested Design Crack Width EC2 (mm) (T127)

Exposure Class	Reinforced members and prestressed members with **unbonded** tendons	Prestressed members with **bonded** tendons
	Quasi-permanent load combination (Sustained load combination)	Frequent load combination (Total load combination)
X0, XC1	0.4[1]	0.2
XC2, XC3, XC4	0.3	0.2[2]
XD1, XD2, XS1, XS2, XS3		Decompression

Note 1: For X0, XC1 exposure classes, crack width has no influence on durability and this limit is set to guarantee acceptable appearance. In the absence of appearance conditions this limit may be relaxed.

Note 2: For these exposure classes, in addition, decompression should be checked under the quasi-permanent combination of loads.

DURABILITY RECOMMENDATIONS BY LOCATION IN THE US

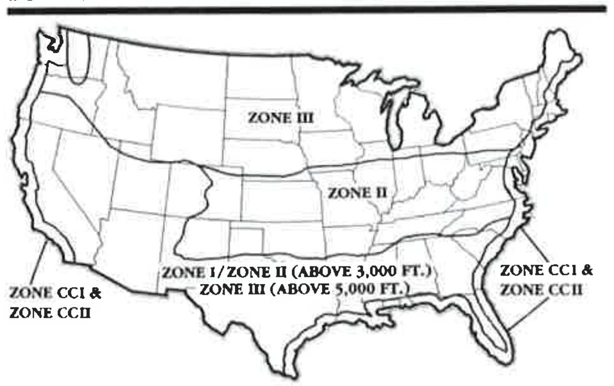


FIGURE 4.4.1.3-3 Map of Durability Zoning of US (P503)

B. Minimum Member Thickness: For slabs, fire resistance is generally achieved through the selection of a minimum thickness for the member and an adequate concrete cover to the reinforcement. The minimum slab thickness requirements for various fire ratings are summarized in TABLE 4.4.2B-1.

C. Design Charts for Fire Resistivity: Sample charts for fire resistivity design or investigation are given in Figs. 4.4.2C-1 through 4.4.2C-3. The first two charts express the percentage loss on the original characteristic values of prestressing steel and concrete at higher temperatures. The last charts give the temperature of reinforcement for different covers and durations of standard fire. The application of the charts is described in part D of this Section.

D. Four Steps of Fire Resistivity Design or Investigation: The steps in design for a given fire resistivity or investigation are:

Step 1: Select the required number of "hours" for fire resistivity. In most cases, it is two hours.

Step 2: For the selected hours of fire resistivity, and the cover to reinforcement, read the temperature of prestressing steel and rebar from the design charts in Section C (Fig. 4.4.2C-3).

Step 3: For the temperature of reinforcement obtained from the previous step and from the graphs of reduction in strength (Fig. 4.4.2C-1), determine the reduced strength of the heated reinforcement.

Step 4: With the reduced strength values from step 3, calculate the reduced design capacity of the member and compare it with the original design demand to determine the safety of the member

4.4.3 Wear

Where the surface of a slab, as in parking structures, is subject to traffic but is not provided with a protective topping, and where there is likelihood of freezing, a good practice is to provide an additional 10 mm (3/8th in.) cover to the slab for wear and irregularities in construction. The contribution of this "wear" thickness to the structural strength of the slab is generally disregarded. Figures 4.4.3-1 a and b illustrate examples of two different parking structures, where irregularities in workmanship and lack of a protective wear have exposed the post-tensioned tendons.

4.5 LOAD PATH

4.5.1 Prerequisites of a Load Path

The steps in selection of a load path for design of floor systems are detailed in Chapter 3.

TABLE 4.4.2A-1 Suggested Concrete Cover for Prestressed Slabs (T128US)

Restrained or unrestrained	Aggregate type	Cover thickness, in (mm) for fire endurance of prestressed slabs		
		2 hr	3 hr	4 hr
Unrestrained	Carbonate Siliceous Lightweight	1.5 (38)	2.0 (51)	--
Restrained	Carbonate Siliceous Lightweight	0.75 (19)	1.0 (25)	1.25 (32)

Figure 4.4.2-A1 shows the partial plan of a floor system and the required fire cover, based on the restrained and unrestrained definitions above and a two-hour fire resistivity.

TABLE 4.4.2A-2 Suggested Concrete Cover for Post Tensioned Beams (T129US)

Restrained or unrestrained	Aggregate	Beam width, in. (mm)	Cover thickness for fire endurance, in. (mm)			
			1 hr	2 hr	3 hr	4 hr
Unrestrained	Carbonate Siliceous Lightweight	8 (200) 12 (305)	1.75 (44) 1.50 (38)	2.5 (64) 2.0 (51)	4.5 (114) 2.5 (64)	-- 3.0 (76)
Restrained	Carbonate Siliceous Lightweight	8 (200) 12 (305)	-- --	1.75 (44) 1.50 (38)	2.0 (51) 1.75 (44)	2.5 (64) 2.0 (51)

Note: Except from International Building Code – 2006, Table 720.1(1)
It shall be permitted to interpolate the values given

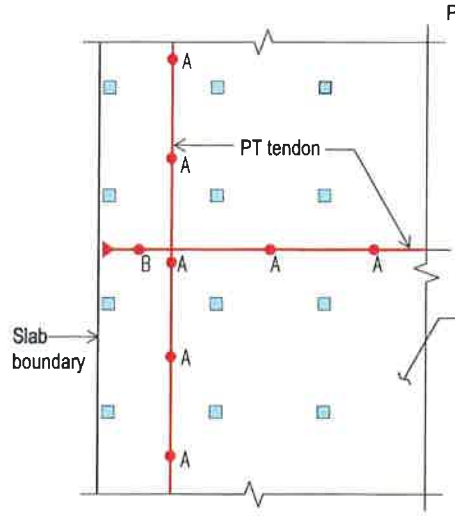


FIGURE 4.4.2-A1 Plan - Bottom Cover for Fire Resistivity

The following Section covers the conceptual background, and its application to conditions other than those previously discussed.

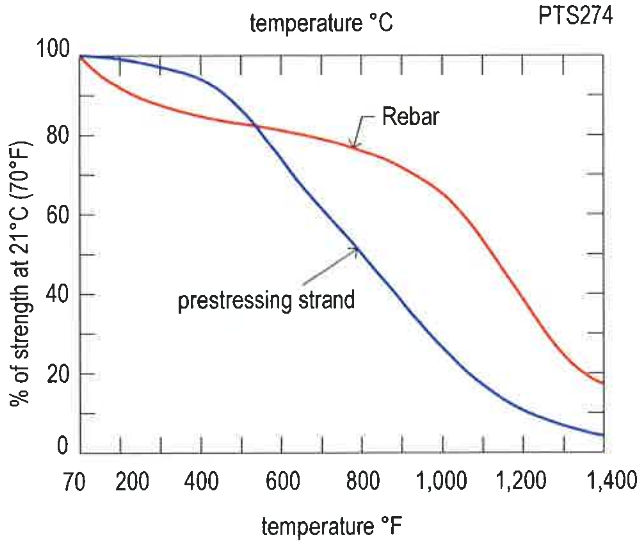
The prerequisites of a safe design are:

(i) There shall be an uninterrupted load path, from the point of application of a load, to where the load is resisted, such as a support or foundation;

(ii) the distribution of forces along the designated load path shall be in static equilibrium with the applied loads; and

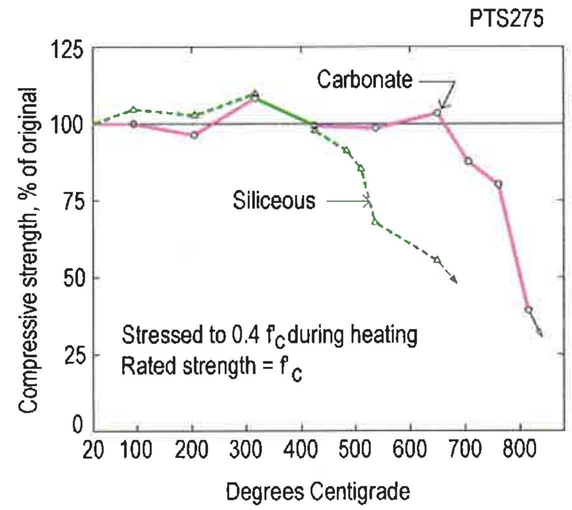
(iii) the structure shall have adequate ductility, to deform on demand, and to redistribute the internal forces, in order to develop the resistance provided along the load path selected at its design in step (i)

Selection of load path – also referred to as "structural system" - is a central step in design.



Tensile Strength of a Prestressing Steel at High Temperature

FIGURE 4.4.2C-1



Compressive Strength of Concrete at High Temperatures

FIGURE 4.4.2C-2

TABLE 4.4.2B-1 Suggested Minimum Concrete Thickness Requirements for Fire Endurance (T130US)

Aggregate type	Slab thickness for the fire endurance indicated, in. (mm)				
	1 hr	1.5 hr	2 hr	3 hr	4 hr
Carbonate	3.2(81)	4.0(102)	4.6(117)	5.7(145)	6.6(168)
Siliceous	3.5(89)	4.3(109)	5.0(127)	6.2(157)	7.0(178)
Lightweight	2.5 (64)	3.1(79)	3.6(91)	4.4(112)	5.1(130)

Note: Except for International Building Code - 2006. Table 721.2.3(4)

(a) Parking structure deck with exposed tendon (Palo Alto, CA; P264)

(b) Exposed tendon on parking deck (San Jose, CA; P706a)

FIGURE 4.4.3-1 Post-Tensioning Tendons Exposed Due to Lack of Adequate Cover on Parking Structure Decks

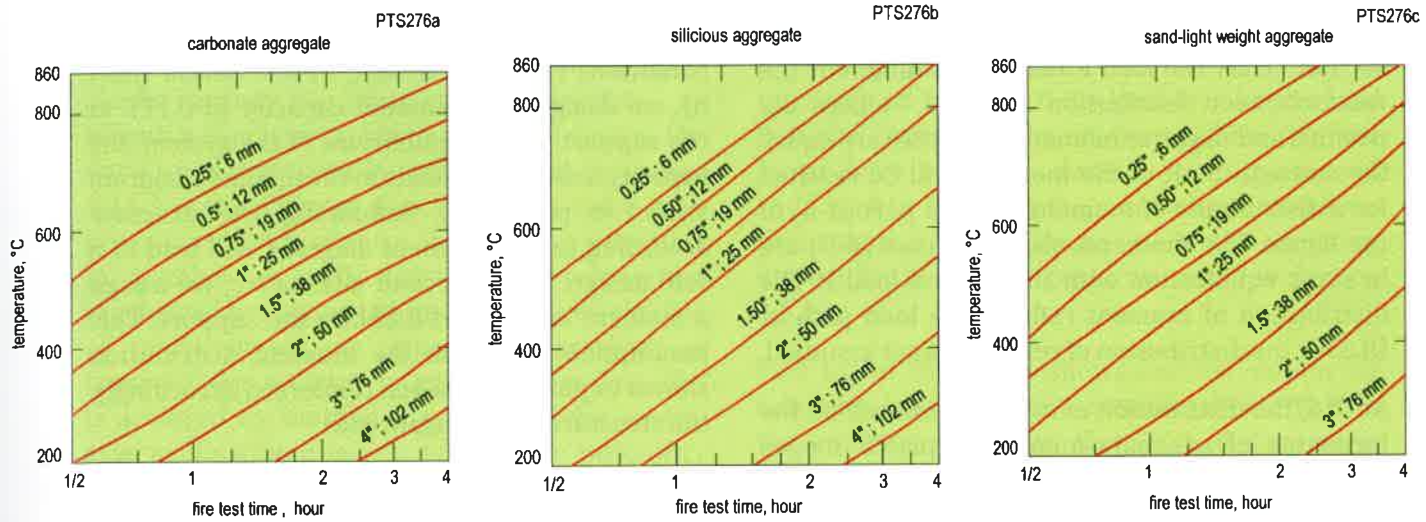
Illustrative Example for Load Path

Unlike the skeletal structures, such as floors constructed with steel girders and beams, where the selection of load path is implicit in the articulated members of the floor, in the continuum of floor slabs the load path is not evident. For floor slabs, the load path for the strength limit state is generally envisaged by the designer within the outline of a floor slab's continuum, and reinforced accordingly. Being a critical step in design of concrete floors, the following simple example is used to crystallize several of the underlying concepts.

We review the example of a propped cantilever, and highlight the following:

- ❖ For service condition (SLS), there is generally a SINGLE and unique load path. The load path is determined by the pre-cracking "elastic" response of the structure to the applied load.
- ❖ For the safety of the structure (ULS), there can be more than one load path.
- ❖ The load path(s) for ULS are generally not the same as that for the SLS.
- ❖ The load path for ULS is generally determined by the designer at design time, and reinforced accordingly.

Consider the propped cantilever beam shown in Fig. 4.5.1-1a. The cantilever has a uniform cross section; rests on roller support at the propped end; and is fixed at the other end. It is subject to a single load P. We review the design of this cantilever, on the assumption that the effect of selfweight is negligible compared to the applied load P. This does not change the process of design, nor the conclusions arrived at. Part (b) of the figure shows the free body diagram of the cantilever.



(a) (b) (c)

FIGURE 4.4.2C-3 Concrete Temperature for Standard Fire Duration Times and Different Aggregates

The distribution of moment, based on linear elastic material properties, is shown in part (c) of the figure. The deflection of the cantilever, as well as the distribution of stress in the member and the initiation of first crack are associated with this bending moment. In service, the cracks if any are assumed not to be extensive. The elastic response of the structure under the applied load is calculated using the gross cross-sectional dimensions of concrete and its material properties. While it is recognized that the presence of reinforcement has an influence on the deflection and crack formation of the member, due to its small impact in most cases, the presence of reinforcement is generally not accounted for in the SLS design.

Deflection of a member and crack formation under service load are part of the serviceability check (SLS). There is only one elastic solution for the applied load shown – hence one load path. The distribution of moment shown in part (c) of the figure, defines the load path for SLS.

Let the reinforcement provided in this propped cantilever be two identical layers, one at the top and one at the bottom, each extending over the entire length of the member. For a rectangular section, the provided reinforcement results in a constant resistance capacity for both the positive and the negative moments over the entire length of the member.

Increase in the applied load P, beyond the elastic limit of the member, results at first in the post-elastic response of the connection at the fixed support, where the demand moment is most. Upon further increase in load, the rise in the moment demand will be shifted from the support

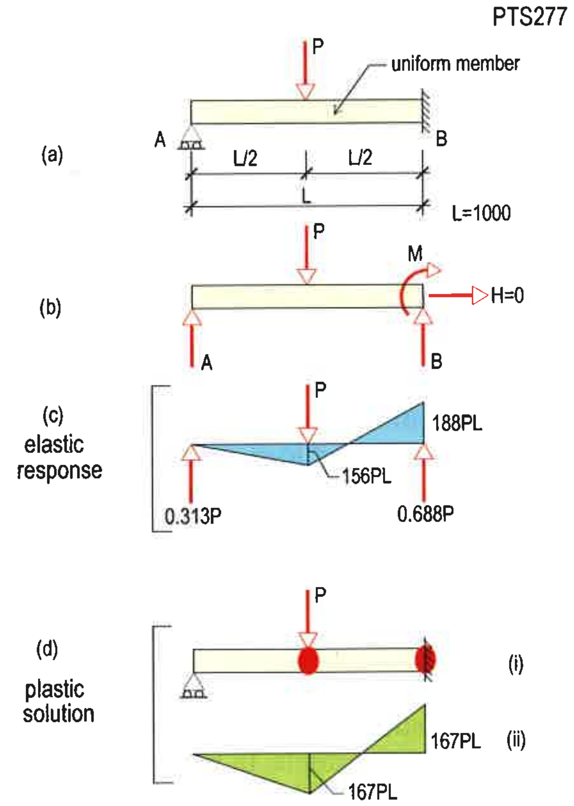


FIGURE 4.5.1 -1 Propped Cantilever; Failure Mode Options

to the rest of the beam, until the resistance of the section below the load P reaches its limit. For the reinforcement distribution assumed, where the positive and negative moment capacities are equal, the strength limit of the member will be reached for a distribution of moment given in part (d-ii) of the figure. The moments shown in part (d-ii) are in static equilibrium with the applied load P. The distribution of moment reflects the load path at ULS for the distribution of reinforcement assumed.

At ULS, the distribution of moment is based on the formation of adequate number of plastic hinges that lead to collapse of the member. Observe that the distribution of moment at (ULS) is different from the elastic response in part (c) of the figure. In other words, at ULS the load is carried by a different load path that is governed by the distribution of moment and hinge formation shown in part (d). The observed load path is based on the resistance provided in the member– namely the "assigned" distribution of reinforcement.

Alternatively, if the reinforcement provided is only at the bottom. The structure can still be safe, provided the bottom reinforcement can resist the distribution of moment associated with a single hinge below the load.

In summary, for the propped cantilever, we conclude that there is a single load path for its in-service response, and several load path options for its safety. Each load path at the ULS is function of the distribution, and amount of the provided reinforcement.

Further the premise for a valid load path at ULS is that (i) the distribution of moment at ULS shall be in equilibrium with the applied load, (ii) at each point along the assumed load path, the member shall be provided with resistance adequate to meet the distribution of actions assigned to it on the load path; and finally (iii) the member shall have adequate ductility to undergo post-elastic deformation and re-distribute the actions to the selected load path.

Figure 4.5.1-2 illustrates two other load path options for ULS of the same example. In both options, the distribution of moment is in equilibrium with the applied load P. The objective of this second figure is to re-emphasize the multi load path possibilities of design at ULS.

Part (a) of the figure is the re-statement of conditions for a safe design. In one option (part b), we designate a moment capacity of 0.1PL at the support. From equilibrium of the system, the capacity assignment leads to the moment diagram shown in part (b-ii). Reinforcing the structure according to this moment diagram, will lead to a safe design. In the second alternative, we assign a moment capacity of 0.2PL to the support. This assumption results in the moment distribution shown in part (c-ii). When reinforced accordingly, the structure will be again safe.

It is reiterated that for the safety of the structure (ULS) there can be more than one load path. Each path follows our assumption of capacity assignment (provision of reinforcement). The central point is that, at ULS the objective is not to replicate the "elastic response" of the structure. Rather, the objective is to provide a safe load path that often will be different from the elastic (initial) response of the structure.

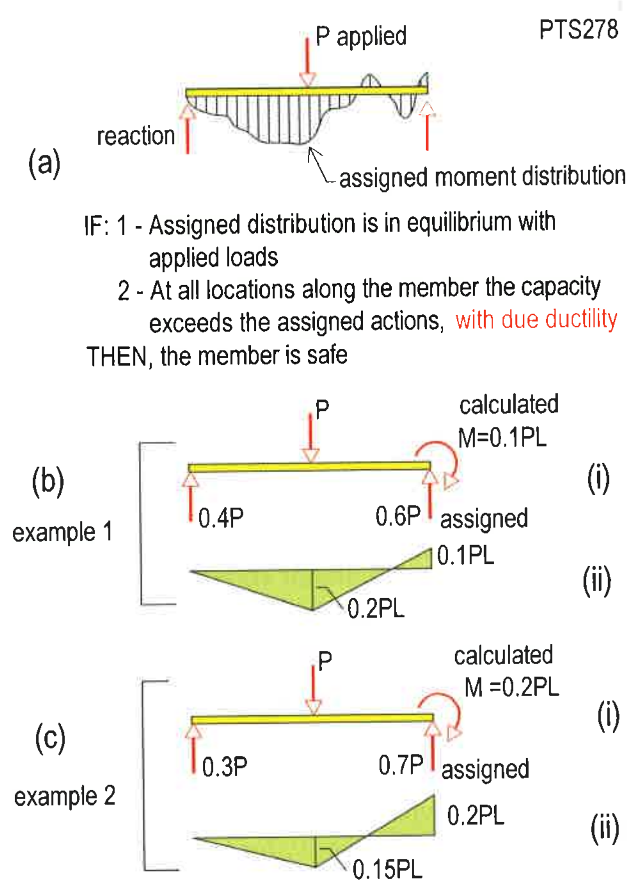


FIGURE 4.5.1-2 Examples of Load Path Options for Safe Design of Propped Cantilever

The ability of a concrete section to undergo limited rotation past its elastic limit, while maintaining its elastic capacity is a measure of its ductility - an essential characteristic in floor system design. The ductility is discussed in greater detail in Section 4.11.3 of this Chapter. Major building codes have provisions that guarantee the necessary ductility for member design. Ductility of a concrete section is achieved by forcing failure to initiate by yielding of reinforcement, as opposed to crushing of concrete. For sections in primarily bending, this is achieved by limiting the depth of the neutral axis, as illustrated in Fig. 4.5.1-3. The ratio of c/d_t is controlled to initiate yielding in tension reinforcement.

4.5.2 Strip Method

In this section we review the application of the strip method to concrete slabs. Strip method is the backbone of the contemporary floor system designs that rest on automated finite element procedures. Strip method has been in use for over a century, forming the basis of design of many structures.

Consider the design of a single concrete panel supported on four columns, and subjected to a central concentrated load (Fig. 4.5.2-1). Let the selfweight to be small relative to the applied load. Hence, the design will be governed by the effects of the central load.

One design option is to consider a single slab band extending between two diagonal columns to resist the load (Fig. 4.5.2-2b). The moment demand on this slab band will be that necessary to sustain the entire load on the design strip. The outcome will be concentration of the entire reinforcement along the band between the selected diagonal columns. The reinforcement needed for the remainder of the panel will be that necessary to bring the selfweight of the slab to the designated slab band. We refer to the slab band as "design strip."

Another load path option is to select two slab bands, one along each of the diagonals as shown in part (c) of the figure. The two diagonals will be designed to share the resistance necessary to sustain the applied load. From standpoint of safety of the structure (ULS), it is up to the designer to determine what fraction of load P will be resisted by which of the two strips. We may assume 20% of the load to be taken by one of the strips, and the remainder 80% by the other. Each strip will then have to be designed, and reinforced according to the assigned fraction of the load. It is emphasized that the concept relies on the premise that the strips have adequate ductility to displace and shed load in excess of their capacity to members that can resist them. The necessity and adequacy of ductility in our designs is discussed in Section 4.5.1.

Part (d) of the figure shows another option for selecting load path and reinforcement. This is the common option in floor slab design, as is expounded later on.

4.5.3 Slab as a Continuum

The advent of the finite element method (FEM) and its application to the analysis and design of concrete floor systems led to the treatment of floor slabs as a continuum compared to the skeletal modeling used in strip method. Unlike the practice of design engineers, where strength reinforcement was placed according to engineer's choice of load path, the limitations in the first generation of FEM led to the use of the same load path for SLS and ULS. The process is clarified through the Roya balcony shown in Fig. 4.5.3-1. The figure illustrates the distribution of reinforcement intensity along the two principal directions, following the style of the first generation FEM-based designs. The line contour for the intensity of reinforcement is determined from the moment contour of the balcony in each of the two principal directions. Unlike the strip option used in Fig 3.3B-1 where the primary reinforcement is placed either in up-down, or left-right direction, in the first FEM generation the reinforcement is placed in both directions – a departure from the traditional

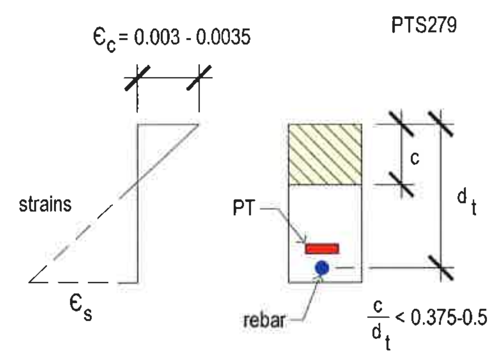


FIGURE 4.5.1-3 Example of Depth of Compression Limit for Ductility

design practice. The natural option for many engineers using the first generation of FEM has been to select prefabricated wire mesh to envelope the reinforcement intensities reported in the FEM-based intensity contours.

As it is outlined in Chapter 3, the current FEM-based design technology can recognize the user selected load path for the ULS design of a floor, and provide the reinforcement accordingly. Hence, in contemporary FEM-based designs the SLS and ULS load paths no longer need be the same.

4.5.4 One-Way and Two-Way Systems

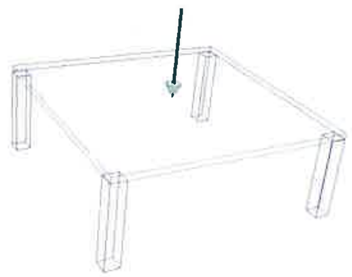
The categorization of a floor system as one-way, or two-way is necessary for two reasons. First, the allocation of design load to a design strip depends on whether the system is one-way or two-way. This is explained further in this Section. Next, ACI 318 requirements for allowable stresses, amount of reinforcement, and several other design considerations are based on whether a floor system is identified as one-way or two-way. In the following, we start by reviewing the categorization of the structural system as one of the two options. The code compliance for each is dealt with later.

We begin with the one-way and two-way concept for skeletal structures. Fig. 4.5.4-1 illustrates a typical one way system, where the beam AB carries the entire force F along its length AB. In part (b) of the figure, on the other hand, the force F can be shared between the two beams AB and CD. Under service condition (SLS) prior to cracking and onset of plasticity, the force is distributed between the two beams according to their relative stiffness. This is determined by the commonly used linear elastic material analysis of the structure, in which the compatibility of deformation between the two beams at the intersection is used to arrive at the solution.

At its strength limit (ULS), each beam can carry the force determined by the amount of its reinforcement. At this stage, the breakdown of the force between the two beams is no longer a function of their initial stiffnesses.

Conversely, at design stage, one can assign the percentage of the load that should be carried by each beam, and design the structure for its safety limit (ULS) based on the designer-based allocation of the force. In this case the fractions of the force

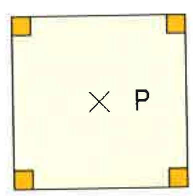
(a) Loaded panel on four columns (P266)



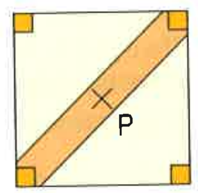
(b) See-through view of panel (P267)

FIGURE 4.5.2-1 Panel on Four Columns Subject to a Central Point Load

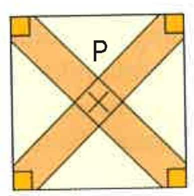
PTS586



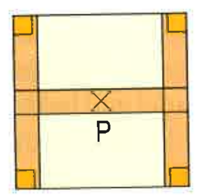
(a) Plan of slab



(b) Single diagonal strip



(c) Two diagonal strips



(c) Orthogonal two-way system

FIGURE 4.5.2-2 Plan of a Column-Supported Single Panel; Load Path Options for Strength Design

that are assigned to the constituent parts must add up to the total force. If one beam is assigned to carry 30% of the load, the other beam must be designed for the 70% balance.

Consider now a similar situation as shown in Fig. 4.5.4-2 (i). It shows the plan of two intersecting beams under load *P*. Similar to the previous case, if beam AB is viewed in isolation, it can be designed for a fraction "*a*" of the force *P*, leaving beam CD to be designed for the balance of the fraction, namely *(1-a)P*, as shown in the figure.

Next, view the structure in up-down direction as shown in part (iii) of the figure. In this view, support A falls behind support C, and support D behind B. Viewing the structure in this fashion we observe a span with length (*L*) under a central point load *P*. The resistance of the structure to sustain the load along the span L in this view must be adequate for the entire force *P* - not a fraction of it. Similarly, the resistance necessary in the up-down direction for a similar view in the perpendicular direction needs to be for the entire load *P*.

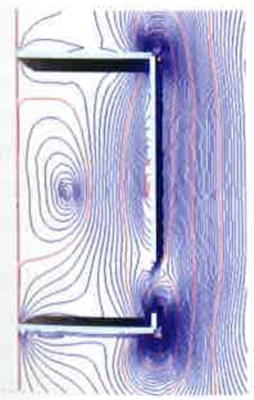
Part (b) of the figure is a single slab panel supported on four columns and subject to a central load *P*. If the resistance to the load is provided by reinforcement in X-X (left-right) direction, and Y-Y direction, similar to part (a) of the example, the reinforcement in each direction should provide resistance for the entire force on the panel, namely the actions for design of the slab should consider 100% of the load in each direction – not less than 100 %.

The orientation of the reinforcement in this example, renders the structure a "two-way system." The interpretation of the term "two-way" system is that the structure has to be designed to resist the entire (100%) load in each of the principal directions, contrary to the intersecting one-way systems, where each member can be assigned for a fraction of the load, as long as the entire load is accounted for. The concrete panel of part (b) of the figure can be designed as one-way, if the reinforcement is placed along one or both of the diagonals.

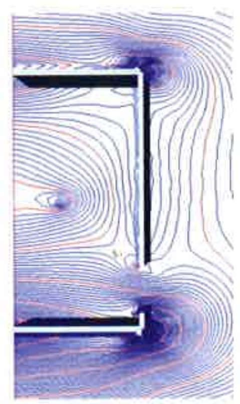
It is concluded that in addition to the support conditions, the selection of load path by design engineer- reflected in the orientation of reinforcement in the structure. In this case the orientation determines whether a structure is a "one-way," or a "two-way" system.

An extension to the categorization explained above is illustrated in the example of Fig. 4.5.4-3, where a panel supported on six columns is subject to a central load P.

Viewing the panel in three different directions along parallel lines that join adjacent supports, such as the view shown in part (b) of the figure, indicates that in each case, the reinforcement provided between the supports shown must be designed to resist the entire force P on the panel. In this case, three independent views can be identified, each of which is required to resist 100% of the load. Following the preceding definition of one-way and two-way systems, the current slab

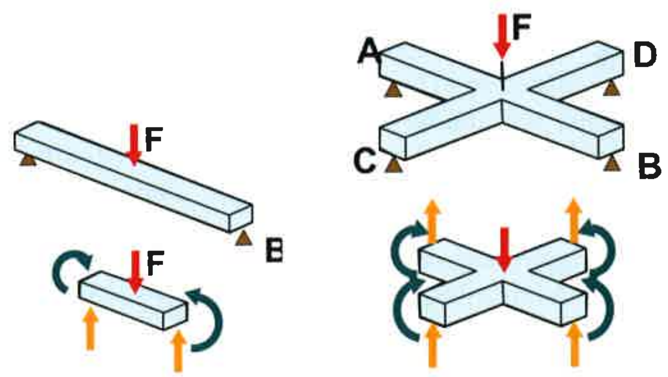


Reinforcement along X-X



Reinforcement along Y-Y

FIGURE 4.5.3-1 Reinforcement Contour in Two Directions (P268; P269)



(a) One-way system

(b) Two-way system

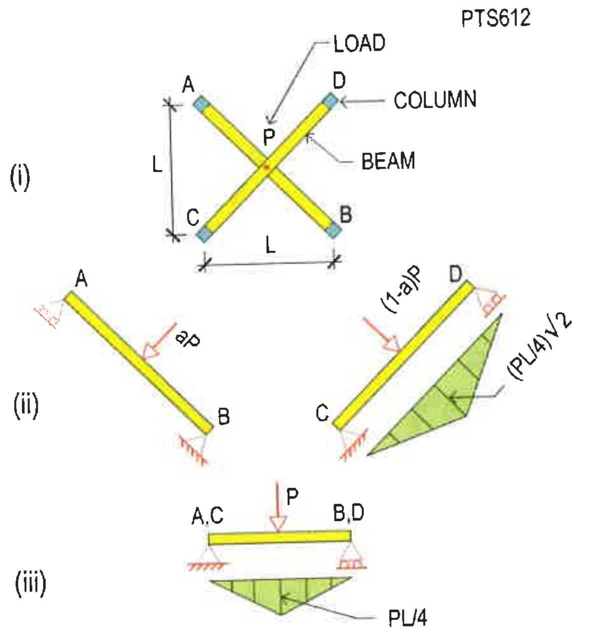
FIGURE 4.5.4-1 One-way and Two-way Skeletal Structures (P505) can be viewed as a "three-way" system. However, this is a rare condition. In practice the structures fall into the categories of either one- or two-way system.

Figure 4.5.4-4 shows plans of several floor systems and their designation as to one-way, or two-way floors. In the one-way system example, non-prestressed reinforcement is used to close the gap between the wall supports normal to the tendons.

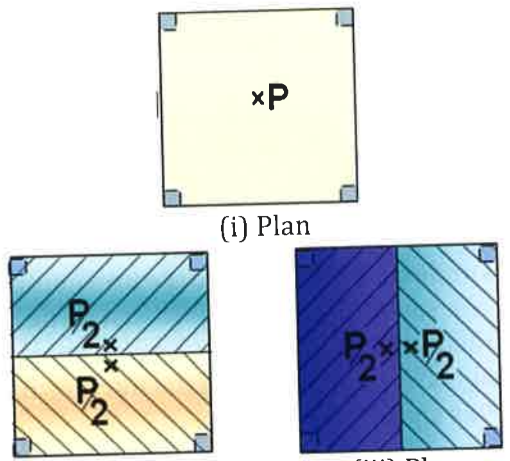
4.6 STRUCTURAL SYSTEM

Prior to initiation of analysis, a designer assigns a fabric of interconnected load paths that are intended to provide resistance to the anticipated loads of the entire structure. The aggregate of the load paths forms the "structural system" of the building. The structural system is made up of structural members, such as beams, slabs, columns, column drops and walls. It also includes the connectivity of the members, such as whether a slab is fully fixed to a wall, or simply transfers gravity loads and not shear; type of support to the foundation, such as hinged, fixed, or otherwise; assignment of stiffnesses, such as whether a member is assigned to resist torsion or not; or fraction of resistance that is dictated by a member's gross cross-sectional geometry.

The structural system must be fully defined prior to analysis. One outcome of the analysis is the forces that each member of the structural system must resist. The structural "analysis" is followed by structural design of the members, and finally by its detailing.



(a) Intersecting beams under central load



(i) Plan

(ii) Plan (iii) Plan

(b) Panel modeling with "design strips" (P507)

FIGURE 4.5.4-2 Intersecting Beams and Solid Panel on Corner Supports

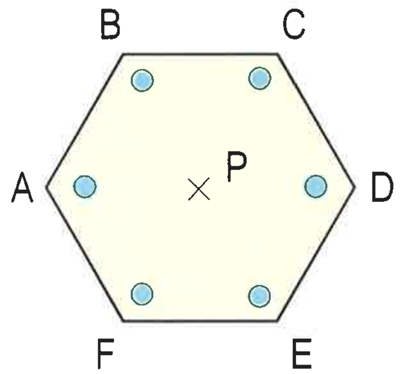
There need not be a single structural system envisaged by a designer to resist the applied loads. In regions of high seismic risk, it is common to have two systems, one intended to resist the gravity loads, and the other the combination of gravity and lateral loads- or only the lateral loads. In this Chapter, the focus of the presentation is the structural system for gravity, and the design of the members that resist gravity loads. The structural system for lateral loads is scheduled for a different volume, that covers the design of concrete buildings for seismic and wind forces.

4.6.1 Slab Systems

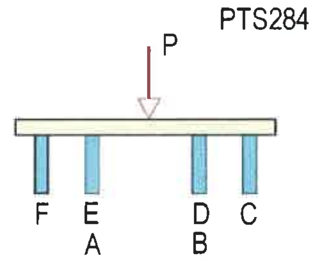
A. Geometry: Floor systems come in a variety of different shapes, each suited for a particular span, load, and support conditions. In addition, the relative cost of concrete, reinforcement and forming, as well as the skill of construction crew, and the availability of construction equipment, each play a role in the selection of a specific floor system. Fig. 4.6.1A-1 [Aalami, 1989] illustrates a variety of cast-in-place floor types in common use.

As an example, in the US, where cost of forming can exceed one-third of the cost of a floor, a simple geometry, such as flat slab as shown in block 1 of Fig. 4.6.1A-1 is advantageous. In California, where provision of resistance for seismic effects plays a great role, again a light and thin floor such as the one shown in block 1 is the choice, since

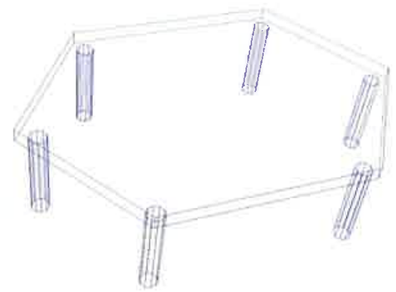
the thin slab, coupled with elimination of beams reduces the weight and height of a building. In another example, in parts of India, where cement is expensive and labor is readily available, a waffle construction, such as the one shown in block e and illustrated in Fig. 4.6.1A-2 is the answer. In the figure, the voids are created by placing bricks on the form. Fallen bricks subsequent to removal of forms create the void and savings in cement consumption.



(a) Plan of plate



(b) Elevation of plate



(c) 3D view of plate (P270)

(d) Deflection of plate under selfweight (P271)

FIGURE 4.5.4-3 Concrete Plate on Six Supports

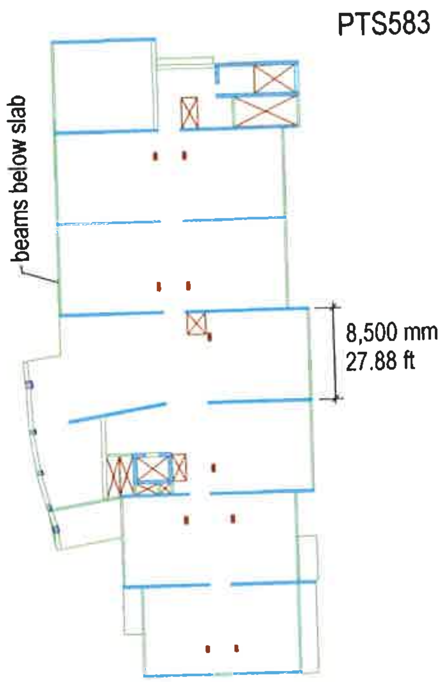
B. Examples of Slab Construction: Figures 4.6.1B-2 through 6 are several examples of different floor slab construction.

Each of the systems shown above can be supplemented with post-tensioning for longer spans, thinner slabs and improved performance. Slabs with filler foams and voided slabs are advantageous, where material cost is significant.

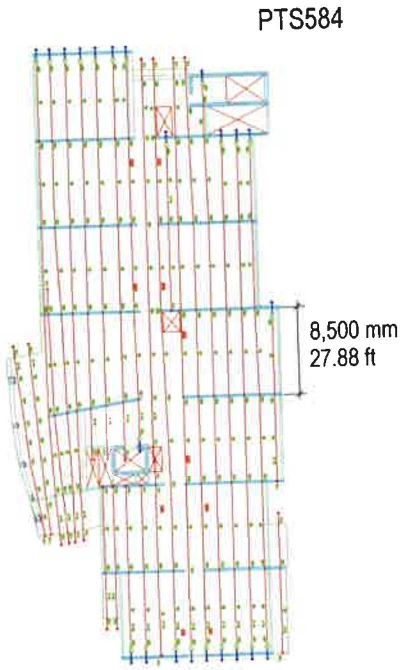
C. Analysis and Design: The slab systems shown above form a continuum, unlike the articulated systems common in steel construction. As discussed in Section 4.5.3, the structural systems for the in-service (SLS) and ultimate limit state (ULS) are generally assumed to be different. For service condition of conventionally reinforced slabs, it is adequate to consider the slab to act as a "plate" member, with three actions on each face, namely normal shear (normal to the plane of the slab), bending and twisting as shown in Fig. 4.6.1C-1a. [Aalami et al, 1975]

For floor that are post-tensioned, the distribution of axial force in the slab requires the consideration of the membrane actions as shown in part (b) of the figure. Also, where other factors such as restraint of supports, temperature, creep, shrinkage and changes in thickness are required to be considered in design, the inclusion of membrane forces (part b) becomes necessary. The combined consideration of the membrane actions shown in part (b) of the figure, along with those shown in part (a) of the figure is expressed through the "flat shell" formulation. For post-tensioned floor systems, as a minimum, it is necessary to use a flat shell representation of the slab, in order to correctly capture the floor system's response to applied loads[11].

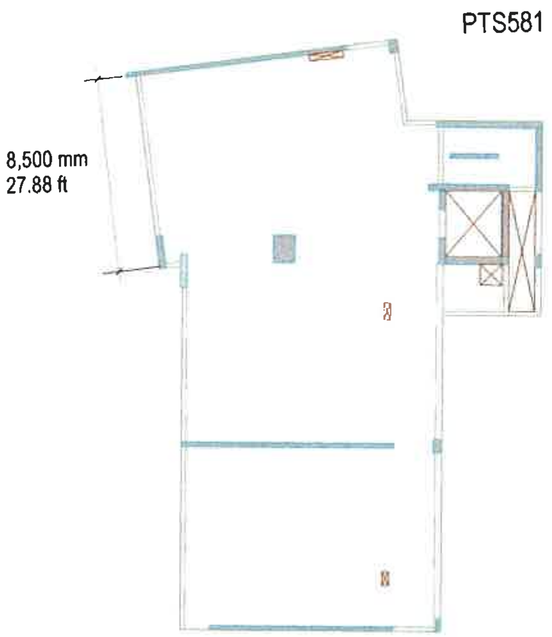
[11] ADAPT Floor Pro; www.adaptsoft.com



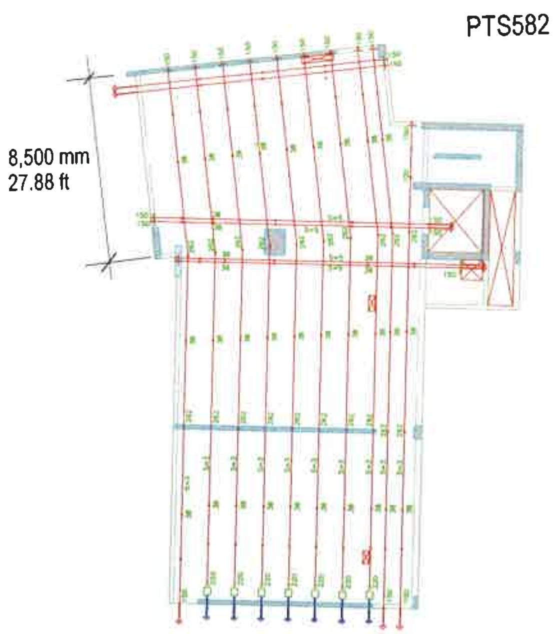
(a) Plan; One-Way Floor System; Slab Thickness 230 mm (9")



(b) Plan; One-Way Tendon Layout



(c) Plan; Mixed Floor System; One-Way and Two-Way Systems



(d) Plan; Mixed Tendon Layout; One-Way and Two-Way Systems

FIGURE 4.5.4-4 Examples of Floor System Classifications (Burj View, bonded system, KSA)

The actions on an element of plate shown in Fig. 4.6.1C-1 can be grouped into the following:

Flexural Actions

- M_y = Moment about the y-axis
- M_x = Moment about the x-axis
- M_{xy} = Twisting moment
- Q_x = Shear on the element face perpendicular to the x-axis
- Q_y = Shear on the element face perpendicular to the y-axis

Membrane Actions

- N_x = Axial force along the x-axis
- N_y = Axial force along the y-axis
- N_{xy} = In-plane shear

(a) One-Way Floor System (JBR) Dubai, grouted (P508)

(b) Two-way floor system, unbonded (Busan, Korea, Freyssinet) (P514)

FIGURE 4.5.4-5 Examples of Post-Tensioned Floor Systems under Construction

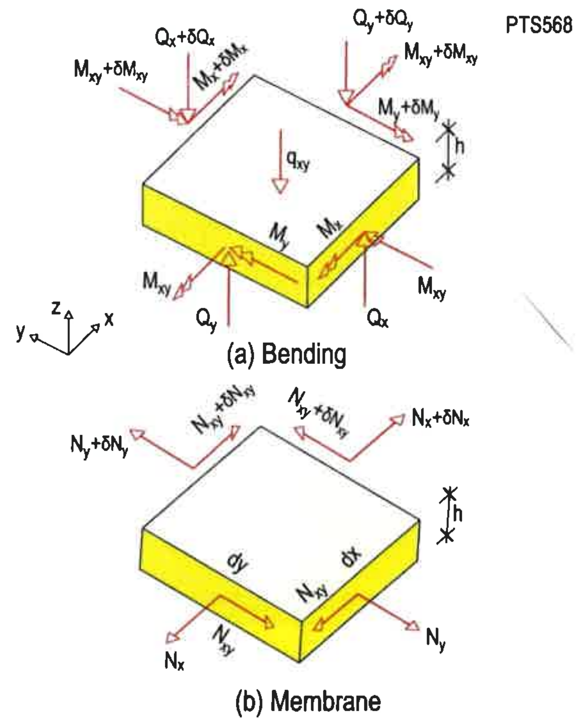


FIGURE 4.6.1C-1 Plate Element; Identification of Bending (Flexural) and Membrane (Inplane) Actions on Slabs

4.6.2 Slab Bands

Where support spacing in one direction is longer than the other, the longer span governs the thickness, if a slab of uniform depth is selected. In post-tensioned slab construction, however, the adverse effects of the longer span can be reduced, if the tendons in the long-direction are grouped together and placed with an increased drape below the slab to provide larger upward forces. This arrangement allows for a reduced slab thickness to be based on the span in the short direction- thus maintaining the economy of post-tensioning.

Slab bands (Figs 4.6.2-1 through 4) are thickening of slab along the column lines of the longer span to accommodate the additional drape of post-tensioning tendons. The restriction on geometry of the band does not qualify it as a "beam" for design purposes. Rather, the thickening is viewed as an extension of the slab. The localized stiffening of the slab band does not significantly impact the biaxial action of the panel – and does not draw an inordinate share of shear forces to the band to render the band as a beam.

Slab band dimensions commonly used in industry are: band depth "h" not exceeding the slab thickness, and band width not less than three times the slab

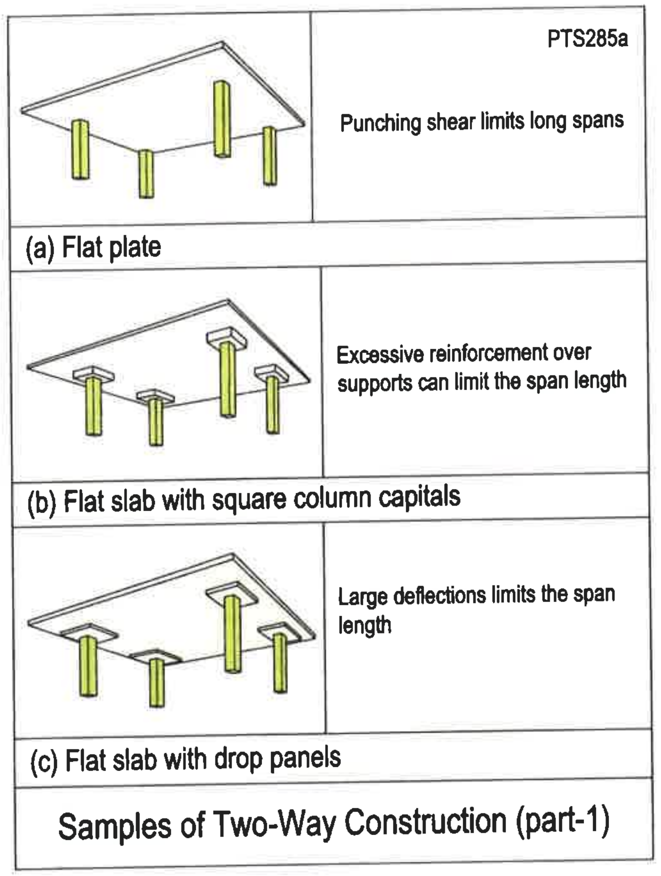


FIGURE 4.6.1A-1a

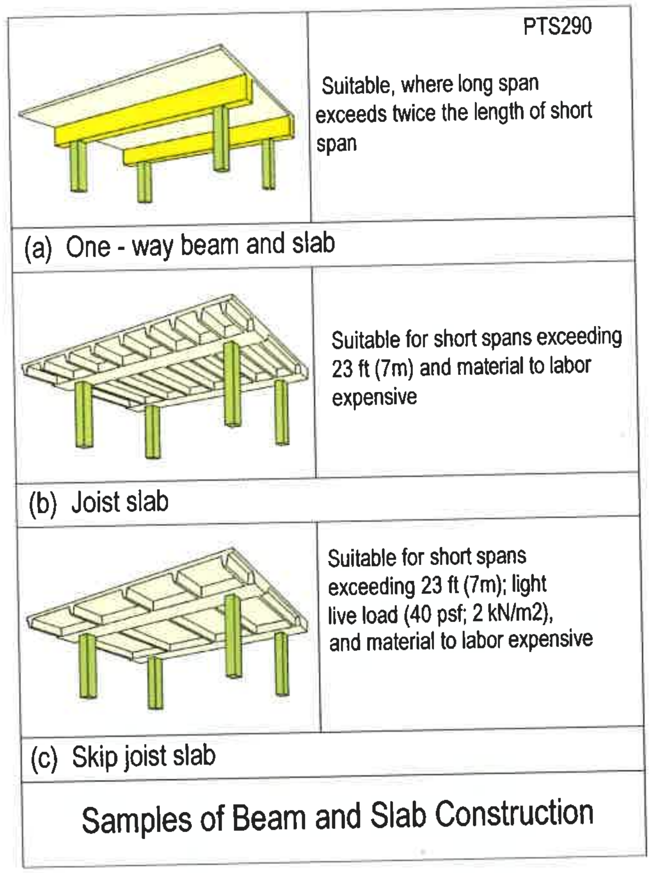


FIGURE 4.6.1A-1c Geometry of Common Floor System Types

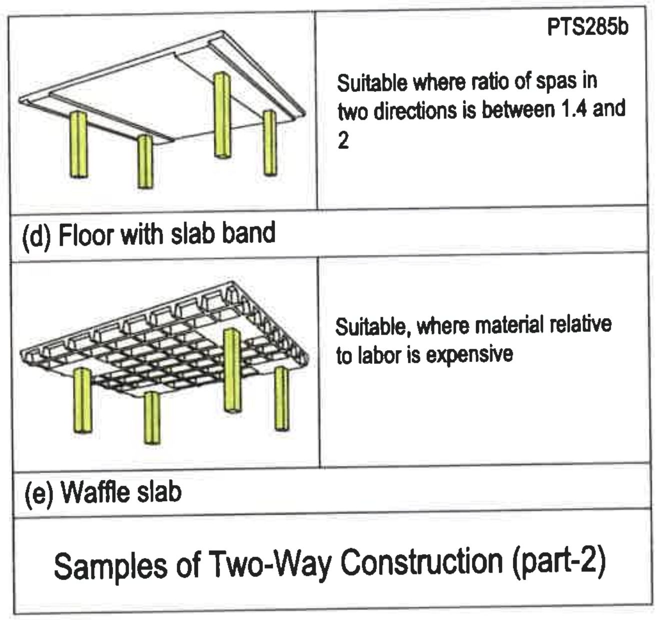


FIGURE 4.6.1A-1b

FIGURE 4.6.1A-2 Waffle Slab Construction in Himalayas (India; P276). Bricks on form provide the voids. Bricks that stick to cast concrete are left in place.

(a) Floor slab with plastic void generators (P279a)

(b) View of a plastic void generator

FIGURE 4.6.1B-1 Slab with Plastic Voids for Conventionally Reinforced, or Post-Tensioned Construction. The raised box provides a smooth soffit (Fortaleza; Brazil; courtesy Ing Caracas,Impacto; P279b)

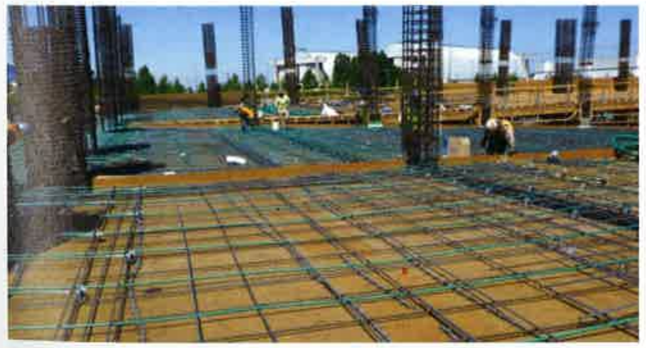


FIGURE 4.6.1B-2 View of a Typical One-Way Slab and Beam Construction for a Parking Structure in California. (beam are 65 ft (19.81m) long; 20 ft (6.10 m) apart; slab is 5.5" (140 mm) thick. Long plastic strips support the tendons at design height (P277)

FIGURE 4.6.1B-3 Voided Slab Construction (P278)

Spherical voids afford a greater slab depth, and consequently moment arm for increased strength, with reduced weight compared to a solid slab alternative of the same concrete material.

FIGURE 4.6.1B-4 Example of a Conventionally Reinforced Flat Slab Construction Using Foam Infills (Ryadth KSA; P280)

FIGURE 4.6.1B-5 Example of a Post-tensioned Floor System with Slab Band and Void Creating Foams (P281)

thickness. The dimensions are summarized in Fig. 4.6.2-5. Suggested plan geometry for use of slab bands is shown in Fig. 4.6.2-6. When using slab bands, tendons in the long direction are grouped along the longer span and lowered into the band. Tendons in the orthogonal direction are spaced uniformly in the short direction.

Results of finite element analyses indicate that the shear in the slab is resisted essentially by the entire cross section of the floor, as opposed to the band "stem," which would be the case of a "beam." For this configuration, column supports are designed for punching shear, as opposed to one-way shear normally checked for beam stems. A computer model of a floor system with slab bands is shown in Fig. 4.6.2-7.

4.6.3 Column Drops (capitals)/Drop Panels

Where added strength over a support is required, local thickening of slab can be used for increased capacity (Fig. 4.6.3-1). A *column drop*, also referred to as a *column capital*, or *drop cap*, is a small enlargement of the column-slab interface, with the primary objective to enhance the punching shear capacity. A drop panel is a thickening of generally smaller depth than column drop, but covers a larger area of slab soffit. The primary design objective of a drop panel is to reduce deflection and increase the negative moment capacity over the support region.

While both measures afford an increase in span length beyond that practical without drops, they do so for different reasons. Based on the ACI 318[12], for conventionally reinforced floors, a drop panel shall extend not less than one-sixth of the clear span in each direction, measured from the centerline of support, and project below the slab soffit not more than quarter of slab thickness[13]. The added thickness of drop panels can be included in the computation of the moment capacity at the face-of-support, while the primary contribution of column drop is considered to be an increase in the punching shear capacity. There is no such stipulation in ACI 318-11 for post-tensioned floors. A column/slab connection can also be constructed with a combination of drop cap and drop panel. Figure 4.6.3-2 illustrates several of the more common configurations of column capitals and drop panels used in building construction.

Based on EC2; each thickening in a floor system, be it a column drop, drop panel, or otherwise will be handled according to its contribution in stiffness; stress and strength, as determined by analysis and designed for.

The designation of thickened slab areas as drop caps or drop panels based on the foregoing dimensional ratios in ACI 318, and their treatment as distinct members are the legacy of the limitations in the analytical tools of former years. With today's analytical capability for floor slabs, both thickenings, as well as other changes in geometry of a floor system, can be treated in the same manner, each allowed for to duly participate in the response of a floor and to be designed for accordingly.

For analysis and design of post-tensioned slabs, with local thickenings around column supports, the following guidelines apply:

(i) Deflections and moments are calculated taking into account the geometry, and the associated stiffness of each thickening, regardless of the categorization of the geometry as a drop cap or drop panel.

(ii) For computation of the hypothetical stresses used in serviceability compliance of the code, actions computed based on the actual geometry are applied to the actual cross-sectional shape of the associated design sections. In other words, the column drop/drop panel designations do not influence the serviceability check of post-tensioned members

(iii) For strength computations, the added thickness is accounted for to contribute to the strength of a section.

Figure 4.6.3-3 shows the example of a column drop that in the presence of the beams at the same location does not measure up to the cost of its construction. It can be eliminated.

FIGURE 4.6.2-1 View of a Post-Tensioned Slab Band (P282)

12 ACI 318-11, Section 13.2.5

13 ACI 318-11, Section 13.3.7

FIGURE 4.6.2-2 View of a Slab Band Extending Over the Long Spans (P283)

FIGURE 4.6.2-3 View of a Slab Band Using Unbonded Tendons (P284)

FIGURE 4.6.2-4 Construction View of a Slab Band Using Grouted Tendons (P285)

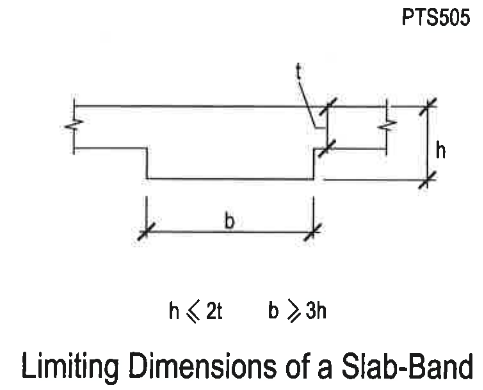


FIGURE 4.6.2-5 Dimensional Parameters of a Slab Band

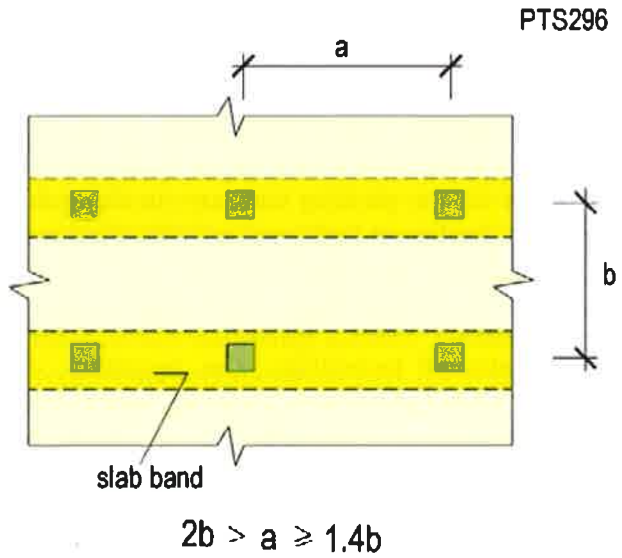


FIGURE 4.6.2-6 Parameters for Efficient Application of Slab Bands

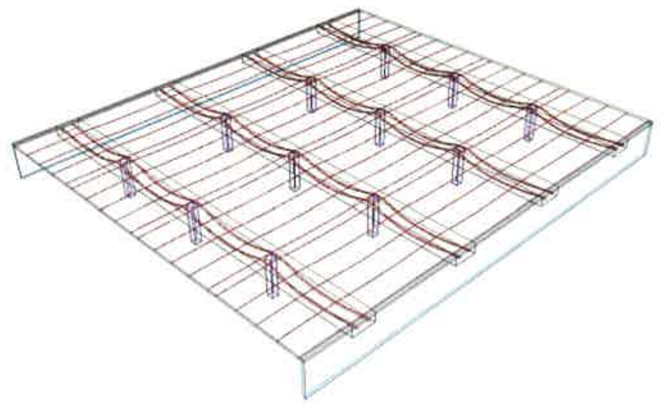


FIGURE 4.6.2-7 Analytical Model of a Floor System with Slab Band (HK Building Department validation floor) (P465)

4.6.4 Waffle Slabs

Waffle slabs are used extensively in many parts of the world, where the relatively lower cost of labor with respect to material provides an advantage over a solid slab alternative. Waffle slabs have also been used as an architectural accent for exposed ceilings. Designers distribute the tendons among the stems of the waffles in each direction, as illustrated in 4.6.4-1a, or group the tendons in each direction, and place the group tendons in solids along the line of columns (part b)[14].

A. Geometry and Construction: Fig. 4.6.4A-1 illustrates the cross-sectional geometry of a typical waffle slab unit. The waffles are mostly made solid around the columns, and along the lines of supports in longer spans.

An alternative arrangement for panels of fairly square and long spans is to break the panel up into an equal grid work of beams as shown in Fig. 4.6.4A-3 for the parking structure in Hong Kong. In effect, the layout replicates an over-sized waffle construction.

B. Analysis and Design: Using current computational technology, the geometry of a waffle slab can be modeled faithfully, where stems, solids, and the reinforcement can be represented in true form, size and location, such as in the example shown in Fig. 4.6.4B-1

At ULS, the load path selected for a waffle slab construction is the same as that of a solid slab. For design, the resistance capacity of all the waffles that fall within a design strip are considered to be available. As in flat slab construction, design strips typically extend to the tributary of column supports.

For simplified calculations, such as analysis and design based on extracted design strips from a floor system, the extent of simplification in modeling depends on whether the floor system is conventionally reinforced, or post-tensioned.

(i) Simplified waffle slab modeling for conventionally reinforced concrete floors. Where the analysis and design are based on the plate action of the floor (Section 4.6.1C), the primary targets of the analysis are deflections under service condition, and reinforcement at ULS. Deflection is function of waffle's modulus of elasticity (E), and its second moment of area (I). The computation of the reinforcement requires the correct concrete material (f'c; or fck), and the depth of the section. For this condition, by modifying the modulus of elasticity of concrete Ec, it is possible to model a waffle unit by an equivalent solid slab of the same depth and the same second moment of area, and design the substitute slab for deflection and computation of reinforcement.

(ii) Simplified waffle slab modeling for post-tensioned floor systems using EFM[15]. In using the Equivalent Frame Method for modeling of waffle slabs, the ribs in the direction of the frame and perpendicular to it are combined into representative solid members. Consider the solid cap/drop shown in Fig. 4.6.4B-2. Part (a) of the figure displays the contact surface of the ribs to the solid support regions. In the direction of the frame, the stems reaching the cap are combined into a single T-section (part

(a) View of a square column drop used for punching shear (P286)

(b) View of a tapered column drop used for punching shear (Kolkata) (P287)

[14] This tendon layout does not meet the ACI 318-11 stipulations, but is practiced, where ACI does not govern.

(c) View of a drop panel for improved negative moment capacity at support (P288)

(d) Strengthening around a non-standard column support configuration for improved shear transfer (P509)

Figure 4.6.3-1 Examples of Column-Support Strengthening

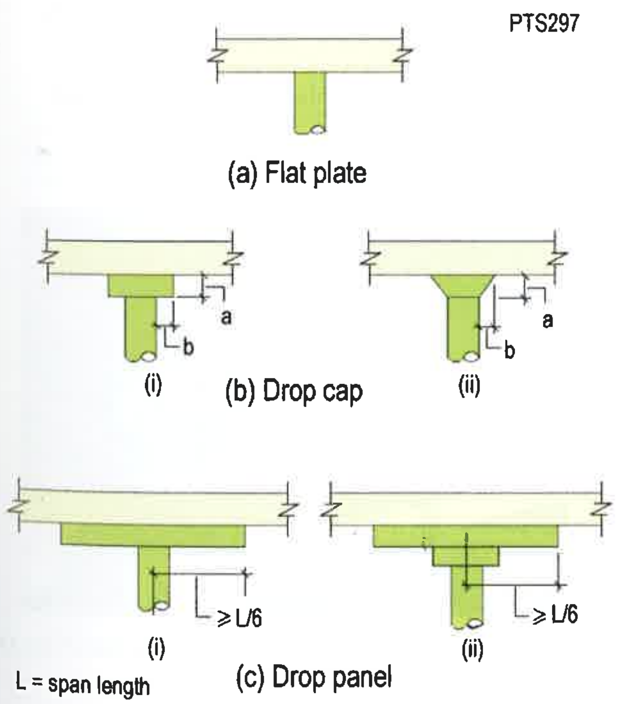


FIGURE 4.6.3-2 ACI318 Designation of Column Drop/Panel for Conventionally Reinforced Slabs

FIGURE 4.6.3-3 Example of a Non-effective Column Drop to Enhance Punching Shear Capacity (Trump Tower, Panama; P510)

b) with stem width equal to the sum of the individual stems of the associated ribs. Thus, the second moment of area of the region joining the solid sections of the two adjacent columns is kept unchanged. In the transverse direction, the stems in contact with the solid region are combined into a torsional member having the same cross-sectional torsional constant J as the sum of the individual stems reaching the solid block. This modeling scheme results in the representation shown in part (c) of the figure for a typical interior support. The modeled structure is treated as a two-way system using EFM.

The application of the modeling scheme proposed for use with the EFM is illustrated through the following numerical example.

EXAMPLE

Use the geometry parameters of the waffle slab panel given below, and idealize it for analysis and design using the Equivalent Frame Method (EFM). Refer to Figs 4.6.4B-4, using Figs 4.6.4B-2 and 3 for definition of parameters.

Structure:

As shown in Fig. 4.6.4B-4.

c = 1200 mm; h = 600 mm; bw = 250 mm (averaged); hf = 125 mm

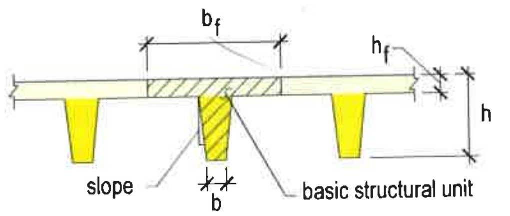
[15] EFM refers to the "Equivalent Frame Method" modeling scheme proposed by ACI 318, where a linear design strip is modeled to account for the biaxial response of the prototype floor. It is implemented in a number of commercially available software, such as ADAPT-PT

(a) Tendons Distributed among Waffle Stems (P512)

(b) Tendons grouped along the line of columns (P711)

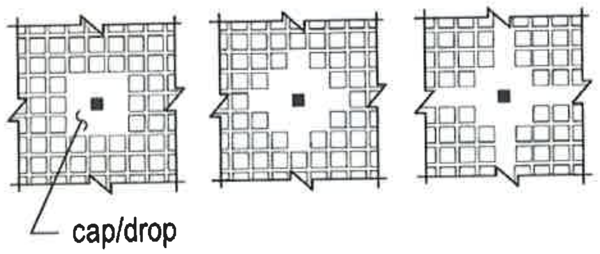
FIGURE 4.6.4-1 Views of Waffle Slab Construction
(Brazil)

PTS298



(a) Basic geometry of w... ...

PTS484



(b) Solid cap (c) Extended solid cap (d) Solid cap and band

FIGURE 4.6.4A-1 Geometry of a Typical Waffle Unit and Arrangement Options of Solids at Support

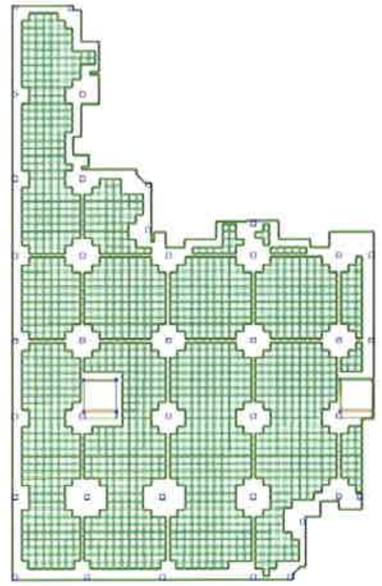
Two views of post-tensioned waffle slab construction are shown in Figs. 4.6.4A-2.

(a) Construction of a post-tensioned waffle slab
(Moscone Center San Francisco;P290)

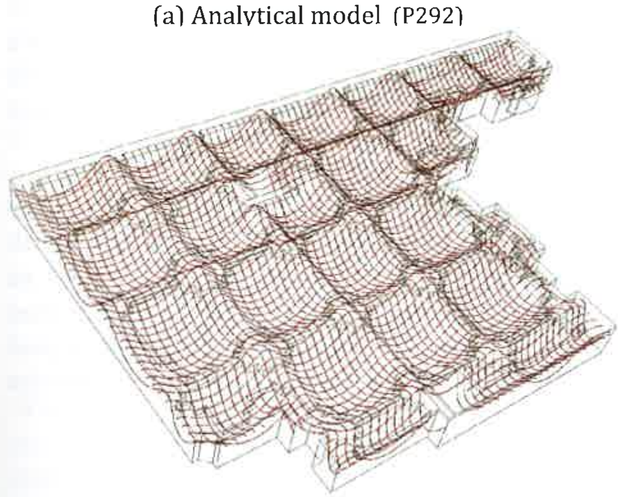
(b) Construction of a post-tensioned waffle slab
(South Africa; Vstructural; Courtesy Graemer; P291)

FIGURE 4.6.4A-2 Examples of Post-Tensioned Waffle Slab Construction

FIGURE 4.6.4A-4 Slab Panel Supported by a Square Grid of Beams (Hong Kong; P659)



(a) Analytical model (P292)



(b) Tendon layout of the waffle slab (P293)

FIGURE 4.6.4B-1 Analytical Model of a Post-Tensioned Waffle Floor System Construction
(BBR, India, Floor-Pro)

Required:
Calculate the effective width, b_e, of the stem for the torsional model member

Three ribs are in contact with the solid block at support:
Alternative (i): $b_e = (\sum b_w^3)^{1/3}$

$b_e = (3 \times 250^3)^{1/3} = 360.5$ mm
$b_e = 360.5$ mm $< (h - h_f) = 600 - 125 = 475$mm

Use b_e as the equivalent width of the torsional member in equivalent frame modeling.

4.6.5 Joist Slabs

Joist slabs (Fig. 4.6.5-1) are an alternative to waffle slab construction, where a rectangular arrangement of supports favors spanning the floor in one direction. The stems of the joists typically have the same cross-sectional dimensions as waffle slabs (Fig.4.6.4A-1). Provisions in ACI 318 for joist slabs relieve a number of provisions that are intended for beams not to apply to joists. For the structural modeling and analysis of post-tensioned joist slabs, ideally each joist would be represented on its own to arrive at an efficient design.

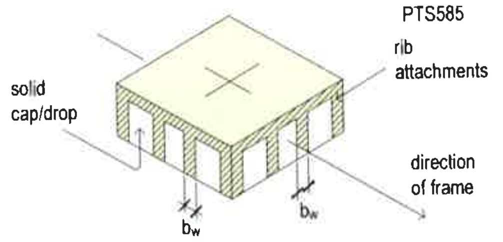
Figure 4.6.5-2 illustrates the computer model for the structural analysis of a joist slab floor system. Joists are selected in direction of longer spans. Typically, each of the joists is provided with one or two post-tensioning strands, as illustrated in part (b) of the figure.

Each joist along with its own flange tributary is designated as a "design strip." Figure 4.6.5-3a highlights four such design strips. In the longitudinal direction, the design strips selected each cover one half the width of the floor system (part b of the figure). The function of the five longitudinal joists is to distribute the loads among the transverse members. The tendon along the longitudinal joists is straight and positioned at the centroid of the floor system.

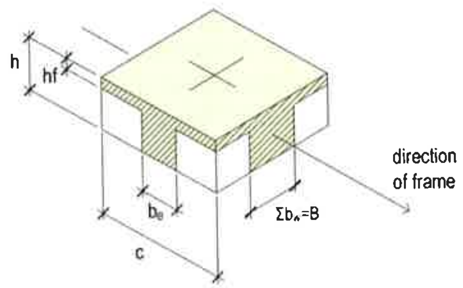
4.6.6 Beams

A. Construction: From structural standpoint and post-tensioning, choice of beams as structural members of a floor system is economically justified, where spans in one direction exceed approximately twice the length of the shorter side. For spans with aspect ratios from 1.4 to 2, slab bands, as described in Section 4.6.2 can be more advantageous. A practical and efficient application of beams is shown in Fig. 4.6.6A-1, where in one direction the spans of the parking structure are about 3.5 times the orthogonal direction. A beam and one-way slab construction with dimensions shown in Fig. 4.3.3-1is a frequent selection for the US parking structure projects.

The construction practice in a number of countries is to opt for beams between adjacent columns, irrespective of the structural necessity and the span dimensions. The universal inclusion of beams is viewed to provide the framing necessary to support the floor slabs, and to resist the lateral forces. The concept of column-supported slabs is not universally adopted. Figures 4.6.6A-2 and 3 show two examples, where modern construction practice would eliminate the beams between the closely spaced columns. Where the seismic risk is high, and slabs are not considered to be part of the lateral force resisting system, one option is to provide beams at selected locations to form moment frames with the adjoining columns.

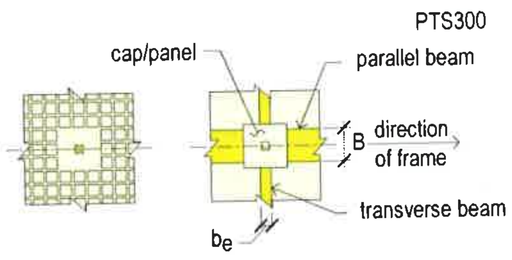


(a) Cap/drop with rib attachments



(b) Structural model

FIGURE 4.6.4B-2 Analytical Model of the Solid Region of a Waffle Slab Floor



(a) Prototype (b) Structural model

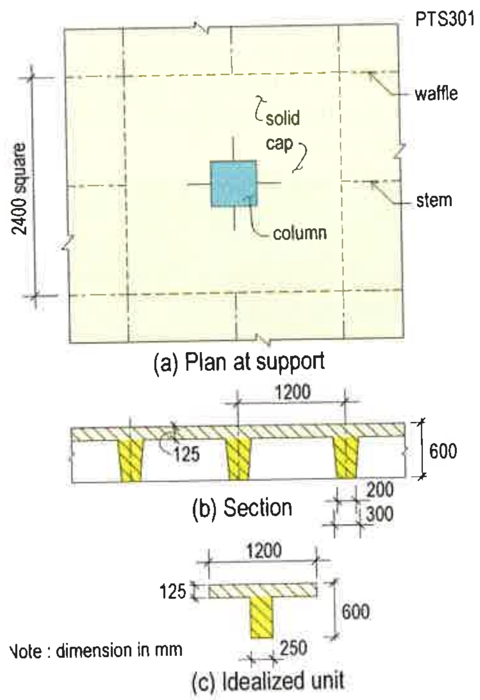
FIGURE 4.6.4B-3 Idealization of Waffle Slab Construction Using EFM of Analysis

Where the lateral force resisting system of a concrete frame is through shear walls, it is neither required, nor is it practiced in the US to interconnect the shear walls to the columns of the structure by a grid of interconnecting beams.

In buildings, where shear walls are designated to resist the entire lateral forces, beams where necessary will be designed for gravity loads only. Figure 4.6.6A-4 illustrates a concrete frame with shear wall construction in San Francisco Bay Area – a high seismic region of the US. Note that no beams are provided between the columns.

B. Analysis and Design: Beams can be downturned, upturned, or mixed as shown in part (a) of Fig. 4.6.6B-1a. Early generations of FEM analysis tools, and a large number of currently available software, formulate the beam in the background as is shown in part (b). Modeling a beam as illustrated in part (b) significantly simplifies the analytical formulation. Such a modeling scheme may be considered permissible for deflection computation of conventionally reinforced concrete floors, since the analytical model underestimates the stiffness of the structure, and results in conservatively larger deflections.

Subsequent to the analysis, at design stage, early FEM formulations are based on assigning an arbitrary width of the adjoining slab to act as flange of the beam stem in resisting the computed forces. Alternatively, the beam stem is designed to resist the entire forces computed for it from the



(a) Plan at support

(b) Section

(c) Idealized unit

FIGURE 4.6.4B-4 Example of a Waffle Slab Modeling for Analysis

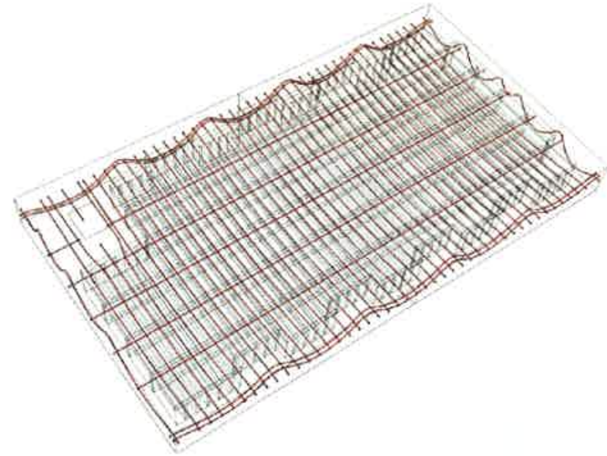
FIGURE 4.6.5-1 View of a Joist Slab Construction (P295)

analysis model. The objective of the simplification in the analytical model is to align the centroid of the beam with that of the slab, as shown in part (b). The alignment simplifies the expression for the equilibrium equations between the slab and the beam for transfer of force from one to the other (handshake of force between two adjoining elements). The physical representation of the structure as shown in part (a) reflects a disruption in the continuity of the centroids of the adjoining parts (part c) of the figure – a condition that leads to extended formulation.

When dealing with post-tensioned floors, it is the eccentricity of the axial force with respect to the centroid of a section that provides the beneficial post-tensioning moments. A modeling scheme as shown in part (a) of the figure fails the favorable effects of post-tensioning. Chapter 8 offers detailed discussion on the analytical modeling schemes of post-tensioning.

Another approach adopted in a number of commercially available software is to assume the beam stems disconnected from the slab as shown in part (b) of Fig. 4.6.6B-2. In this approximation, the analysis will conclude with" beams" subjected to pure bending, with zero axial force. Proper analysis considers the continuity of the stem with the flange (part a), in which case the compression in the flange will result in a net tension in the beam stem (part a-ii). The design of the beam will involve the bending of the stem, along with the tensile force in the beam stem. The tensile force in the stem is necessary to balance the compression in the flange. The inclusion of tension in the design of the beam stem leads to a universally correct determination of the beam reinforcement,

(a) Soffit view of joist slab construction (P296)

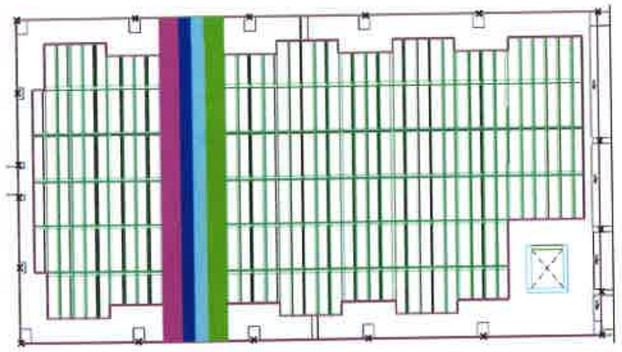


(b) See-through view showing tendon layout (P297)

FIGURE 4.6.5-2 Views of Analysis Model of a Joist Slab Floor Construction (Ferca Norte; Chile)

irrespective of the extent of the flange envisaged by a designer to be applicable as "effective flange," and act with the beam stem. In effect, the question of "effective width" for strength design does not arise. This concept is illustrated by way of several numerical examples in Section 4.8.3.3.

In summary, the proper formulation of the analysis model for flanged beams, based on part (a) of Fig. 4.6.6B-2, will not require the definition of an "effective width" to act with the beam stem, in order to determine the correct design reinforcement. Where a design requires the definition of "effective width", in order to determine the strength reinforcement, it is due to simplifying assumptions made in the underlying analysis procedure.



(a) Typical design strips in joist direction (P298)

(b) Design strips for the longitudinal direction (P299)

FIGURE 4.6.5-3 Design Strips in the Two Orthogonal Directions (Ferca Norte; Chile)

(a) Interior view of a beam and one-way slab construction (P300)

(b) View of a parking structure (P301)

FIGURE 4.6.6A-1 View of a Parking Structure with Beam and One-Way Slab Construction (Moffett Towers Parking Structure, DSI, California)

FIGURE 4.6.6A-2 Example of a Beam and Slab Construction (Peru; P304)

FIGURE 4.6.6A-3 Example of a Beam and Slab Construction in Turkey (P305)

The discussion on "effective width" is further continued in Section 4.8.3 of this Chapter.

Figure 4.6.6B-3 is another attempt to illustrate the difference in response of a beam/slab connection where the two are modeled disjointly.

4.6.7 Support Conditions; Releases and Stiffness Assignment

The shape of a structure is mostly determined by its expected function, and the architectural considerations. If analyzed in its physical form, as it appears in its external shape, the distribution of forces among the members of the structure will be based on the volumetric gross geometry of its members, and the material properties of each. An analysis based on the physical geometry and connectivity of the members of a concrete frame might not lead to the most efficient distribution

FIGURE 4.6.6A-4 View of a Beamless Floor System in a High Seismic Risk Region. Shear Walls Provide the Lateral Force Resisting System (San Francisco Bay Area; P306)

of forces among the members of the load path selected by the structural engineer. Prior to the initiation of analysis, as a central step in the load path assignment, a design engineer will select support conditions; specify releases at selected connections; or assign stiffnesses to the members different from the values dictated by the physical outline of the members, in order to arrive at a force distribution through the concrete frame that is more amenable to a successful design.

A. Support Connections: Analytical assumption regarding the connection of a concrete frame to foundation impacts the distribution of forces at the first elevated deck and to a lesser degree at the next few levels above it. Where fixed connections are not necessary to develop the required strength, hinged connections may be beneficial. In particular, at the perimeter of a building's footprint hinged connections can lead to a better performance of the first elevated floor. In this case, the forces generated from the shortening of a post-tensioned floor at the first elevated deck will be reduced. Assumption of hinged connection is common in bridge construction, such as the example shown in Fig. 4.6.7-1.

B. Member Connectivity and Stiffness Assignment: In structure frames, such as the one shown in Fig. 4.6.7B-1, a more efficient design can be obtained, if the connection between the uppermost exterior column and the beam it supports is assumed hinged. The stiffness of the long-span beams, governed by their cross-sectional geometry, is several times that of the

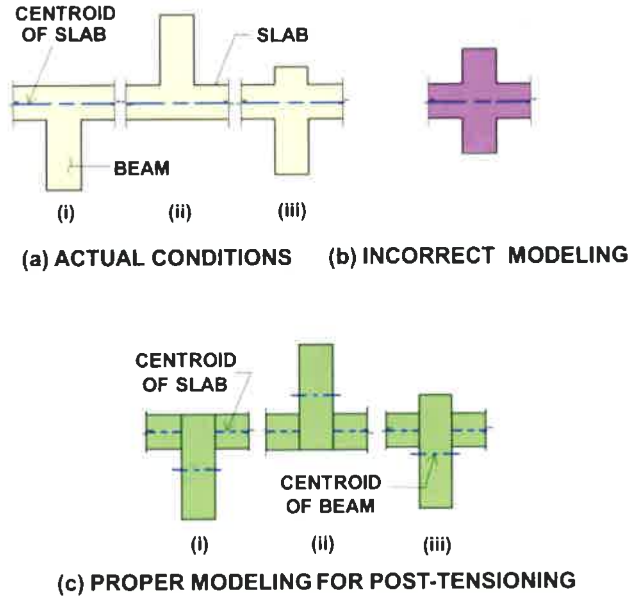


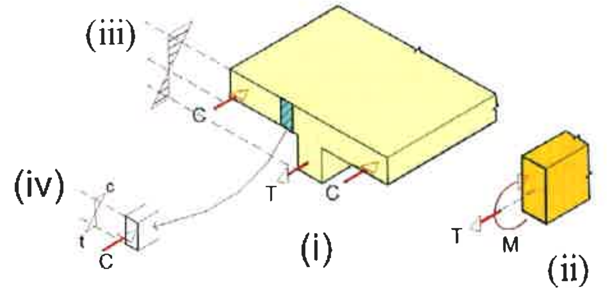
FIGURE 4.6.6B-1 Beam Modeling Schemes Used in Analysis Algorithms (P658)

uppermost column. This results in a large moment at the column connection, without adequate axial force from above. It is not uncommon among designers to assume a hinge at the connection between the uppermost column and the beam (part b). Alternatively, a lower stiffness value is assigned to the uppermost column prior to the analysis.

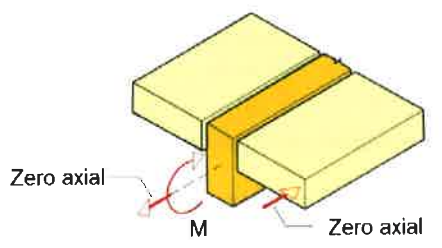
It is not common to follow the hinge assumption at analysis, through its physical implementation in construction. The understanding is that, the lack of adequate moment resisting reinforcement resulting from the hinge assumption in the analysis is likely to lead to hairline cracks around the monolithically cast connection and relieve the moment. However, where critical, physical hinges are detailed for construction (Fig. 4.6.7B-2). The connection shown in the figure features a bond breaker at the top of the column and a central dowel passing through the column into the beam.

To divert a lesser design force to a member than dictated by the member's geometry is handled through assignment of a reduced stiffness to the target member prior to analysis. Several commercially available software allow for stiffness assignments by the designer[16]. As an example,

[16] ADAPT-PT for post-tensioned slabs and beam frames; www.adaptsoft.com



(a) Flanged beam with compatibility of deformation between the stem and flange (P660)



(b)Flanged beam without compatibility of deformation between stem and flanges (P660)

FIGURE 4.6.6B-2 Examples of Rigorous and Approximate Flanged Beam Modeling for Analysis and/or Design

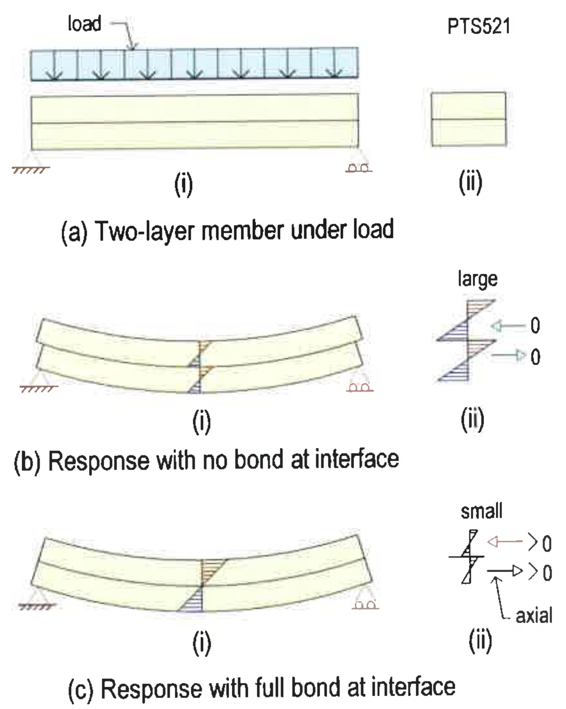


FIGURE 4.6.6B-3 Compatibility Among Components of Multi-Part Members

for the uppermost columns of parking structure frames, such as the one snown in Fig. 4.6.7B-1, an alternative to assignment of hinge at the connection is selection of a column stiffness 30 to 50 percent of its gross value.

Observations from the last major earthquake in California (Northridge, 1994) concluded that stubby members, such as short columns between ramps of parking structures, are likely to develop large end moments. The moments result from imposed horizontal displacements, when these non-seismic short columns follow the movement of members assigned to resist earthquake. This, in turn, leads to high shear values in the short columns, and possibility of column failure. An option to mitigate the potential failure is the provision of physical releases at one or both ends of the column connections as detailed in Fig. 4.6.7B-3. The alternative is to detail the short columns for ductile response to the imposed design displacement.

For post-tensioned slabs that are not part of the primary lateral force resisting system of a concrete frame, assume no moment transfer between the slab and the walls at analysis stage. But, detail the connection for a "ductile" response, as opposed for a computed moment transfer. Figure 4.6.7B-4a illustrates a slab-core wall detail that is shown experimentally [Klemencic et al, 2006] to possess a rotational capacity (ductility) in excess of that likely to be experienced from the maximum drift of design earthquake. In this detail tendons terminate at the face of the core wall.

Figure 4.6.7B-4b, shows a similar slab/wall connection, in which the tendons are placed within a cut made in the thickness of the core wall. Detail in part (a) of the figure is simpler to construct and is shown to exceed the design requirements.

C. Release to Allow for Slab Shortening from Post-Tensioning: At the lowest floor level of post-tensioned buildings, and to a lesser extent at the second level, the foundations can significantly restrain the shortening of a floor slab from post-tensioning. Recognizing that the shortening is prerequisite to the development of axial compression necessary for the design-intended performance of the floor, various schemes have been developed to allow adequate shortening to take place. Figure 4.6.7C-1 shows two options used in the US for low rise buildings. Part (a) of

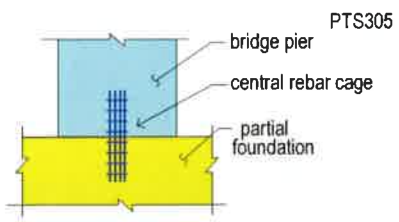


FIGURE 4.6.7A-1 Hinge Simulation at Bridge Pier/Foundation Joint

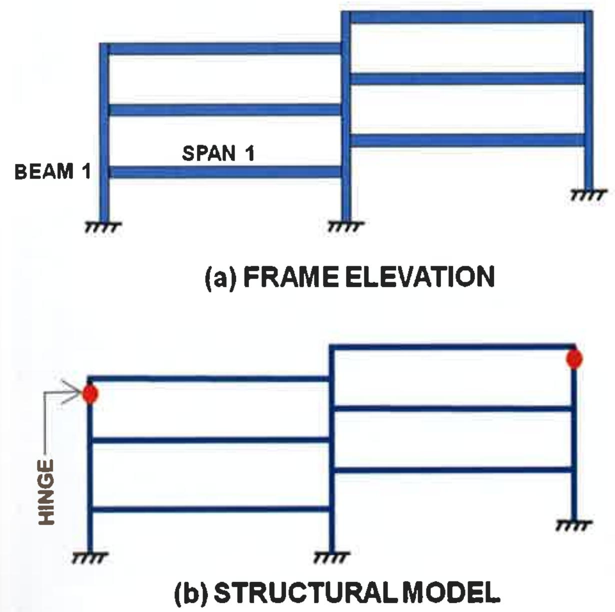
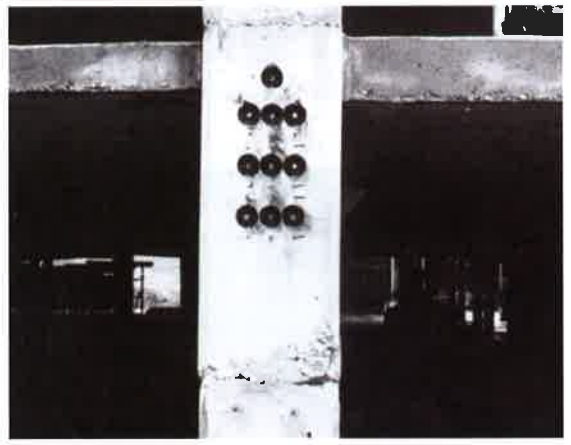


FIGURE 4.6.7B-1 Example of Hinge Assignment at Selected Joints (P662, P663)

the figure illustrates an interior bearing wall that supports a post-tensioned slab, but is not intended to restrain its shortening along the wall. A bond breaker at the top of the wall and compressible material over the vertical dowels that penetrate into the slab allow the slab to move relative to its supporting wall. Part (b) of the figure is a similar condition with a bond breaker over the wall and compressible material over the dowels. However, in this case, the dowels extend through the slab to the wall above to provide the continuity required in design.

There are other provisions, such as delayed strips (closure strips), and released joints. A good practice at the connection of a large podium slab to the building tower is to provide closure strips and release at the tower connection as shown in Fig. 4.6.7C-2.

(a) Physical hinge at column-beam connection



(b) Physical hinge at column-beam connection

FIGURE 4.6.7B-2 Physical Hinge at Colum-Beam Connection (P307; P308)
(Grossmont Parking Structure; San Diego, CA, Libby Structural Engineers)

4.6.8 Other Floor System Examples

Developments in higher strength concrete, increase in cost of labor, and innovations in computational technology have led to construction of floor slabs that are simple to form, use less material, and provide greater flexibility in floor layout. Post-Tensioned flat slab construction has fueled the progression of imaginative and daring architectural floor plans. The following are several examples of what can be avoided, and successful examples in selection and construction of concrete floor systems.

Figure 4.6.8-1 is an example of a flat slab in a multistory office building with shear wall construction. The slab thickness is of the order of (span)/45. The building is located in one of the highest seismic zones of the US. There are no edge beams, nor are there beams connecting the shear

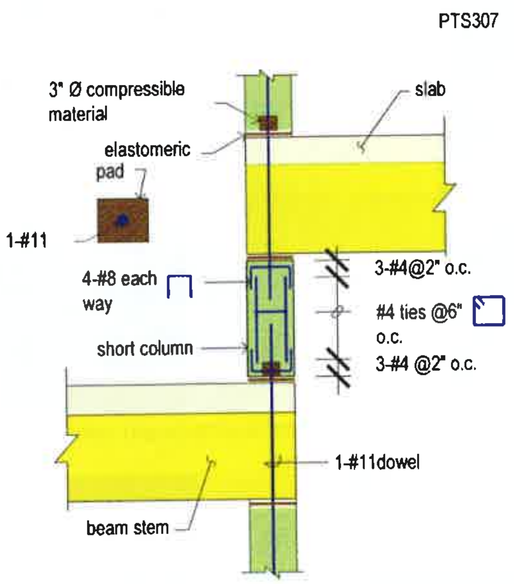


FIGURE 4.6.7B-3 Hinge Creation at Short Ramp Columns of Parking Structures

walls to the columns. Floor cover being typically carpet, tiles, stone, or wood is placed on the cast surface of the floor slab. Partitions are typically light.

Two examples of aspirational floor plans specifically suitable for the flexibility post-tensioning offers in support layout are shown in Fig. 4.6.8-2.

Figure 4.6.8-3 illustrates two examples of complex and inefficient concrete frames for a structure with spans of about 4m (13 ft). The practice among contractors in several regions of the world is to frame and cast beams between the column, to be followed by forming and casting the slab to be supported on the beams. Ideally, the beams should be eliminated altogether.

The necessity of beam between the columns of a floor system is deeply engrained in the construction culture in parts of the world. Irrespective of difficulty, beams are provided and cast first. Figure 4.6.8-4 illustrates the effort in construction of beams for a beam and slab construction in a regular floor layout of moderately short spans.

The closely spaced multiple beams of the post-tensioned floor in Fig. 4.6.8-5a are expensive to form, and not all necessary. Likewise in part (b) of the figure, the horizontal thickening of the slab along its edge between the downward apron and

(a) 301 Mission street, San Francisco, CA (P309)

(b) Marina Tower, Beirut (Dar Al Handasah; P310)

FIGURE 4.6.7B-4 Views of Shear Wall – Slab Connection

the slab does not contribute much to the structure behind the adjoining deeper downturned member along the slab edge.

4.7 LOADING

The primary loads on the structure are selfweight, superimposed dead load, "prestressing" and design live load. Structures are also checked againt snow, wind, earthquake and possibly other effects, such as temperature, creep and shrinkage. Consideration of prestressing as an applied load depends on the design methodology adopted. Chapter 11 describes the available options in detail. In addition, there can be special loads relating to the occupancy of the structure, such as fire trucks. In the following, we review the primary loads, namely selfweight, superimposed dead load, live load and prestressing.

(a) Provision for release of a discontinued supporting wall (P311)

(b) Provision for release of a continuous wall (P312)

FIGURE 4.6.7C-1 Two Examples for Release of a Wall to Allow Slab Shortening

4.7.1 Selfweight

Selfweight is computed from the volume geometry of a floor, and the unit weight of the reinforced concrete used. Typical unit weight values, including the reinforcement and prestressing for floor systems are:

- Normal weight concrete 150 pcf; (2400 kg/m^3)
- Semi-light weight concrete 110 - 140 pcf ; (1750 kg/m^3 - 2240 kg/m^3)
- Lightweight concrete 95 -110 pcf ; (1520 kg/m^3 - 1760 kg/m^3)

4.7.2 Superimposed Dead Load

Superimposed dead load is the permanent cover over or ceiling buildup of a concrete floor. In the US

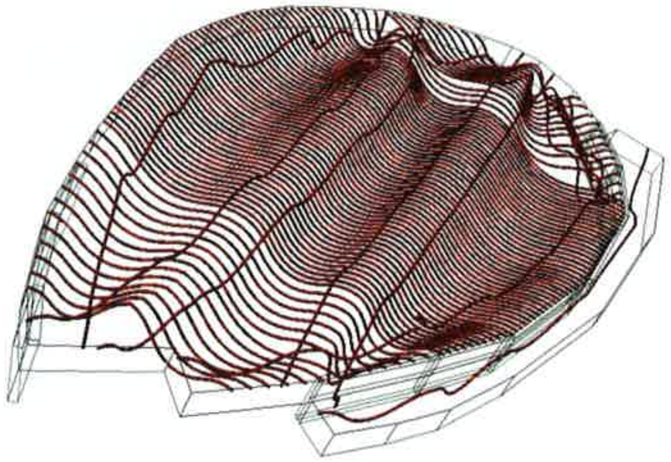
(c) Tower rising above a multilevel podium (P313a)

(b) Podium provided with closure strip to control cracking (P313b)

FIGURE 4.6.7C-2 Release of Podium Slab from Walls and Tower through Delayed Construction Strips (Dubai Tower, Freyssinet Gulf)

the practice in most cases is to finish the surface of a concrete slab level and smooth. Floor covers, such as carpet, wood panels or tiles are mostly placed directly on the finished floor, without additional buildup. The weight of the superimposed load is typically small. However, in most instances a minimum of 20 psf (1 kN/m^2) is assumed to allow for partition loads. Partitions are assumed to be light and subject to change in location.

FIGURE 4.6.8-1 View of a Typical Post-Tensioned Flat Slab Construction in California (P314)



(a)Tendon layout of floor slab (Croop, Florida; P315a)

(b) Floor plan of multi-story building in Florida (P315b)

FIGURE 4.6.8-2 Examples of Floor Plans of Two Buildings in Florida

(a) Framing to support floor slab (P316)

(b) Completed framing and floor slab (P317)

FIGURE 4.6.8-3 Example of Inefficient Geometry of Floor System Framing (Spans approximately 12' (3.6m))

In a number of regions, such as UAE, often the residential and commercial building floors are finished with a rough surface. The surface is then covered with one inch or more of mortar and leveled before placing stone or tile. The specified superimposed load can be as high as 60 to 70 psf (3 to 3.5 kN/m2). In other locations, such as South Korea, it is not uncommon to add up to 7" (175 mm) of topping over the finished structural surface of floor to cover heating and other installations.

For parking structures, where the structural surface is exposed, common practice in the US is to assume a superimposed dead load of 5 psf (0.25 kN/m^2) to account for mechanical installations.

4.7.3 Live Load

The intensity of live load is defined based on the intended function of a floor system. The recommended minimum values for the common cases of residential and commercial construction based on IBC[17] are:

[17] IBC 2009 Table 1607

(a) Beam under construction (P664)

(b)Beam under construction (P665)

FIGURE 4.6.8-4 Example of a Beam and Slab Floor System under Construction (Mexico)

Suggested values for "design" live load:

- ❖ Residential 40 psf (2 kN/m^2)
- ❖ Office 50 psf (2.5 kN/m^2)
- ❖ Shopping malls ; stores 75 psf (3.75 kN/m^2)

The application of "design" live load is generally broken down to: (i) fraction of "design" live load that is likely to be present most of the time during the normal use of the structure, and (ii) live load that the structure is likely to experience occasionally. The former is the primary cause of time-dependent effects, such as long-term deflection. In both instances, the structure is expected to remain serviceable. ACI 318 refers to these two as "sustained load" and "total load" respectively. The "sustained" load being the one causing time-dependent long-term effects, and

(a) Superfluous beam framing for a post-tensioned parking structure floor (P666)

(b) Redundant beam between slab an downturned apron (Dubai; P319)

FIGURE 4.6.8-5 Examples of Concrete Frames with Potential for Improvement

the "total" load generally having short term effects. The corresponding terminology used in EC2 is "quasi permanent," for "sustained" and "frequent" for "total." The ratio of sustained load to total load assumed is generally between 0.2 and 0.3. For certain load combinations ASCE recommends a ratio as high as 0.5. The live load values listed above refer to "design live load," which is the same as "total" or "frequent" loads. The "sustained" live loads will be a fraction of the values quoted.

The "sustained" live load is used to check deflections, stresses and crack width in a floor. These are associated with a floor's function under normal conditions. Wind and earthquake, as well as other transient scenarios, are not included as part of the in-service deflection and stress checks. Transient loads are accounted for in load combinations for strength check (ULS), and drift of a building, where applicable. Common load combinations for code compliance are further discussed in Sections 4.10.1 and 4.11.1 of this Chapter.

Rare vehicular loads, such as a fire truck in emergency situations are viewed to be of the same category as loads from wind or earthquake. Due to rare occurrence and the transient nature, deflection, stress and cracking checks are not performed for fire truck loading. However, the adequacy of the floor is checked for its strength limit (ULS).

4.7.4 Prestressing

In the traditional analysis procedures used for post-tensioned buildings, and in particular when using the "load balancing" procedure [Aalami, 1990], the prestressing is viewed as an" applied load," similar to dead and live loads. Contemporary analysis procedures, however, view the prestressing as a component that resists the applied loads, somewhat similar to rebar [Aalami, 2000]. In the latter scheme, post-tensioning tendons are treated as "discrete" constituent of a member. Section 4.8.1 outlines the principles and application of load balancing, where post-tensioning is treated similar to other loads. The discrete tendon modeling is detailed in Chapter 11.

Using load balancing, the forces from prestressing are broken down into two groups, namely: those that cause bending in the member, and those that result in axial precompression. It is assumed that the effects of the two groups of forces are uncoupled. Forces from prestressing that cause flexure in the member are used to calculate the moments and shears. The precompression due to prestressing is then superimposed on the stresses obtained from the flexural analysis. Further, moments and shears in the member from prestressing may induce reactions at the connection of the member to its supports. These reactions are referred to as "hyperstatic" or "secondary" actions. Their influence in the ultimate strength (ULS) of a member is accounted for.

4.7.5 Wind/Earthquake/Special Loads

Where a floor slab is considered to be part of the primary lateral force resisting system of a concrete frame, there are specific design steps and detailing to follow. The treatment of slabs as part of the primary lateral force resisting system of a concrete building is beyond the scope of this volume, where the focus is the presentation of basic concepts and design for gravity loads. Volume II covers the design of floor systems under lateral loads.

4.8 PRESTRESSING

This Section begins with the description of the widely used load balancing method for design of post-tensioned members. The introduction is followed by specific aspects of prestressing design concepts and application. The Section concludes with practical recommendations on tendon layout, and the differentiating aspects of the two principal post-tensioning systems, namely " bonded" and "unbonded" options. The contemporary modeling of post-tensioned members is dealt with in Chapter 11.

4.8.1 Load Balancing

Load balancing is a simple procedure for design of post-tensioned members. Load balancing procedure enables the engineers to handle the structural design of post-tensioned beams and slabs within the limits of the knowledge and tools that are used for the familiar design of dead and live loads. Simple load balancing was first introduced by T.Y. Lin [Lin, 1963] and later generalized by Aalami [Aalami, 1990]. The introduction of load balancing fueled the extensive application of post-tensioning in building construction in the US in early 1960s, and beyond. The application of simple load balancing is limited to common floor construction with simple floor geometry and uniform slab thickness. Extended load balancing (Sub-section C) or other design schemes for post-tensioning (Chapter 11) are used for geometrically complex floor systems and bridge construction.

The following reviews both "simple" load balancing for members of uniform section, and "extended" load balancing for members of general cross-sectional geometry.

While the concept of load balancing applies to tendons of general shape, its common application is based on the following premises.

- ❖ Tendon force along its length is constant.
- ❖ The effects of bending and axial forces in a member are de-coupled. Each can be calculated separately, and the outcome added up for the final result.

- ❖ Tendon follows the shape of a parabola or a straight line

The last item is for ease of computational effort, as opposed to restriction in the load balancing concept.

A. Basic Tendon Profile: We start by reviewing the shape of a tendon and the associated forces. Figure 4.8.1A-1 illustrates the basic components of a tendon in shape of a half parabola, extending from a low point with zero slope to a high point distance L from the low point. This is the basic shape from which other parabolic tendon profiles can be constructed. The free body force diagram of this half-parabola tendon is also shown in the figure. The tendon force P is assumed to be constant. For the geometry and the force P shown, the governing shape and force relationships are also given in Exp 4.8.1A-1 through 4. The lateral distribution of the force w_b shown is in equilibrium with the tendon forces. The tendon exerts a uniform upward force to the concrete member that contains the tendon.

In the following W_b is the total load, and w_b is the intensity of the balanced load per unit length of the member

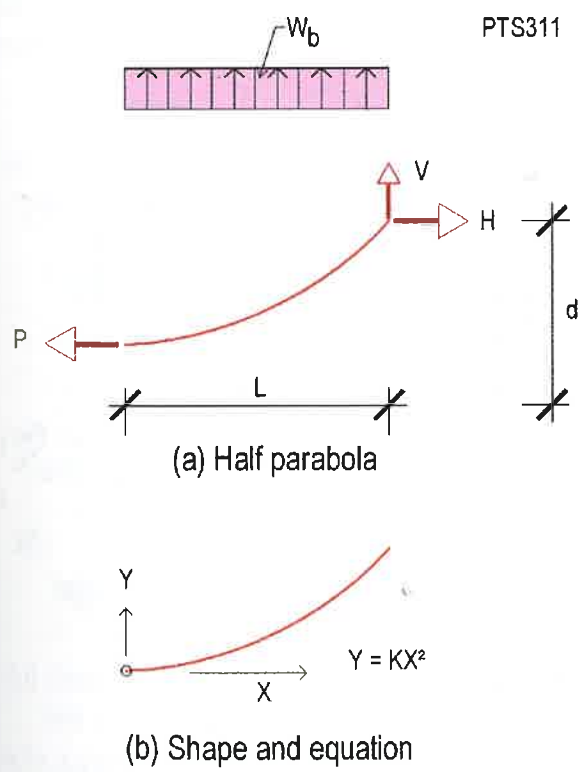


FIGURE 4.8.1A-1 Basic Unit of a Parabolic Tendon

$$y = \left(\frac{d}{L^2}\right)x^2 \qquad \text{(Exp 4.8.1A-1)}$$

$$P = H \qquad \text{(Exp 4.8.1A-2)}$$

$$W_b = \frac{2Pd}{L} \qquad \text{(Exp 4.8.1A-3)}$$

$$w_b = \frac{W_b}{L} \qquad \text{(Exp 4.8.1A-4)}$$

Figure 4.8.1A-2 illustrates several other shapes that are formed from combinations of simple parabola units.

The uplift (balanced load) w_b of the shapes shown in Fig. 4.8.1A-2 are:

- ❖ For symmetrical and continuous tendon (part a) of the figure, the intensity of the balanced load w_b is:

$$w_b = \frac{8Pa}{L^2} \qquad \text{(Exp 4.8.1A-5)}$$

- ❖ For the non-symmetrical and continuous (part b) of the figure, first we calculate distance "c," and use this distance to determine w_b

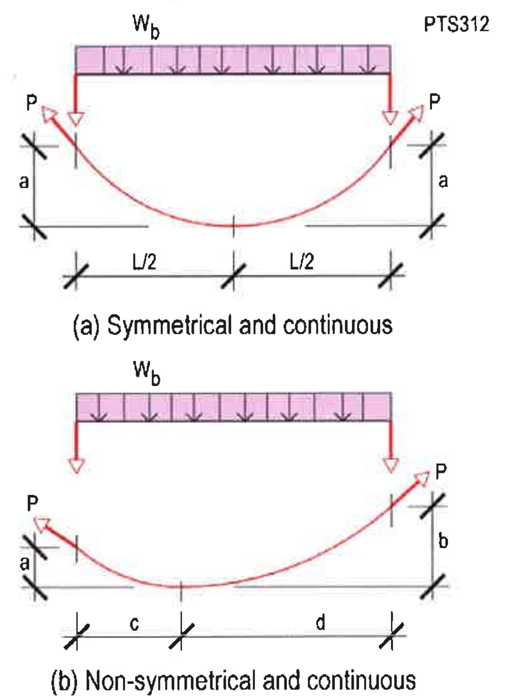


FIGURE 4.8.1A-2 Free Body Diagrams of Continuous Parabolic Tendons

$$c = L\frac{\sqrt{a/b}}{1+\sqrt{a/b}} \qquad \text{(Exp 4.8.1A-6)}$$

Where, L is the total length of tendon span (shown in part a).

$$w_b = \frac{2Pa}{c^2} \qquad \text{(Exp 4.8.1A-7)}$$

To facilitate tendon placing on job site, the low point of a tendon is often selected to be at mid length between the two ends (Fig. 4.8.1A-3). For this configuration, each half tendon will have its own parabolic unit and the associated forces. Using the values given in Fig. 4.8.1A-1, there will be two sets of upward forces as shown in the following figure.

In the above figure:

$$w_{b1} = \frac{8Pa}{L^2} \qquad \text{(Exp 4.8.1A-8)}$$

$$w_{b2} = \frac{8Pb}{L^2} \qquad \text{(Exp 4.8.1A-9)}$$

Evidently, tendons cannot accommodate sharp bends along their length. Over the supports tendons are generally placed with zero slope with respect of the member's centroidal axis (Fig. 4.8.1A4). The shape is referred to as reversed Parabola.

Each half portion of the reversed parabola has its own upward or downward force as shown in Fig. 4.8.1A-5. The reversed parabola has an inflection point as marked in the figure. The inflexion point falls on the line that joints the two ends of the reversed parabola. The total upward force W_1 is

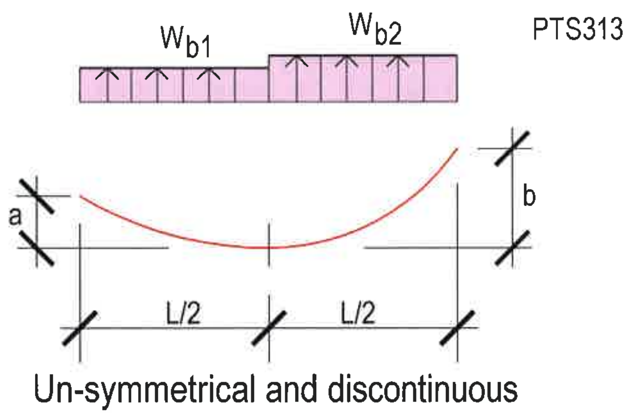


FIGURE 4.8.1A-3 Uplift Force Diagram of Un-symmetrical Parabola with Low Point at Center

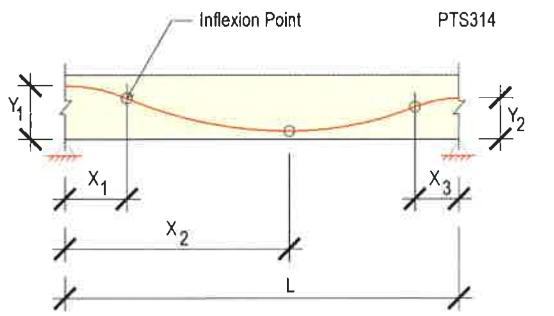


FIGURE 4.8.1A-4 Reversed Parabola in Place at Interior Span

equal to the total downward force W_2. Irrespective of the value "a" for the distance to the inflection point, the total of the upward force (W_1) will equal the downward force (W_2).

B. Simple Load Balancing: Figure 4.8.1B-1a illustrates a simply supported member of uniform cross-section under the externally applied dead (P_D) and live (P_L) loads and a post-tensioning tendon in shape of a simple parabola. The force in tendon is constant and equal to P. The tendon is anchored at the centroid of the member at each end.

In the load balancing approach, the post-tensioning tendon is assumed removed from its duct and

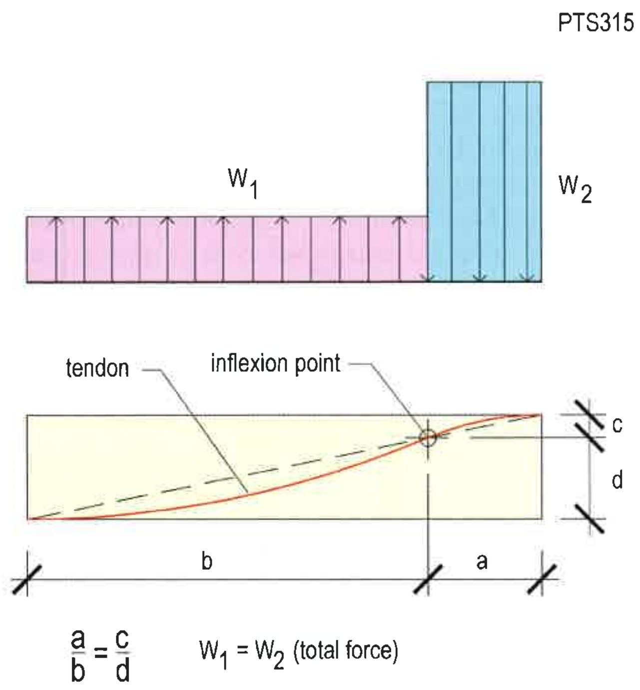


FIGURE 4.8.1A-5 Uplift Force and Geometry of a Continuous Half Reversed Parabola

replaced by the forces that the tendon exerts on the structure when in place. Part (b) and (c) of the figure illustrate the removed tendon and the associated forces. For the purpose of analysis, the post-tensioning forces on the structure (part b) are broken into two parts: those that are perpendicular to the member (w_b), and those that act along the centroid of the member. The grouping of the actions is shown in parts (d) and (e) of the figure. The loading perpendicular to the member is what is referred to as the balanced loading (w_b). The force (P) along the member is the component of axial compression due to post-tensioning. The axial component of the tendon force P acting at the centroid of the member in part (e) results in a uniform compression. The forces w_b along with the dead and live loads will result in bending of the member. The breakdown of the tendon force into bending and axial effects is shown more explicitly in Fig. 4.8.1B-2.

The two load systems, namely bending and axial are analyzed separately and the outcome combined.

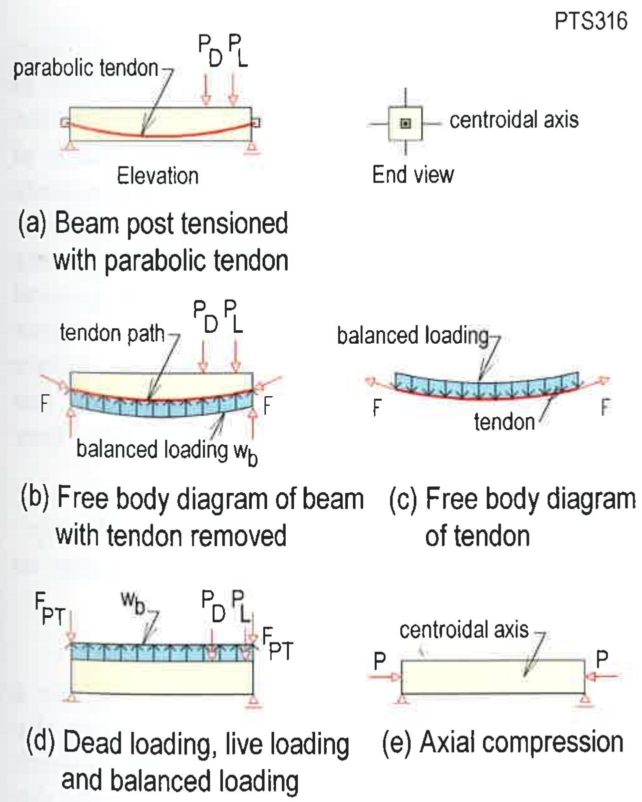


FIGURE 4.8.1B-1 Presentation of Balanced Loading

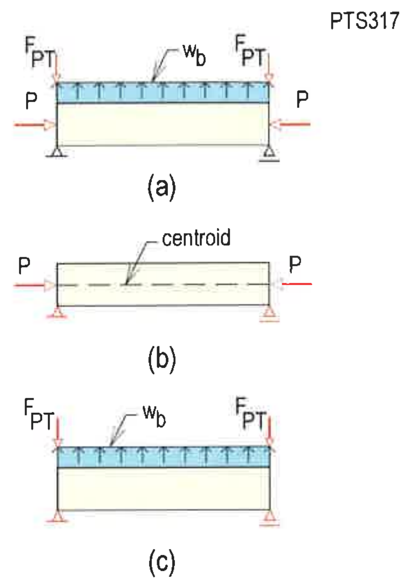


FIGURE 4.8.1B-2 Separation of Tendon Force into Axial (P) and Transverse (Wb) Components

The substitution of tendon by an equivalent force enables the structure to be treated as a simple, non-prestressed member, acted upon by the balanced loading (w_b). The axial load (P) at the centroid of the member will result in a uniform compression. The uniform compression will be added to the stresses obtained from the balanced loading.

The value and distribution of balanced loading (w_b) depend on such factors as the tendon profile and post-tensioning force. Figure 4.8.1B-3 illustrates the tendon forces on the structure for two examples of tendon profiles. For the distributions illustrated, the sum of upward and downward forces add up to zero.

As a guide for design, it is common to determine the sum of all upward forces over the length of a span, and express this value as a percentage of the dead load on the member. This ratio is commonly labeled the "percentage of dead load balanced." For example, for the two members shown in Fig. 4.8.1B-3, the percentage of the dead load balanced is expressed by the following:

$$\left(\frac{Total\ upward\ force}{Total\ span\ dead\ load}\right)\times 100 = \frac{W_2+W_3}{(Span\ dead\ load)\times 100}$$

(Exp 4.8.1B-1)

C. Extended Load Balancing: A central premise of the simple load balancing, as outlined in the preceding, is the decoupling of the bending and

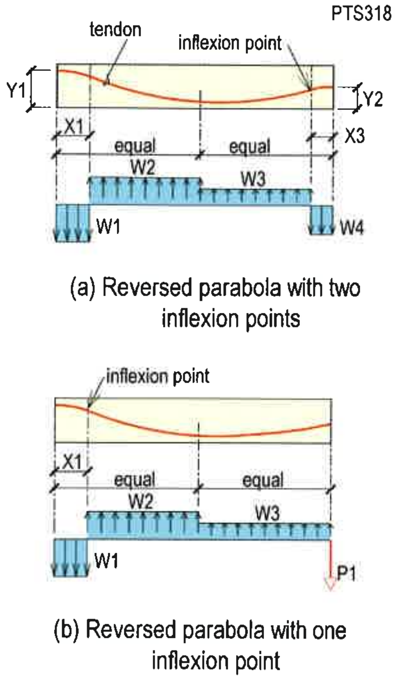


FIGURE 4.8.1B-3 Two Examples for Tendon Profiles and Balanced Loading

axial effects. The axial effect is considered to result in a uniform compression in the member. For members of uniform cross-section, this can be achieved by simply substituting a tendon by the forces that associate with tendon's geometry and its location at the anchorage points.

Consider the member in Fig. 4.8.1C-1a. When in place, the forces that the tendon exerts to the member are shown in part (b) of the figure, where tendon is removed and replaced by its effects. The axial forces *P* at the two ends of the member do not line up to balance each other. The change in member's thickness results in a shift (m) between the centroids of the two parts. Consequently, in addition to the axial load "*P*," the member is subject to a couple "*Pm*." The bending that results from the couple violates the central assumption of decoupling of bending and axial effects. The premise of uniform compression can be restored, however, by envisaging a clockwise and counterclockwise couple each equal to *Pm* and acting at the change in the centroid (part c). This does not disturb the equilibrium of the system. One of the couples can be considered to act with the axial loads of part (b) of the figure to restore the premise of uniform compression shown in part (d) of the figure. The second couple is added to the flexural loads from the tendon part (e). The concept is introduced and explained in detail in ref [Aalami, 1990]

Figure 4.8.1C-2 is the example of a slab with thickening at support. The thickening can be a column drop, drop panel or slab band. Irrespective of the function of the thickening, the change in the centroidal location of the section in elevation will require the addition of compensating moments as shown in part (b) of the figure. Part (c) of the figure shows the distribution of the ensuing axial loads necessary to provide uniform compression in the member.

4.8.2 Force Selection

Among the many differences between the design of a conventionally reinforced (RC) member and its post-tensioned (PT) counterpart, the "non-deterministic" nature of PT design poses a challenge to engineers. The following explains:

Once the geometry, loading, support conditions, and material properties of a conventionally reinforced member (Fig. 3.4.3-a) are established, the information leads to a unique value for the required area of reinforcement (A_s). For the same member, different designers are expected to arrive at the same minimum area of required steel. There is no room for optimization, once the aforementioned design parameters given.

For a post-tensioned member, shown in part (b) of Fig. 3.4.3, however, there is a multitude of acceptable designs; each arrived at based on the entry values that a designer assumes. Each of the designs while satisfying the requirements of serviceability and safety, can have a different amount of prestressing, or ratio of prestressing to non-prestressed reinforcement. In the general case, for a post-tensioned member, a designer must make at least two entry assumptions, before initiating a design. The two assumptions can be expressed in different forms. Generally they include two of the following three.

- Average precompression (prestressing force);
- Percentage of load to balance (uplift due to tendon drape); and
- Tendon profile (shape and drape).

From the many possible design solutions for a post-tensioned member, the one that meets the Code requirements for serviceability and strength and is the least expensive to build is usually the preferred solution. Generally, for a given member dimension, loading, and construction method, less material means a more economical design. There

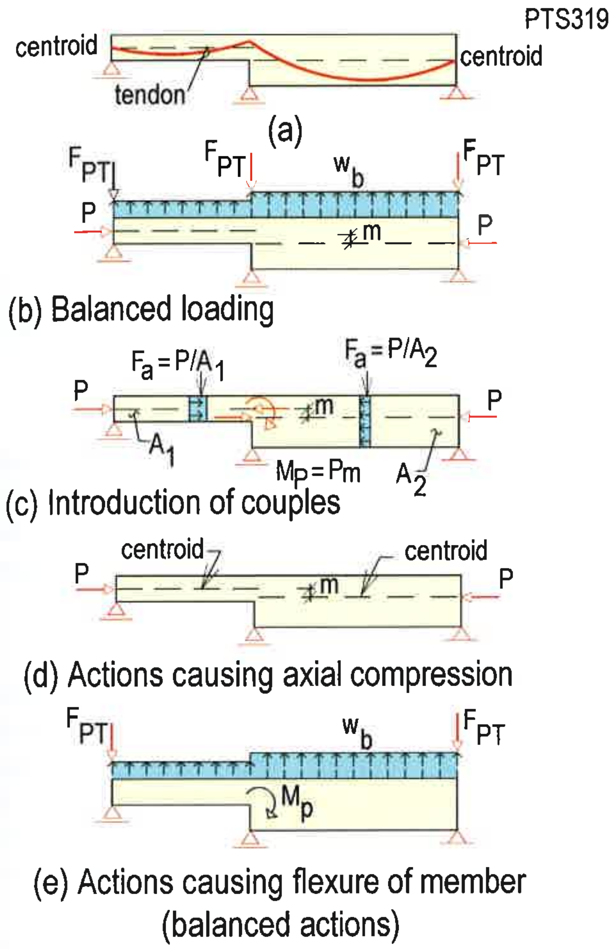


FIGURE 4.8.1C-1 Balanced Loading and Change in Member Centroid

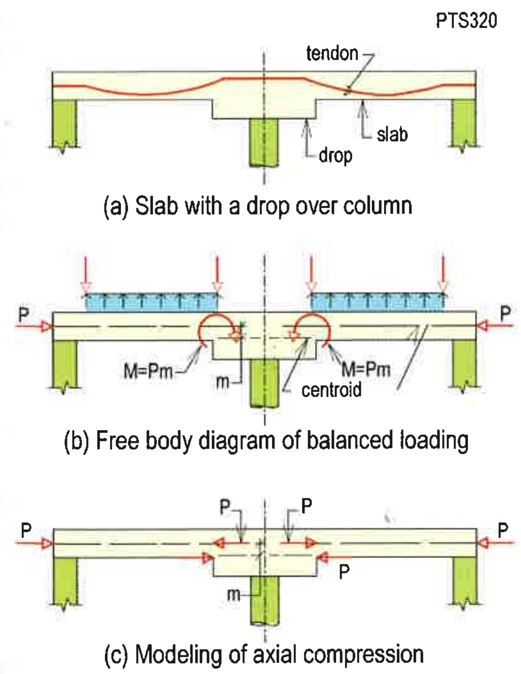


FIGURE 4.8.1C-2 Balanced Loading at Drop Cap/Panel

is a unique value for the design moment (M_u) of the conventionally reinforced member shown in part (a) of the figure, which leads to a unique value for the required area of steel (A_s). For the post-tensioned alternative shown in part (b) of the figure, the in-service behavior depends on the amount of force and the profile of the post-tensioning tendon. The design moment for ULS in general includes the secondary (hyperstatic) effects that are a function of the post-tensioning selected.

The three parameters listed above can generally be grouped into two independent items that must be specified, before an analysis and design can be carried out. The design determines the required amount of the supplemental reinforcement, if any. The two entry parameters are generally (i) post-tensioning force, and (ii) the tendon profile. Many engineers estimate the initial value of post-tensioning for design assuming an average prestressing (*P/A*), along with the percentage of dead load to balance.

For example, in Roya balcony shown in Fig. 3.4-2, subsequent to the selection of load path for tendons between the two walls A and B, the necessity and amount of reinforcement depends on the (i) profile of tendons, and (ii) the force in them.

A. Force Selection Options. The preceding concludes that the economy and performance of a post-tensioning design depend on initial assumptions – generally, the amount and profile of post-tensioning. Several computer programs tailored for design of post-tensioned members have implemented iterative schemes, whereby the selection of post-tensioning can be optimized[18]. When using hand calculation, or software that does not include automatic optimization, designers have to select the entry values based on engineering experience, or other guidelines. In such cases, the entry values are mostly based on an average precompression, and the percentage of dead load to balance. These are expounded in the following.

B. Percentage of Load to Balance: Post-tensioning may be viewed as a system of forces that is configured to counteract other loads, such as dead load, on the structure. For design purposes, it is convenient to express this as the ratio

[18] ADAPT-TN415

(percentage) of the dead load that is balanced by post-tensioning. "Balancing" a percentage of dead load means the post-tensioning provides a force in direction opposite to dead load.

For slabs, a good choice is to balance between 60 and 80% of the dead load. For beams, this is usually increased to about 80. To determine the required post-tensioning force, start with the critical span—generally, this is the longest span. Using the maximum permissible tendon drape in this span as one limiting criterion, and the minimum precompression as the other, determine the post-tensioning force and the fraction of the dead load that it balances. For the spans adjacent to the critical span, a lower percentage of the dead load should generally be balanced, since less upward force in an adjacent span helps to reduce the design values of the critical span. To accomplish this keep the tendon profile at its maximum drape, and reduce the force, where not needed. This reduces the amount of post-tensioning required. If it is not practical to reduce the tendon force by terminating a tendon in the span that is not needed, raise the tendon in span to reduce its drape.

For the member shown in Fig. 4.8.2B-1, an economical design is obtained (see Chapter 7) by balancing 60% of the dead load in the first span and reducing the post-tensioning to balance only 50% of the dead load in the second span. The tendon is straight (it has no curvature) in the third span, where it does not balance any dead load. The design can be further optimized using a tendon profile that exerts a downward force in the third span, since the load of the adjacent span tends to lift that last span.

In summary, the example illustrates that balancing all the spans of a continuous member to the same percentage of dead load, and using continuous tendons through the entire length of a member is not always the best choice.

C. Post-Tensioning Force Selection Flow Chart: The following two flow charts provide a guideline for the selection of post-tensioning tendons prior to analysis and design. The first flow chart (F4.8.2C-1a) covers the two-way floor systems following ACI 318, where the design is required to maintain a minimum precompression, and there is a threshold for allowable stresses. The second

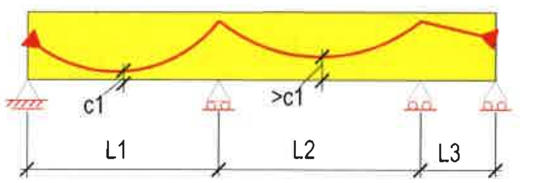


FIGURE 4.8.2B-1 Member with Largely Different Spans

flow chart (F4.8.2C-2) applies to one-way slabs and beams, also to be designed according to ACI 318. The second flow chart applies to both one-way, two-way and beam members to be designed based on EC2.

4.8.3 Effective Flange Width of T-Beams

Common understanding and treatment of effective width for flanged beams are rooted on its historical development coupled with the application of simple analysis methods. With progress in structural engineering, and advent of new analysis options, the "effective width" concept is either no longer applicable, or has to be looked at differently. This Section begins with its historical background, and concludes with its proper application in today's design practice. The review covers the "effective width" to serviceability design (SLS) of post-tensioned members, deflections for both conventionally reinforced and prestressed members, and the requirements for the ultimate strength (ULS) of a member.

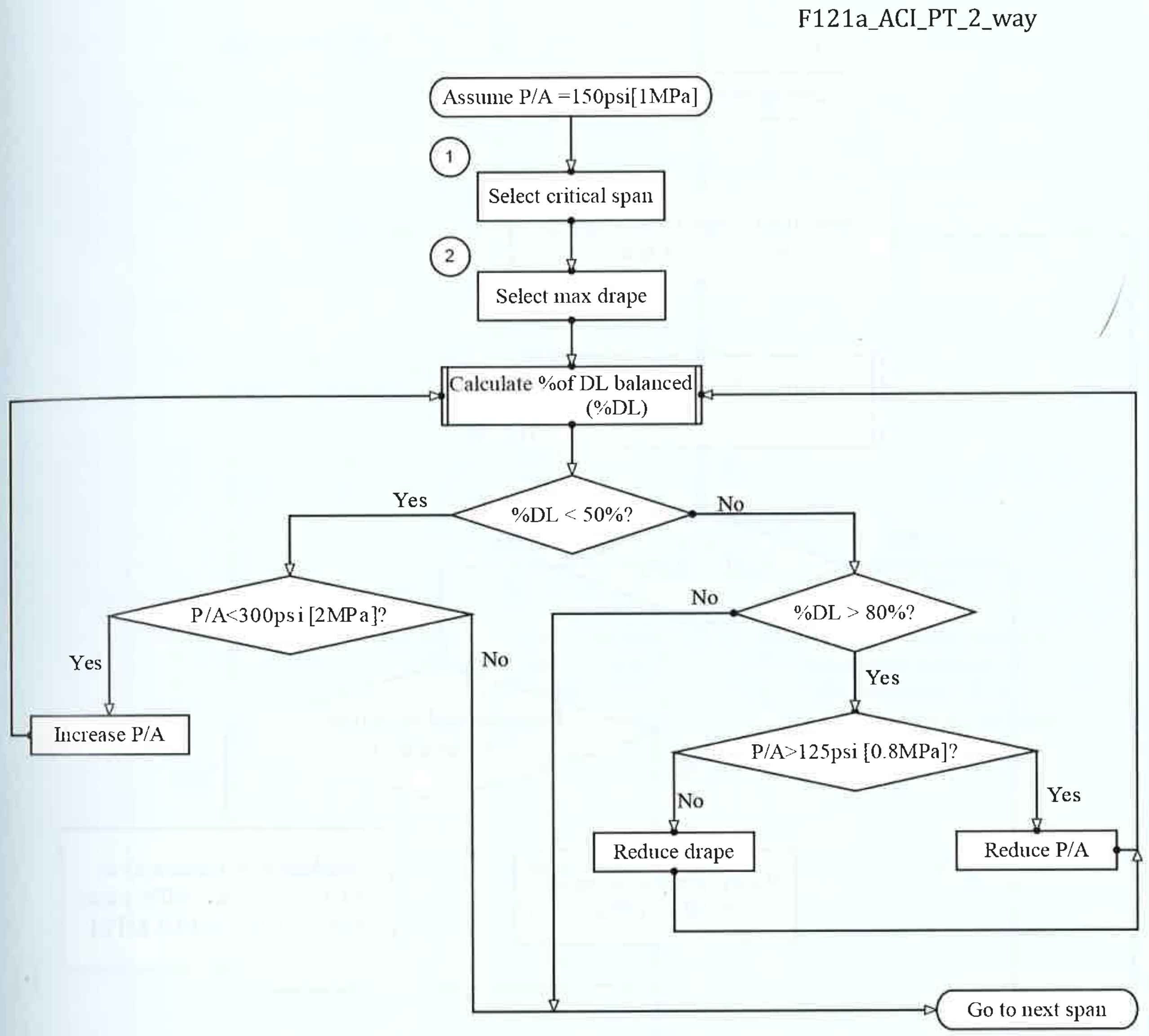
4.8.3.1 Effective Width Concept: Historically, the effective width concept was introduced to estimate the bending stress in the flange of a T-beam, using the simple beam theory (Exp 4.8.3.1-1).

$$f = (M/I)c \qquad \text{(Exp 4.8.3.1-1)}$$

Where

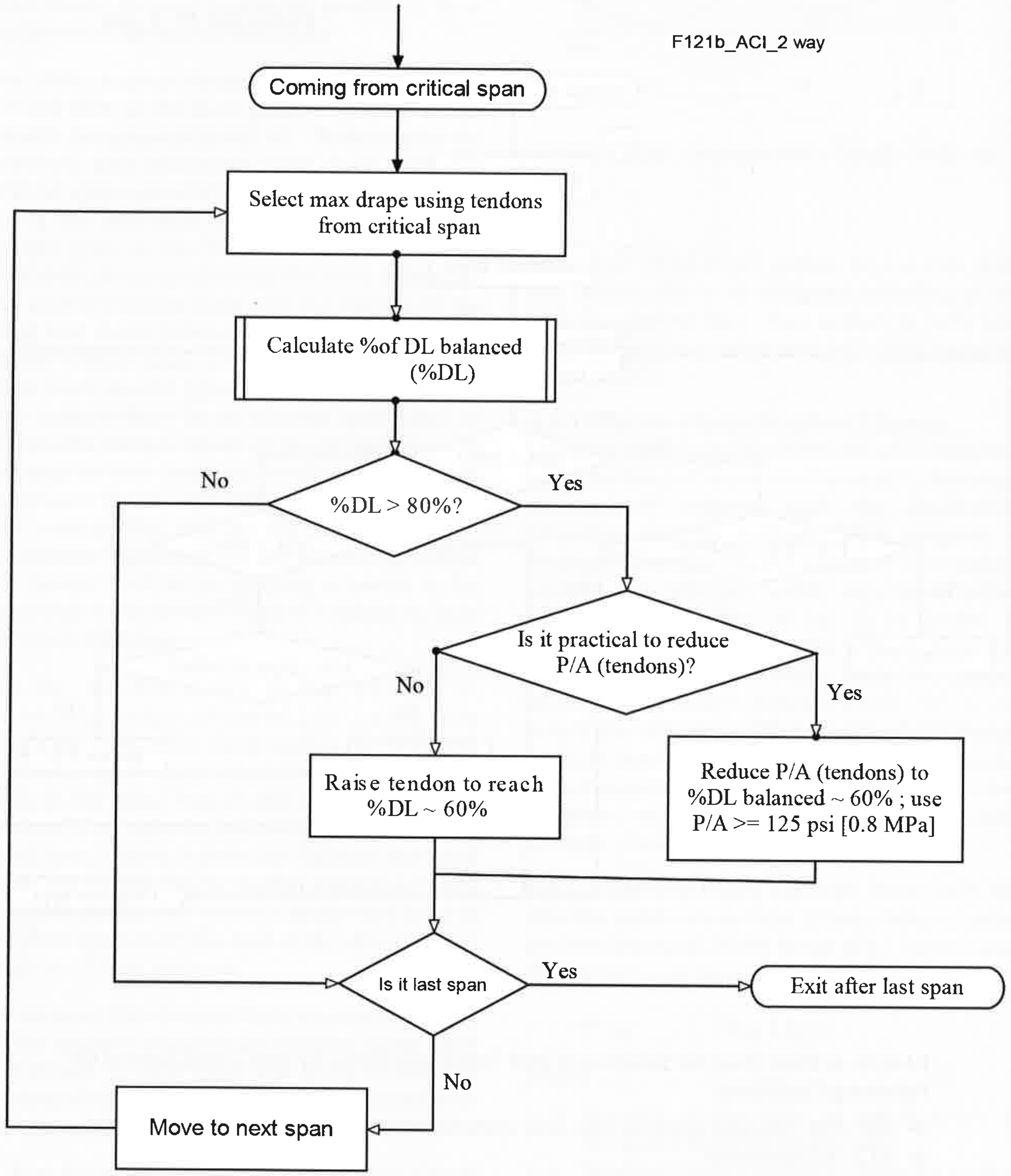
c = distance of the fiber at which stress is calculated from the neutral axis;
f = bending stress at distance c from the neutral axis;
I = second moment of area; and
M = applied bending moment.

Consider the simply supported flanged beam under uniform loading shown in Fig. 4.8.3.1-1. Using a two- dimensional linear elastic theory, it is observed that the distribution of compression

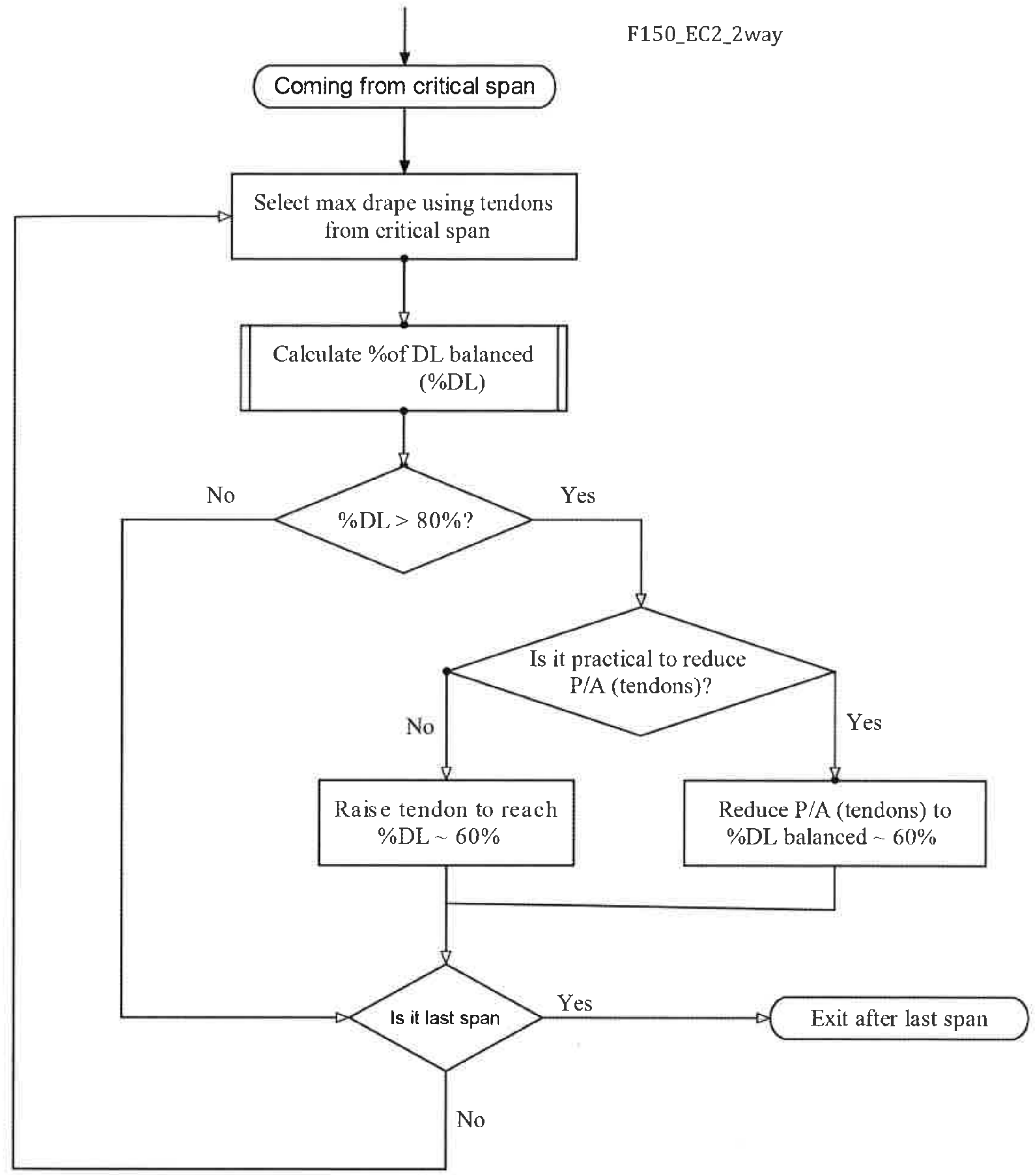


F4.8.2C-1a Flow Chart for Selection of Post Tensioning Force for the Critical Span of the Following Conditions:

- ACI 318: Two-way systems; one -way systems, and beams
- EC2: All concitions



F4.8.2C-1b Flow Chart for Selection of Post-Tensioning for Two-Way Systems Based on ACI 318



F4.8.2C-2 Flow Chart for Selection of Post-Tensioning for One-Way Systems Based on ACI 318, and All Members Based on EC2

stress in the flange is tapered as shown in part (c) of the figure. The stress is maximum at the stem-flange interface and tapers off with distance from the stem [Girkman, 1963]. The simple bending formula (Exp 4.8.3.1-1) is based on the assumption that the stress at a distance "c" from the neutral axis is constant; hence it cannot capture the reduction of force in flange from the tapered distribution of stress. To correct the stresses calculated using the simple beam formula, the tapered distribution of stress must be substituted by a rectangular stress block that provides the same flange force, and has the same maximum stress as the tapered distribution. The width of the rectangular block of equal force is termed "effective width" of the flange. For the geometry and loading shown in the figure, most references in the literature suggest a value between 8 to 12 times the flange thickness on each side of the stem, when dealing with concrete T-beams. The exact value depends on the parameters of the cross-section used.

Effective width varies with the configuration of the applied load and a beam's boundary conditions. Figure 4.8.3.1-2 illustrates several examples

[Girkman, 1963]. Observation of variations in the effective width shown in the figure implies that the application of a fixed multiple of flange width for effective width is not realistic – it is an approximation.

A. Effective Width and FEM: A first corollary to the foregoing is that where stresses are calculated using a formulation that captures the non-uniform distribution of stress in the flange of a beam, the effective width concept does not apply, since a correction to the value of the calculated stress is no longer necessary. Properly formulated FEM analysis tools recognize the interaction of a beam stem and its flange, resulting in a tapered distribution of stress in latter. The maximum stress reported where it occurs. Hence, the correction intended for flanged beams by way of an "effective width," is no longer applicable.

B. Allowable Stress Design: In early days, concrete members were designed using the "Allowable Stress" procedure, where the computation of stress was an integral part of design. Coupled with the fact that finite element technology was not available, the "effective width"

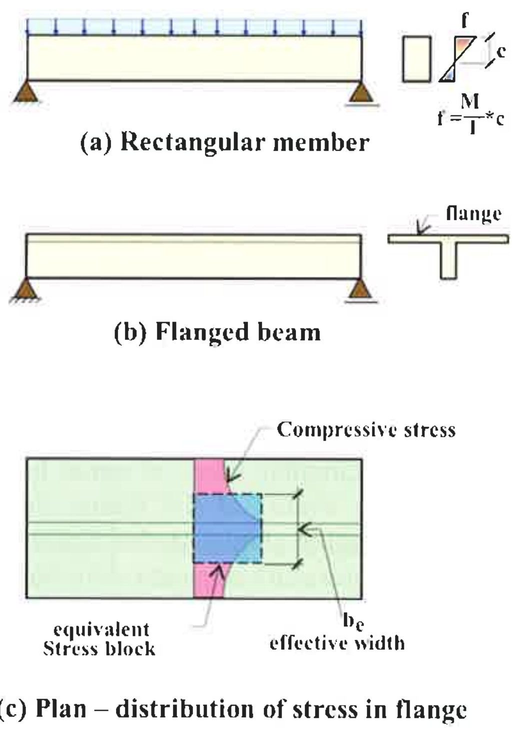


FIGURE 4.8.3.1-1 Illustration of Effective Width for a Simply Supported Flanged Beam under Uniform Loading (P539)

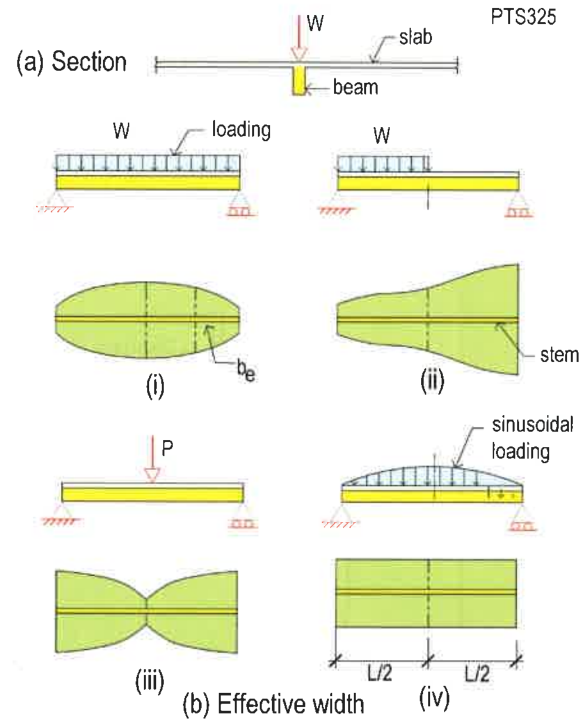


FIGURE 4.8.3.1 -2 Variation of Effective Width (b_e) with Load

became a central parameter in design of flanged beams. Its introduction led to the determination of "stresses" necessary in RC design. Today, concrete design is based on factored loads, used to estimate of a member's ultimate strength. The safety of a flanged beam rests on formation of a hinge line across its width, where depending on the configuration, a width as much as the beam's tributary can be mobilized to resist the applied loads.

C. Service Design of Post-Tensioned Members: One consideration in the service design (SLS) of post-tensioned members is the computation of a "hypothetical" stress for crack control. ACI 318[19] explicitly disallows the application of the "effective width," as used for conventionally reinforced sections to be applied to post-tensioned members. For post-tensioned members, the stress at a point is a combination of bending and axial effects. The ratio of bending to axial stress varies from point to point along the same member. Further, axial stresses distribute with a constant value over the entire cross-section, while bending stresses can have a tapered distribution across the flange.

[19] ACI 318-11, Section 18.1.3; commentary R18.1.3 – Sections 8.12.2, 3 and 4

Since the ratio of contribution of bending and axial to the total stress at a point is not constant, it is impractical to arrive at a single universally applicable "effective width" for a post-tensioned flanged beam, even where simple beam theory is to be used. Where a designer wishes to use the effective width concept, the bending and axial effects each should enter the analysis with its own effective value. In other words, a single value of "effective width" does not apply to both effects. The following Section explains further.

4.8.3.2 Effective Width for Axial Loads: Post-tensioned tendons apply an axial force on the member they act upon. In addition, they tend to flex the member if the prestressing force is eccentric with respect to the member's centroidal axis. In this Section, we concentrate on the distribution of axial force from post-tensioning. To crystallize the concept, we first consider the example of the familiar column shown in Fig. 4.8.3.2-1. The forces applied at the end of the column are intended to simulate those from tendons anchored at member ends.

Let the cross-sectional area of the column be the irregular shape shown in part (a) of the figure. An irregular shape is selected to emphasize that the outcome applies to all cross-sectional shapes. In part (b) of the figure, the column is loaded with an axial force *P* applied at the centroid of the section. The physical property of the centroid is that an axial force applied on it disperses uniformly over the entire section. As illustrated in the figure, the uniform distribution of stress takes place a distance away from the discontinuity at the ends, where the force is applied. It is concluded that the entire section, regardless of the shape of its geometry, is equally stressed to resist the applied load at distances far enough from the point of application of the force.

Consider part (c-i) of the figure, where the axial load is placed with an eccentricity "*e*." This represents the case, where a tendon is anchored eccentric to the centroid of a section. We simply use the common procedure of substituting the applied load by a force at the centroid (part c-ii) and a moment equal to *Pe*. The response of the column to the axial force will again be a uniform distribution of stress at a distance away from the ends. The distribution is identical to part (b) of the figure. The distribution of stress due to moment,

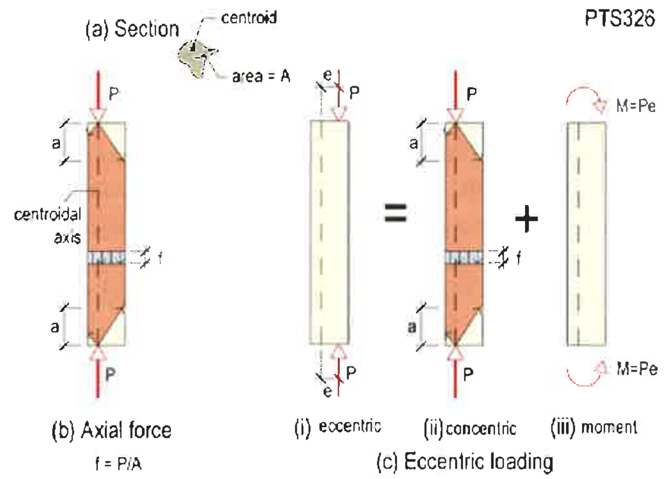


FIGURE 4.8.3.2-1 Dispersion of Force in End-Loaded Member

however, can possibly be subject to an "effective width" concept, depending on the formulation we might use to evaluate its effects.

The example illustrates that (i) irrespective of where a tendon is anchored at the ends of a member, and (ii) irrespective of the cross-sectional shape of a member, the axial force from a post-tensioned tendon distributes uniformly over the entire cross-section away from the point of application of the force.

In summary, unlike the case of "bending effects," there is no "effective width" for axial loads, since the entire width of a section becomes mobilized. In post-tensioned members, the local stresses are a combination of bending and axial effects. Since the contribution of each effect is not known a-priori, it is not practical to define a unified "effective width" for the combined stress. As mentioned earlier, each effect has to be considered separately and the outcome combined. This is true, when simple analysis procedures such as SFM or EFM are used. In a properly formulated multi-dimensional FEM analysis, the combined stress is calculated without the necessity to use the "effective width" concept.

A. Precompression in Beam and Slab Construction: Precompression is defined as the average compressive stress over a section. It is the value of the stress at centroid of a section. When dealing with axial load only, precompression will be a uniform constant force over the entire cross-section. The following is a practical application of the conclusion arrived at in the preceding.

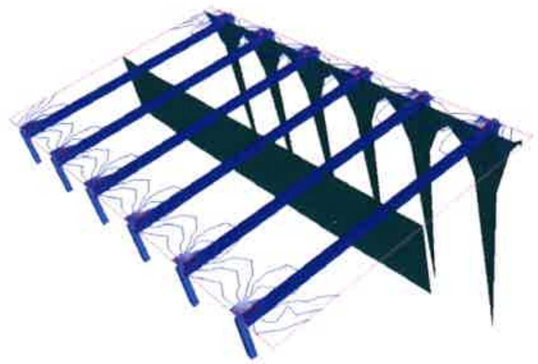
Consider a beam and slab construction, such as shown in Fig. 4.8.3.2A-1. In part (a) the slab over the beams is displayed transparent for better visualization. The structure represents the typical geometry common in the post-tensioned parking structures built in the US. Let the tendons be anchored at the end of the beams. Since there is no effective width applicable to axial loads, as illustrated in the foregoing, the distribution of the force in the structure will result in a uniform compressive stress at distances adequately away from the point of application of the axial loads. Part (a) of the figure shows the distribution of precompression at two sections. One section is close to the beam ends, where tendons are anchored, and the second at mid-span. Near the beam ends where the tendons are anchored, the stress has a peak. At mid-span, the stress is essentially uniform. The same phenomenon is reflected in part (b) of the figure, where the distribution of axial stress in the flange is plotted. The distribution reflects a rapid dispersion of force into the entire cross-sectional area of the structure, leading to uniform distribution across the entire width of the slab.

Note that the stresses shown in the figures apply to both the slab and the beam. The distribution of axial stress is uniform through the depth and width of the structure. The distribution is uniform, irrespective of the number of spans, shape of the cross-section, or profile of the tendons.

Example 4.8.3.2A-1
Figure 4.8.3.2A-2 illustrates a flanged beam with a concentric straight tendon. The distribution of precompression will be uniform and equal to force (P) divided by the total cross-sectional area (A) as shown in Fig. 4.8.3.2A-2.

Example 4.8.3.2A-2
Figure 4.8.3.2A-3 shows two beams, featuring eccentric and profiled post-tensioning tendons. One of the beams is multi-span. For each of the beams the distribution of precompression will be uniform and equal to force (P) divided by the total cross-sectional area (A) as shown in Fig. 4.8.3.2A-2 parts (c) and (d).

The above assumes that there is no friction loss along the length of the tendons. Friction loss will result in drop of the compressive stress along a member, but does not invalidate the concept. Also, neither the eccentricity of tendon at the anchorage,



(a) Distribution of precompression at mid-span and next to support (P320)

(b) Contour of precompression stress (P321)

FIGURE 4.8.3.2A-1 Distribution of Axial Load through a Beam and Slab Construction

nor the profiling of tendon along the member length changes the conclusion regarding uniform distribution of precompression over a member's entire cross-sectional area.

B. Position of Tendon Anchorage: When designing flanged beams, unless you plan to account for special effects, such as applying moments at member ends to improve the performance of a flanged beam, it is recommended to anchor a tendon at the centroid of the tendon's tributary as shown in part (a) of Fig. 4.8.3.2B-1.

When using "simple analysis procedures," such as SFM, or EFM, the section shown in part (a) of the figure resists the axial force of tendon, while for bending effects the section with reduced width (part b) is used. It is reiterated that when using a properly formulated FEM tool, the selection of effective width and the differentiation between the two effects are no longer applicable.

4.8.3.3 Effective Width for Strength Design: To illustrate the concept, let us review the strength design of a single span simply supported flanged beam using finite elements, where flanged beams are properly modeled as they appear in their physical outline. The stem is offset with respect to the flange and interacts with it through strain compatibility at the interface of the two.

Figure 4.8.3.3-1 shows the geometry of a flanged beam, typical of dimensions used in parking structures in the US, where a parking deck is constructed with parallel beams and one-way slab spanning across the beams.

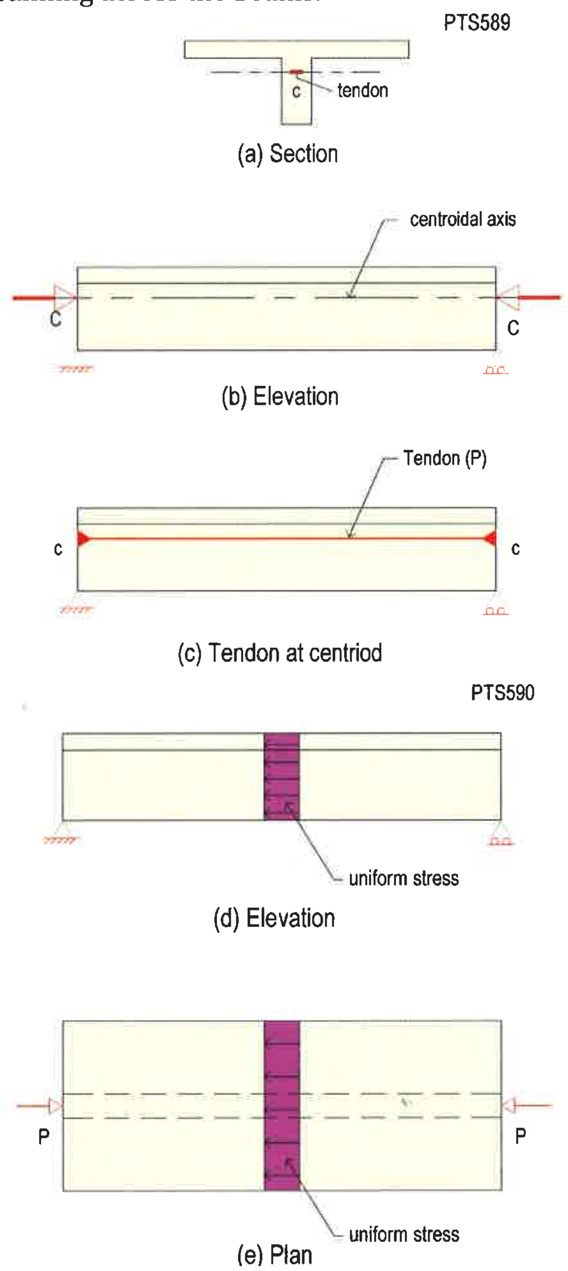


FIGURE 4.8.3.2A-2 Flanged Beam with Eccentric Tendon – Average Precompression

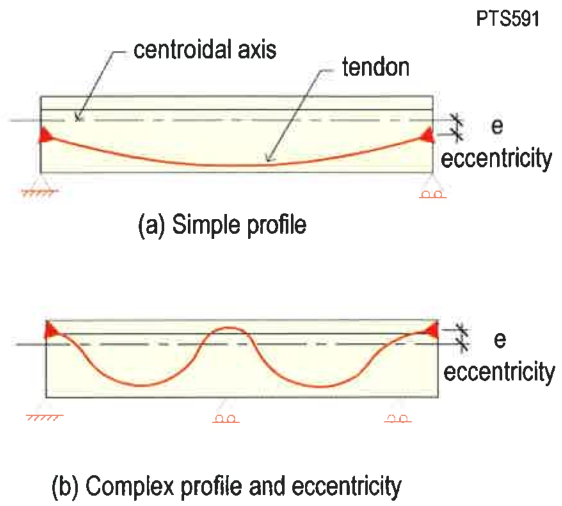


FIGURE 4.8.3.2A-3 Post-Tensioned Members with Arbitrary Tendon Profiles

The geometry and design parameters of the flanged beam are summarized below. The beam is not post-tensioned for simplicity of demonstration, without compromising the principle being demonstrated.

Geometry
- Span = 62 ft (18.90 m)
- Beam = 14 inch wide, 30 inch deep (356x762 mm)
- Flange = 5 inch (127 mm) thick ; 17 ft (5.182 m) wide

Load
- Uniform factored load of 1 k/ft (14.6 kN/m) along the centerline of the beam. The load on the actual structure will be somewhat different. The assumed load is to illustrate the concept.

Material
- Concrete strength f'c = 4000 psi (27 MPa)
- Reinforcement fy = 60 ksi (413 MPa)

Design Criteria
- Centroid of reinforcement to beam soffit = 2.30 inch (58 mm)
- Design code = ACI 318 -11

From the loading and span information, the design values at the beam's midspan are:

- Moment $= (1 \times 62^2/8) =$ 480.50 k-ft (6501.46 kNm)
- Shear = 0 k
- Axial load = 0 k

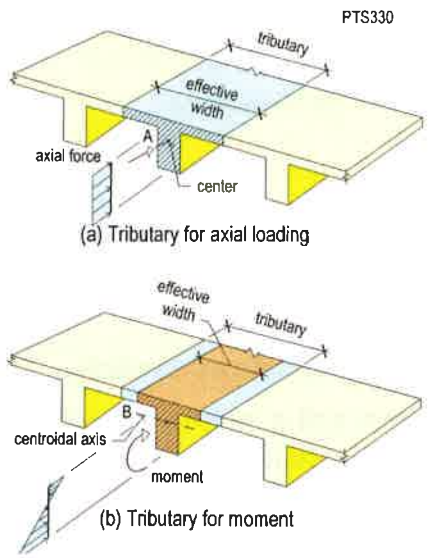


FIGURE 4.8.3.2B-1 Tributaries for Axial and Flexural Actions for Frame Analysis

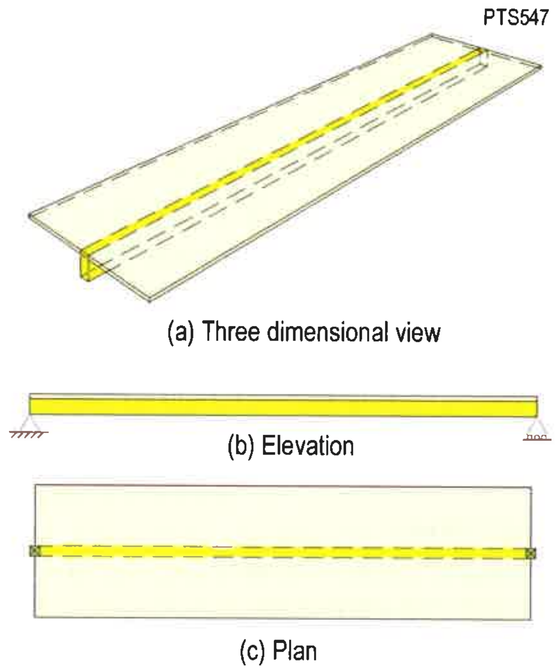


FIGURE 4.8.3.3-1 Simply Supported Flanged Beam under Uniform Loading

Considering the moment resisted by the entire cross-section of the flanged beam, the reinforcement required for the applied moment 480.50 k-ft is 3.95 in^2 (651.46 kNm ; 2,548 mm^2).

Next, we calculate the reinforcement in the beam using several different options, each with a different assumed effective width. The objective is to demonstrate that when properly formulated, the outcome of reinforcement design is independent of a designer's choice of the width of the flange that acts with the stem to resist the applied loads – namely, "effective width." In FEM – based designs, the "design strip" typically includes the beam stem and a strip of flange acting with it, when calculating the amount of reinforcement. The objective of the examples is to demonstrate that the width of the strip does not influence the amount of reinforcement needed for safety of the structure.

We select three different scenarios to illustrate the point (Fig.4.8.3.3-2). In part (a) we select a design section that includes the entire flange; part (b) illustrates an option with a design section that considers only one-half of the flange to act with the stem in resisting the applied load. And, finally, in part (c), we assume that the entire load is carried only by the stem of the flanged beam – the design section includes the stem only.

Table 4.8.3.3-1 reports the design actions generated for each of the design sections, using the proper modeling of the beam as illustrated in Fig.4.6.6-B-1 and a proper finite element formulation. Note that – as expected - the values of bending moment and axial forces reported vary, depending on the amount of the force from the flange that is captured in each design section.

The reinforcement calculated for each of the three design sections is reported in Table 4.8.3.3-2.
The table indicates that the reinforcement reported for the beam stem is essentially the same in all three cases. The variation is about 2.5% for the two extreme cases of full tributary and zero tributary. This is well within the limits of engineering approximation, when considering the two extreme cases of zero and full effective flange inclusion. The small difference is due to the inaccuracies in the numerical computations, rather than the concept.

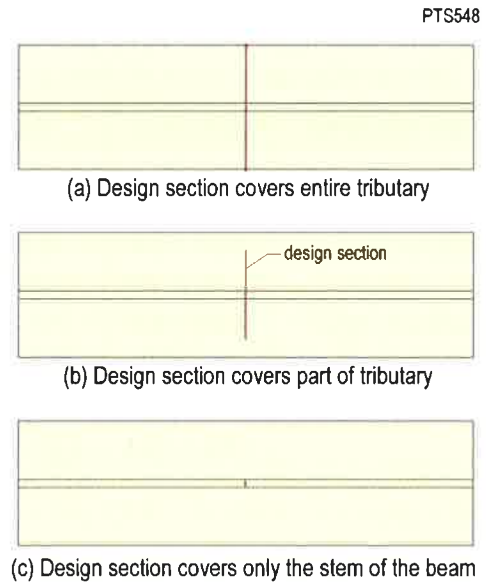


FIGURE.4.8.3.3-2 Plan of Flanged Beam Showing Different Design Section Widths

The recognition of presence, and allowance for the axial force in the beam stem that in each instance balances the compression in the flange tributary to the stem, make the determination of reinforcement for safety of the structure independent of an "assumed flange width." This conclusion is valid for both conventionally reinforced and prestressed members. It clearly demonstrates that using modern analysis tools, the application of "effective width" is redundant. Where a FEM design tool requires the definition of "effective width" to compute the design moments and the associated reinforcement, it signals clear evidence in the underlying modeling of the beams and the computational formulations.

The conclusions arrived at is general and apply to multiple beams too. Consider the partial view of a beam and slab construction shown in Fig. 4.8.3.3-3. The beams have the same dimensions and loading as in the previous example. In the following three design sections are selected: one is a section across the entire structure (4.8.3.3-4a); the next shows a section that contains one beam stem and include a greater flange than the tributary of the stem; finally the last (part c) is the stem with a smaller flange than its associated tributary. The objective of the example is to illustrate that in all three cases, the total reinforcement necessary for the safety of the structure is the same, recognizing that at ultimate strength, a hinge forms across the center of the entire structure folding it down.

The bending moments and the associated axial forces calculated for each of the three design sections selected are listed in Table 4.8.3.3-3. There will be no net axial force for the full section, since the structure is supported on rollers. For the other two sections, the tension reported in one (51.192 k) balances the compression reported in the other. If the sections were each covering the natural tributaries of the beams (mid-distance between the stems), the compression developed in the flange of each unit would have been equal to the tension in the stem. No net axial force would have been developed. Also, note that the sum of moments of the two sections across the width of the entire structure adds up to the moment of the design section that covers the entire width of the structure, namely M = 956.055 k-ft.

The reinforcement obtained for each of the three design sections is listed in Table 4.8.3.3-4. The values of the table illustrate that the total reinforcement for the structure (7.58 in^2) is provided by the sum of the partial designs in the latter two cases. Since a properly formulated FEM analysis determines and accounts for the continuity of the beam stem and the slab, as well as the presence of the axial force in each part of the design section, the reinforcement calculated becomes less sensitive to the width of the design section. More importantly, the total reinforcement

TABLE 4.8.3.3-1- Moments, Axial Force, and Shear at Selected Design Sections (T151)

Design section	Moment	Shear	Axial
	k-ft (kNm)	K (kN)	K (kN)
1 - Full section	480.504 (651.467)	-0.235 (1.045)	-0.005 (0.022)
2 - Partial section	448.203 (607.674)	-0.234 (1.041)	70.526 (313.714)
3 - Beam stem only	214.173 (290.376)	-0.230 (1.023)	243.882 (1084.836)

TABLE 4.8.3.3-2 - Required Reinforcement for Strength (T152)

Design section	*As* top	*As* bottom
	in^2 (mm^2)	in^2 (mm^2)
1 - Full section	0.00	3.95 (2548)
2 - Partial section	0.00	3.86 (2490)
3 - Beam stem only	0.00	4.06 (2619)

reported for the entire width of the structure remains essentially constant, irrespective of the width of the design sections that add up to make the entire width. Again, similar to the moment sum, the total reinforcement across the width of the structure consisting of the two design sections adds up to 7.68 in^2 (4,955 mm^2), which agrees well with the single design section over the entire width of the slab (Table 4.8.3.3-4; As bot = 7.68 in^2; 4,955 mm^2).

The slight difference between the distributions of reinforcement among the two partial designs is due to numerical approximations. The difference is within the limits of engineering computations, and is not of design significance, since the hinge

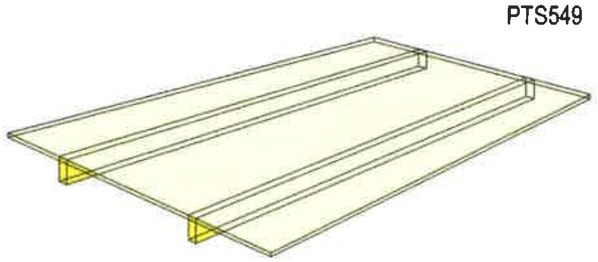


FIGURE 4.8.3.3-3 Beam and One-Way Slab Floor System

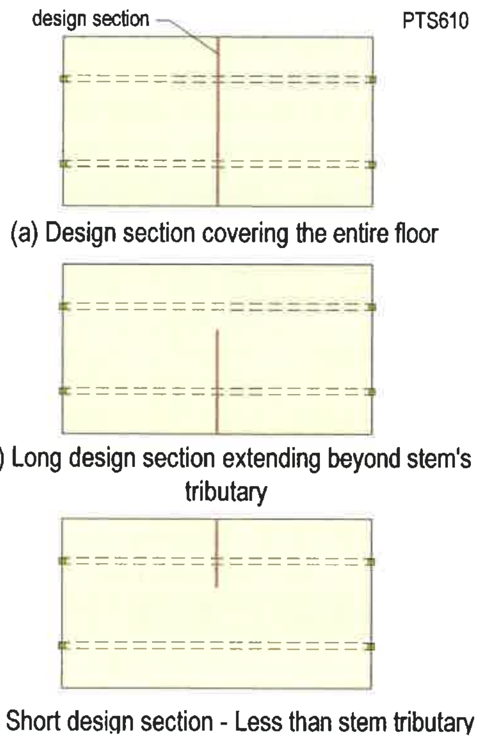


FIGURE 4.8.3.3-4 Design Sections of Different Widths

TABLE 4.8.3.3-3 Resultant of Actions for Different Design Section Widths (T153)

Design section	Moment	Axial
	k-ft (kNm)	k (kN)
1 - Full section	960.994 (1302.92)	0.019 (0.08)
2 - Partial large	497.345 (674.30)	-51.173 (227.63)
3 - Partial small	458.710 (621.92)	51.192 (227.71)

TABLE 4.8.3.3-4 REINFORCEMENT REQUIREMENT (T154)

Design section	*As* top	*As* bottom
	in^2	in^2
1 - Full section	0.00	7.58 (4890)
2 - Partial large	0.00	3.94 (2542)
3 - Partial small	0.00	3.74 (2413)
Sum of rebar of partial sections		7.68 in^2, (4954)

line developed at ultimate state extends across the entire width of the structure, thus mobilizing the reinforcement in each of the two beam stems.

4.8.4 Judicial Placing of Tendons

A non-prestressed reinforcement bar is mobilized on demand. When concrete that contains the bar is stretched or compressed, the bar encased in concrete becomes stressed and resists the applied action. Since non-prestressed rebar develops resistance to an applied force, its presence in concrete is generally beneficial.[20] Engineers refer to non-prestressed bars as passive reinforcement. The bars react and oppose the deformation of the concrete that surrounds them.

The same does not apply to prestressing. A prestressed bar or strand applies a force to the member that contains it, irrespective of the member's demand - a prestressed bar is active. If a prestressed bar or strand is not positioned favorably in a member, its prestressing force can counteract the resistance that a member has to develop in response to an externally applied load. Many engineers consider the prestressing force as

[20] Excessive reinforcement reduces ductility, a topic not covered

an applied load similar to dead and live loading, and position the prestressing such that the force it provides benefits the design objective.

In the following we review a simple example to illustrates how the positioning of prestressing at an unfavorable location can harm the design-intended performance of a structure, whereas addition of nonprestressed reinforcement at the same location would improve it. It is intended to highlight the concept that prestressing is not always beneficial. Its application requires engineering evaluation.

Consider Fig.4.8.4-1. It shows a simply supported reinforced concrete member. In one case, the member is provided with two conventional non-prestressed bars (2 #9; 1,290 mm^2) at the bottom to resist the design load. In the alternative case, four 0.5" (13 mm) post-tensioning strands at the bottom are used. The objective is to review the impact of added top rebar (case 1) or added top prestressing strands (case 2) on the serviceability and safety of the member.

Figure 4.8.4-2 illustrates the outcome. In the case of non-prestressed conventional reinforcement, the addition of two top bars reduces the downward deflection; slightly reduces the potential of cracking at the bottom of the beam; and slightly increases its design moment capacity. In the same figure, the addition of the four post-tensioning strands at the top (i) reduces the safety of the beam by lowering its design capacity, (ii) makes the member develop cracks at a lower level of externally applied loads, and (iii) increases the downward deflection of the member - in summary addition of tendons in this instance is detrimental in both the in-service performance and the safety reserve of the member.[21]

Consider the flanged beam shown in Fig. 4.8.4-3 - typical of beam and one-way slab construction used in post-tensioned parking structures in the US. Tendons positioned in the flange fall above the neutral axis of the beam, when the structure develops a collapsible mechanism at its ultimate strength. The overall impact of the added strands in the flange is the same as illustrated in the previous example. The flange strands reduce the design strength; increase the deflection and promote cracking at the bottom of the beam stem as illustrated in the preceding example.

At incipient ultimate strength failure, the flange tendons counteract the tensile force developed by the tendons at the bottom of the stem (Fig. 4.8.4-4). This leads to a reduction of the available bottom tension to resist the design moment - hence a reduction in beam's design capacity. More reinforcement has to be added at the bottom of the beam stems to compensate tension from the tendons in the flange, and restore the member's design capacity.

In summary, for flanged beams, such as this example, tendons in the flange and parallel to the beam stem harm the in-service performance of the slab-beam combination and reduce its safety.[22]

Slab tendons between beam stems, as illustrated in Fig. 4.8.4-4 are referred to as "temperature" reinforcement and practiced by a number of designers, with the erroneous perception that the tendons compensate the lack of precompression in a narrow strip of slab midway between the beam stems. Section 4.8.3.2 provides additional background.

Another scenario, where in common construction positioning of post-tensioning tendons acts unfavorably, is where a slab, or beam acts with its column support to resist lateral forces from wind or earthquake, and the forces from the lateral actions

[21] The leading parameters of the example are: f'_c = 4000 psi (28 MPa). Grouted tendons were assumed. Loss of capacity for unbonded tendons will be more:

Design capacity without prestressing 313.20 k-ft (424.64 kNm)

Design capacity with prestressing 304.54 k-ft (412.90 kNm)

The bottom fiber of the beam with added top prestressing will crack at a stress 357 psi (2.46 MPa) earlier than if it did not have the added PT.

For a beam span of 62 ft (18.90 m) and cross section given in the figure, the added tendons at the top increase the long-term deflection of the beam by 2.19 in. (56 mm)

[22] The practice is not uncommon in the US, due to the strong lobby of the post-tensioning hardware suppliers in the professional and code organizations. It is an instance, where protecting our industry, overrides engineering logic, and the interest of our clients.

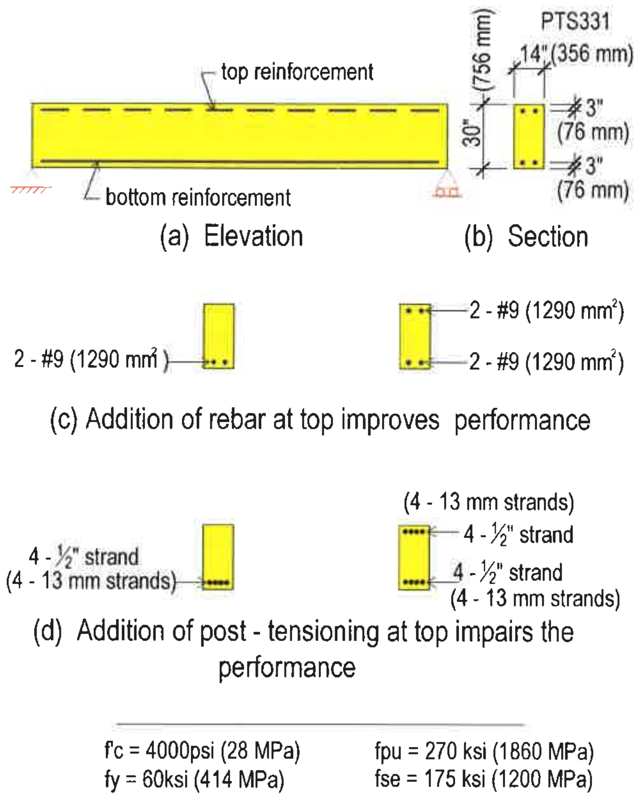


FIGURE 4.8.4-1 Member with Additional (i) Rebar, or (ii) Prestressing at Unfavorable Location

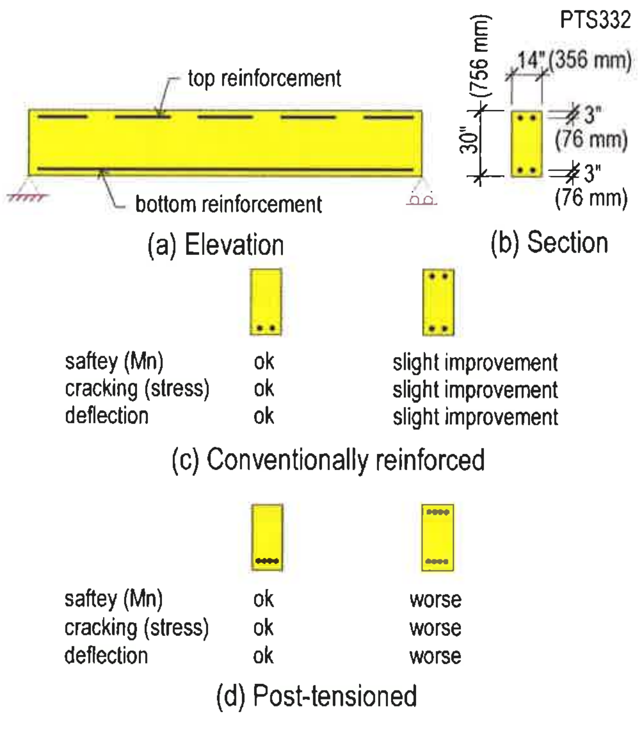


FIGURE 4.8.4-2 Performance of Member with Additional (i) Rebar, or (ii) Prestressing at Unfavorable Location

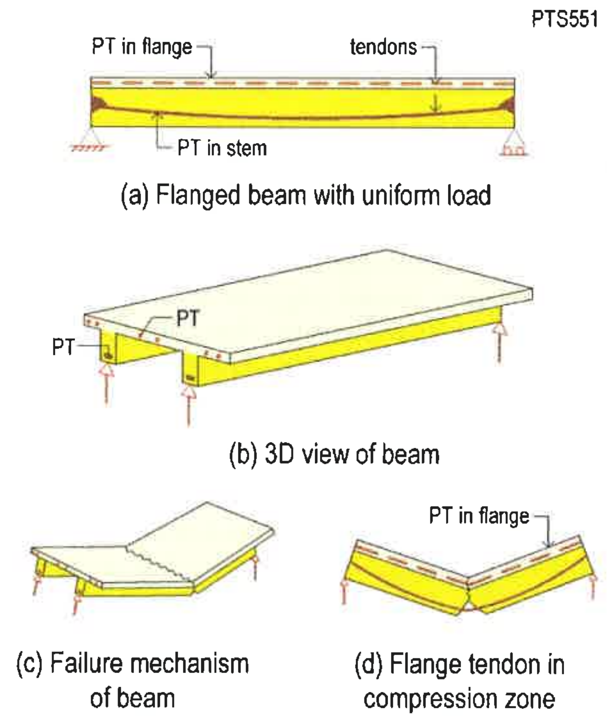


FIGURE 4.8.4-3 Flanged Beam with Tendon in Stem and Flange

result in reversal of moments at the joint (Fig. 4.8.4-5). If tendons fall in the compression zone, a larger amount of non-prestressed reinforcement is necessary in the tensile zone to compensate the adverse effects of the tendons for strength design. Where, reversal of moments at a post-tensioned joint is probable, a more efficient design will be achieved by placing the tendons closer to the centroid of section – farther away from the anticipated compression zone. This will reduce the tendons' adverse effects at reversal of moments.

FIGURE 4.8.4-4 Disadvantageous Slab Tendons between Post-Tensioned Beams (P326)

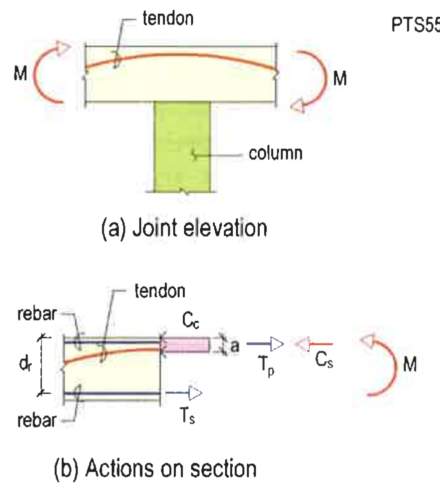


FIGURE 4.8.4-5 Force Distribution in Section with Tendon in Compression Zone

4.8.5 Average Minimum Precompression

"Average minimum precompression," is an ACI 318 provision. It is a notional value that serves as a guide in the selection of prestressing, and indirectly the reinforcement. It is a means for addressing the serviceability of a post-tensioned column-supported floor system, where ACI 318 specifies none other, or little reinforcement. Other major building codes satisfy the intent of the minimum average precompression through specification of "total reinforcement", or "crack control." Hence, unlike ACI 318, other major codes need not, and do not, specify a minimum precompression.

In this Section we discuss the significance and the application of the "average precompression" in post-tensioned two-way floor systems, where the design is intended to follow ACI 318 (IBC).

4.8.5.1 Historical Background: In early 1970s, the tests that were conducted in the US on flat slabs played a significant role in the post-tensioning recommendations of ACI 318. The prestressing of the test slabs provided an average precompression of about 125 psi (0.86 MPa). This level of precompression was viewed to have had beneficial effects in the punching shear performance of the conducted tests. For satisfactory punching shear design in two-way slab construction, the requirement of 125 psi (0.86 MPa) was considered a useful addition to the Code of the time. The thrust of the minimum precompression was satisfactory performance in punching shear - as it was understood in early 1970s.

Much has developed since early 70s, both in understanding the response of floor systems to punching shear, and design for punching shear with or without reinforcement. Thus the necessity of the minimum average precompression in regards to punching shear design is in question, recognizing that the current ACI 318 includes provisions for punching shear design of post-tensioned members with no, or less average precompression.

4.8.5.2 Current Significance of Minimum Precompression: Today, ACI 318's[23] provision of a minimum average precompression for post-tensioned two-way floor systems primarily serves the means by which the Code guarantees a minimum amount of reinforcement for service condition. Other major building codes, such as European EC2, do so by stipulating a minimum reinforcement, as opposed to minimum precompression. The minimum reinforcement in EC2 can be provided by prestressing steel, or a combination of prestressing steel and non-prestressed reinforcement. There is no such requirement in ACI/IBC, since the minimum average precompression is deemed to serve the purpose.

In principle, a satisfactory design can be achieved for a floor slab with a nominal minimum amount of post-tensioning, much less than the average 125 psi (0.86 MPa), if adequate supplemental non-prestressed reinforcement is provided. It is not the level of precompression that guarantees the serviceability and safety of a slab. Rather, it is the entirety of prestressing and non-prestressed reinforcement, in amount and distribution that safeguards satisfactory performance. Consequently, the requirement of a minimum precompression in the presence of adequate other reinforcement is superfluous.

Unlike other major building codes, the provisions for design of post-tensioned slabs in ACI 318 (Chapter 18) are exclusive – exclusive in the sense that a floor system is classified as either "prestressed" or "conventionally reinforced," depending on whether or not the design satisfies the provisions of Chapter 18. This is contrary to the current trend of thought among leading design

[23] ACI 318-11; Section 18.12

engineers, where there is no strict delineation between prestressed and non-prestressed members. Both are viewed, and designed following the same concepts, targeted to meet the intended serviceability and strength requirements through provision of adequate reinforcement.

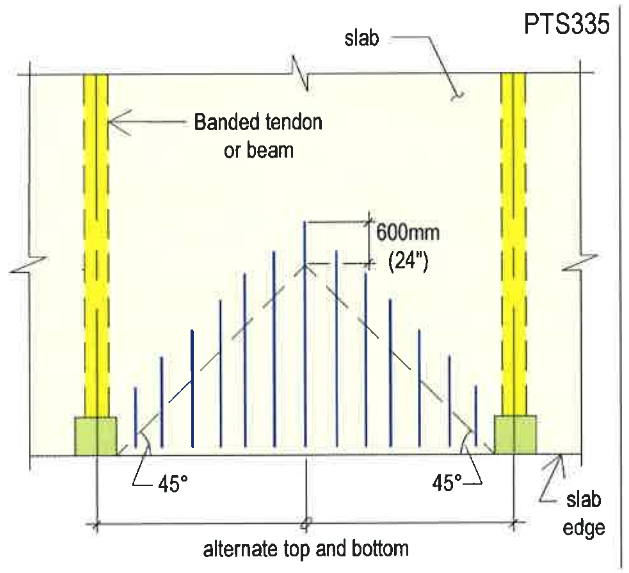
Using ACI 318-11, it is permissible to design a column-supported (two-way) floor slab with no bottom non-prestressed reinforcement. Also, it is permissible to design a slab with bonded post-tensioning system having neither top nor bottom non-prestressed reinforcement. The average precompression in ACI 318 serves to insure a minimum amount of prestressing in the absence of other reinforcement, that otherwise would have contributed in mitigating local cracking. The minimum precompression of ACI 318, in terms of force equivalency, is approximately equivalent to 0.2% of non-prestressed steel for service condition. This is slightly in excess of 0.18% that ACI 318 specifies for creep and shrinkage of conventionally reinforced slabs. Obviously, if in addition to prestressing, other reinforcement is available, such as bottom or top mesh, the presence of other reinforcement can relieve, or eliminate in its entirety, the ACI 318's significance of the minimum precompression for crack control. This is explained next in greater detail.

4.8.5.3 Code Compliance: Since it is impractical to provide precompression at all locations in a floor slab, ACI 318-11 permits to satisfy the requirement by providing either non-prestressed reinforcement, or a combination of prestressed and non-stressed reinforcement, where the intent is to mitigate shrinkage and temperature[24]. The following illustrates two examples, where it is not practical to provide the prescribed precompression. Non-stressed reinforcement is used to meet the objective.

Figure 4.8.5.3-1 recognizes that the precompression does not distribute uniformly over the tributary of a tendon at anchorages. The precompression can be fully absent or much less than the code stipulated minimum. Non-prestressed reinforcement is provided to compensate for the loss in precompression (part b).

Around the openings, such as point A in Fig. 4.8.5.3-2, the precompression normal to the face of the opening invariably falls to a low value, irrespective of the level of prestressing provided and the distribution of precompression away from the opening. Non-prestressed reinforcement generally provided as trim bars around the openings compensate the loss of precompression. In the example of Fig. 4.8.5.3-2 precompression for the location marked A is zero, while the slab may have been provided with adequate precompression for its overall design.

The two preceding examples illustrate that (i) the minimum precompression specified in the code is not intended to be interpreted as stress at a



(a) Beam and slab construction

(b) Reinforcement between grouped tendons (P327)

FIGURE **4.8.5.3-1** Arrangement of Nonprestressed Reinforcement in Lieu of Precompression to Account for Temperature and Shrinkage Requirements

[24] ACI 318-11, Section 7.12.2

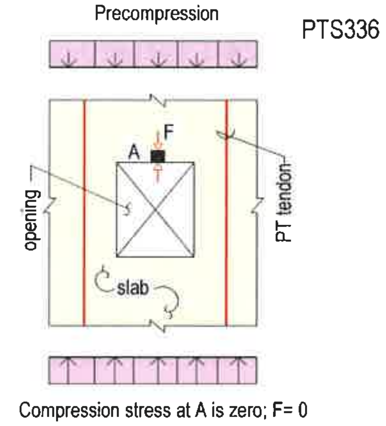


FIGURE . 4.8.5.3-2 Illustration of Loss of Precompression at a Discontinuity

"point," since it is neither practical, nor necessary for a slab's performance. Rather, it is intended for effective tributary of a reinforcement, or group of reinforcement, such as tendons and non-prestressed reinforcement that are placed to serve a given design section.

For compliance with the significance of ACI 318 with respect to minimum average precompression and two-way floor systems, it is adequate if the total area of prestressed and non-prestressed reinforcement in each "design section" is not less than 125 psi (0.86 MPa) or its equivalent expressed as follows:

(a) In the absence of prestressing strands, the reinforcement ratio for each design section shall not be less than 0.002 for steel Grade 60 (420 MPa) or better. This is expressed as follows:

$A_s/A_c \geq 0.002$ (Exp 4.8.5.3-1)

Where;

A_c = area of design section (tributary of a design strip); and
A_s = area of non-prestressed steel in design section.

(b) where prestressing strands are used, the reinforcement ratio for each design section shall not be less than the following

$(A_s + A_{ps})/A_c \geq 0.002$ (Exp 4.8.5.3-2)

Where;

A_{ps} = area of bonded prestressing steel in design section; Note that unbonded tendons do not comply with this requirement, since they are free to slip within their sheathing.

In the above relationship, no advantage is taken for the higher strength of prestressing steel, since at service condition, for crack control the induced stress in reinforcement is low.

4.8.5.4 Calculation of Average Precompression: For compliance with ACI 318, the average precompression is associated with a "design section" – not to a "point." It is important to note that in post-tensioned floors, the actual value of precompression significantly varies from point to point. At discontinuities and around openings the precompression can drop, and possibly reduce to tension in direction of applied prestressing.

Recognizing that at a design section the minimum precompression serves the function of minimum reinforcement; its value is calculated using the "effective area" of both non-prestressed and prestressed reinforcement, applied to the cross-sectional area of the design section (A_c). Relationships Exp 4.8.5.3-1 and 2 in the preceding are used to calculate the effective area. The total effective area is defined as the sum of the area of each reinforcement times the cosine of the angle each makes with normal to the design section (Fig. 4.8.5.4-1). In this figure the effective area of the prestressing and non-prestressed steel shown are respectively $A_{ps} \times \cos\theta_{ps}$ and $A_s \times \cos\theta_s$.

4.8.6 Hyperstatic Actions (Secondary Actions) Hyperstatic (secondary) actions seem to be shrouded in mystery for a number of post-tensioning engineers. It is not without a cause. The necessity for its explicit definition, extraction, and its implementation in design are the outcome of the simplifications, and shortcuts we make in the treatment of post-tensioning, when using variations of "load balancing" methods. In modern analysis and design technology, where tendons are duly viewed as a kind of reinforcement, and modeled as discrete elements, the question of hyperstatic actions, and whether and when they are included in design do not arise (See Chapter 11).

In this Section, for completeness and the benefit of many of our colleagues, who are more familiar with "load balancing," or use design tools based on load balancing and its variations, we define and review the basics of hyperstatic actions. The focus of this Section is to present the topic in a simple, yet complete manner, in order to develop a full appreciation of the concept. The objective is to conclude with clarity on (i) why there are hyperstatic actions; (ii) how they are calculated; and (iii) what to do with them in our designs. The information in this Section is the basic material applicable to all design methods and tools that are based on considering post-tensioning as an "applied load." In Chapter 11 we revisit the topic in greater detail, regarding its application in different design scenarios and modern design technology.

4.8.6.1 Definition of Hyperstatic (Secondary) Actions: Hyperstatic (or secondary) actions develop in a prestressed member from prestressing forces, due to the constraint of the member's supports to the free movement of the member arising from prestressing. If a prestressed member is allowed to displace freely, as in the case of determinate structures or precast members prior to alignment and installation, no hyperstatic (secondary) actions are generated. However, in most cast-in-place construction, where supports constrain the movement of a prestressed member, hyperstatic actions can be significant and impact the outcome of design.

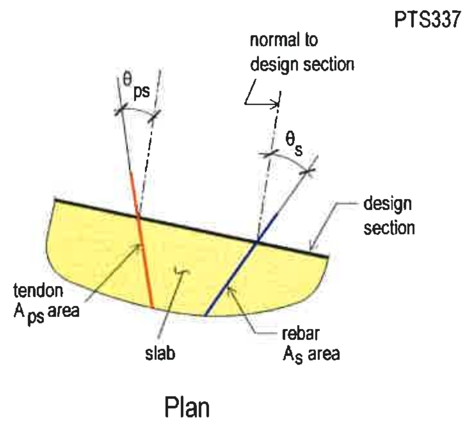


FIGURE 4.8.5.4-1 Plan; Design Section and Intersecting Reinforcement

4.8.6.2 Development of Hyperstatic Actions

We use a simple model to illustrate (i) where and how the hyperstatic actions develop, (ii) how they influence the "service condition (SLS)," and (iii) why and how they enter the "strength check (ULS)," in design. The focus of this section is to illustrate how and why hyperstatic actions impact the safety considerations of a post-tensioned member.

Refer to Fig. 4.8.6.2A-1. For purposes of illustration, we consider the object shown in part (a) to be weightless. The object is made up of a uniform material that can resist compression, but has zero tensile capacity – intended to simulate concrete. We review the "serviceability" and "safety" of this object under several scenarios.

A. No External Loads; No Prestressing; No Supports: The object is free floating with no supports. Regarding its safety, we raise the question: will this object breakup into two parts at a section such as A-A? The breakup of the object into two parts reflects its lack of strength under the specified conditions.

We start our investigation following the standard procedure in mechanics of solids. (i) We make a cut at the location A-A, separating the structure into two parts as shown in part (b) of the figure; (ii) using statics, we calculate the "demand," actions at the face of the cut; these are the forces that exist at the cut section, when the object is intact, and (iii) we investigate whether there is adequate "capacity" at the face of the cut to resist the calculated "demand" at the cut. By demand actions we mean "moments, shears, and axial force." If the structure can provide adequate resistance to the demand at the cut, there will be no failure.

Viewing the right half part of the object (part b), note that there is no demand at the cut A-A, since there are no forces on the cut half to be balanced by a demand at the cut to resist them. The demand is zero. Hence, there is no need to develop capacity at the cut. Consequently, the structure will not break at A-A – it is safe.

If the temperature of the floating body changes, it will deform (part c). However, since there is no external constraint on the object, the conclusion we arrived at in the preceding is still valid. The structure remains safe.

B. External Loads; No Prestressing; No Supports: Figure 4.8.6.2B-1a is the same object as in the previous case, but it is subject to a number of external loads that are in "equilibrium." The object is in static equilibrium. For simplicity, without compromising the concept, only forces in the plane of image are shown. Again, in connection with the safety of this structure, we explore whether the object is likely to break apart at section A-A. We follow the same standard procedure of determining

the "demand," and finding out whether there is adequate capacity at the cut section to meet the demand.

We make a cut at line A-A (part b) and break the structure in two parts. To restore equilibrium, there will be actions at the face of the cut. The actions at the cut balance the forces on the respective halves. For example, the shear shown at the cut is the resultant of the forces on the face of the cut to balance the other two externally applied forces acting on the body of the object. Next, we need to determine whether there is adequate capacity at the face of the cut to resist the demand.

Part (c) of the figure shows the demand moment and one option of how resistance could be developed at the face of the cut. For the type of material we have considered (no tension capacity), we need a tensile force that is generally provided by steel bars (rebar or prestressing), and a compression zone as shown in the figure. The structure is safe if the design capacity is not less than the demand.

From this example, we conclude that if there are "externally applied loads" on an object, in the general case, we need "reinforcement" for the safety of the structure. Note that for simplicity of discussion we did not include selfweight. Otherwise, selfweight would have been grouped with the externally applied force to the structure.

C. No External Loads; Prestressing Added; No Supports: Figure 4.8.6.2C-1a is the same object as in Case A, but it contains post-tensioning tendons that are stressed. The objective is to investigate the safety and serviceability of this structure.

First, we review the safety of the object in connection with the possibility of its failure (breakup) at section A-A. We follow the same established procedure, by making a cut at the section of interest and calculating the force demand at the interface of the cut. Part (b) of the figure shows the structure cut in two parts. Using statics, we note that there are no "external" forces at the face of the cut, necessary to restore the equilibrium of the cut piece. The tensile force in tendon shown must be in balance by an equal and opposite internal compressive force resulting from distribution of stress at the face of the cut section. The resultant of the forces at the face of the cut is zero.

Since there are no external forces to be resisted at the cut section, there is no design "demand" on the section. Consequently, like in Case A, there will be no need to develop "resistance" at the interface. The structure will not break in two at section A-A. The structure is safe. We conclude that in this case, the presence of post-tensioning had no impact on the safety of the structure, irrespective of tendon profile, tendon position and prestressing force.

Deformation and crack formation are matters of serviceabilty. Refer to part (b) of the figure. Since the material is assumed to have zero tensile capacity, depending on the location of the tendon on the section (location of the center of action from prestressing tendons), the section may crack. Where cracks develop, they extend to a given depth that is governed by the cross-sectional geometry of the cut-section. Increase in prestressing force widens the crack, but does not extend its depth.

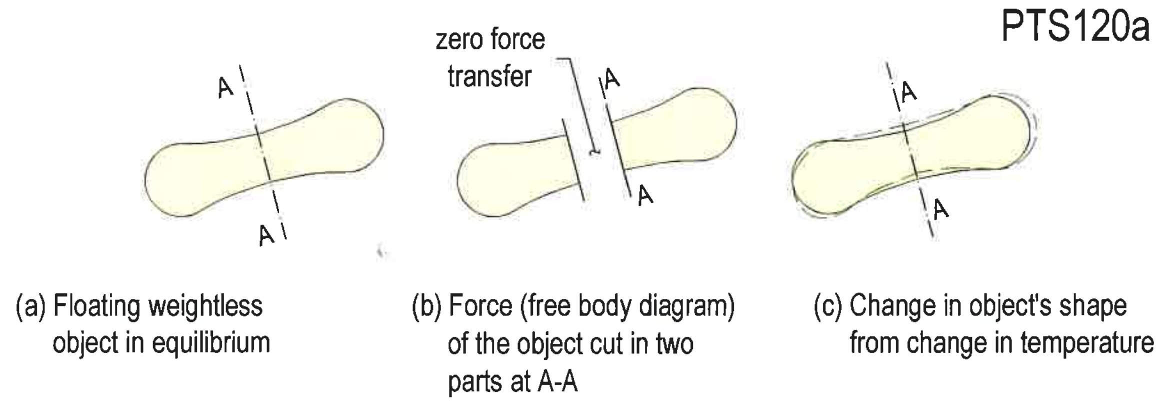
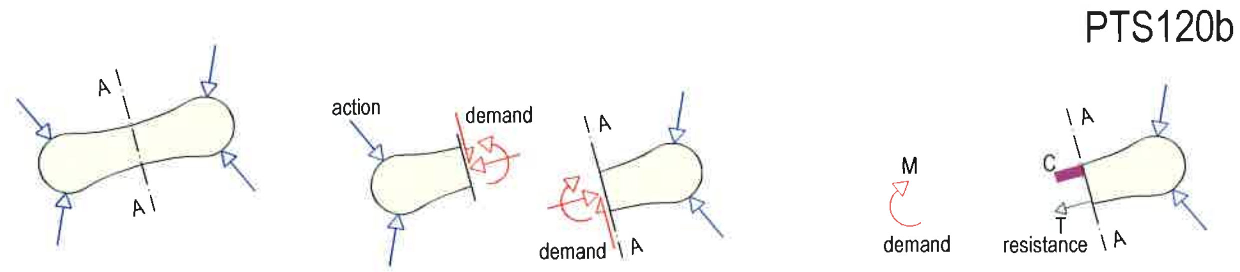


FIGURE 4.8.6.2A-1 Floating Weightless Object with No External Load



(a) Floating weightless object in equilibrium under external loads

(b) Free body diagram of the object cut in two at A-A showing demand generated at the face of cut to sustain external loads

(c) Resistance develops at the face of cut A-A to sustain demand moment M

FIGURE 4.8.6.2B-1 Floating Object with External Load

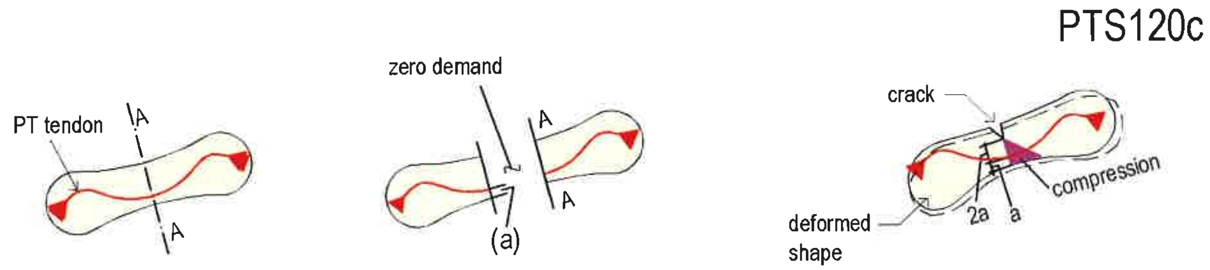
The depth is constant. The following comment explains.

Comment:
The initiation and depth of a crack is based on the distribution of stress on a section. Refer to Fig. 4.8.6.2C-2. It shows the distribution of stress from an off-center axial force on a rectangular section. If the section cannot resist tension, the region of compressive stress on the section will be limited to three times the distance of the axial load to the closest edge. To avoid tension, the axial load should be acting within the middle third core of the section (kern of a rectangular section) as shown in the figure.

Figure 4.8.6.2C-1b shows the distance of tendon to one side as "a." The crack on the opposite face extends to a distance of "2a" from the centorid of prestressing force – if the section is rectangular. Increase in post-tensioning will widen the gap, but will not change its depth. For non-rectangular sections, a different value for the depth of crack would apply. Each section has its own characteristic "kern."

It is concluded that, if a structure is not acted upon by external forces, the post-tensioning on its own can crack the structure (serviceability), but cannot break it up (safety).

D. No External Loads; Added Prestressing; Added Supports: Figure 4.8.6.2D-1a is the same object as in Case C, with the difference that the structure is now supported. There is neither selfweight, nor externally added loads. The objective is to find whether under the action of prestressing alone the structure can break apart along section A-A — whether it is safe.

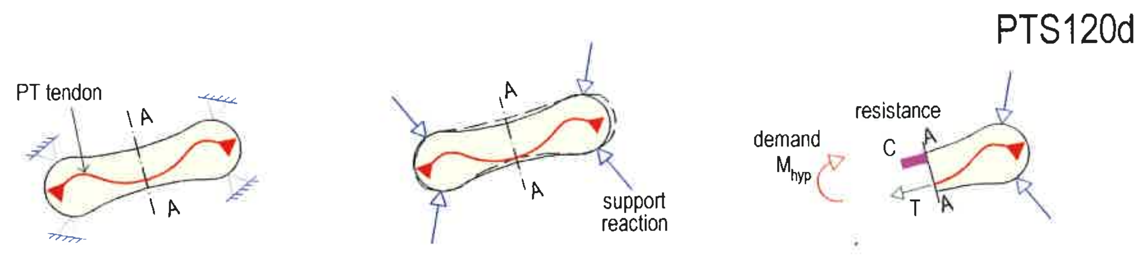


(a) Post-tensioned, floating weightliess object; investigating the possibility of break-up at section A-A

(b) Object severed at A-A. There is no force transfer across the cut. Hence, no resistance. Object does not break at A-A,

(c) Increase in post-tensioning results in cracking of the object, but not its breakup

FIGURE 4.8.6.2C-1 Floating Object with Post-Tensioning and No Supports



(a) Object is held in position (supported). Will it break up at section A-A?

(b) Reactions develop due to restraint of supports to object's change in shape.

(c) Support forces result in demand at A-A. Demand is resisted by "C" and "T."

FIGURE 4.8.6.2D-1 Structure with Prestressing and Supports

Once post-tensioning is applied, the object tends to flex and shorten. If the supports of the structure in number and arrangement are such that they do not freely allow the structure to flex and shorten, restraining forces develop at the supports. The restraining forces developed at the supports are reactions resulting from keeping the structure contained within its bounds at the connection to the supports, while the structure tends to deform from the prestressing forces away from the supports. The reactions developed at the supports are hyperstatic (secondary) forces. If the arrangement and number of the supports are statically determinate, the structure can deform freely despite the presence of supports. That is why the reactions are also referred to as "hyperstatic" forces, since they relate to statically indeterminate support arrangements.

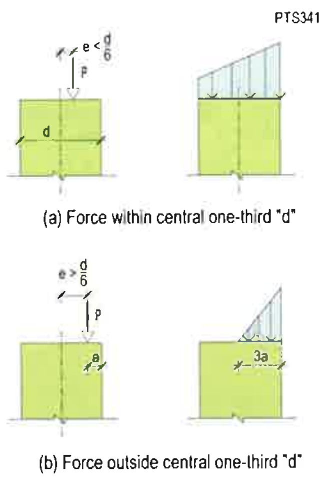


FIGURE 4.8.6.2C-2 Axially Loaded Rectangular Section

Part (b) of the figure shows symbolically the reactions at the supports developed to restrain the deformatoin of the object. Note that the combined reactions must be in static equilibrium, since there is no other external force on the structure.

Next, we investigate the safety of the structure at section A-A shown in part (a) of the figure. Following the standard procedure, we make a cut at A-A (part c) and determine the demand. In this case, the external forces on the structure (reactions from prestressing) are the only forces that exist and result in a demand, such as the moment (M_{hyp}) on the cut section.

For its safety, the demand actions (M_{hyp}) must not exceed the resistance capacity at the face of the cut. In this instance, the prestressing tendon acts as reinforcement and provides a tensile force that along with other forces on the section contribute to the design capacity at the face of the cut. We conclude that it is the actions from hyperstatic forces that create a strength demand in a structure. The post-tensioning acted as a reinforcement resisting the demand.

In summary, (i) in a post-tensioned member, hyperstatic reactions develop, if the supports of the member are statically indeterminate, at the time the post-tensioning is applied; (ii) the hyperstatic actions at the supports are in static equilibrium; and (iii) the hyperstatic actions create a force demand in a member to be resisted by the member's strength capacity.

E. Overview of Load Balancing and Design Checks: The outcome of the foregoing discussion is summarized in Fig. 4.8.6.2E-1 by way of its

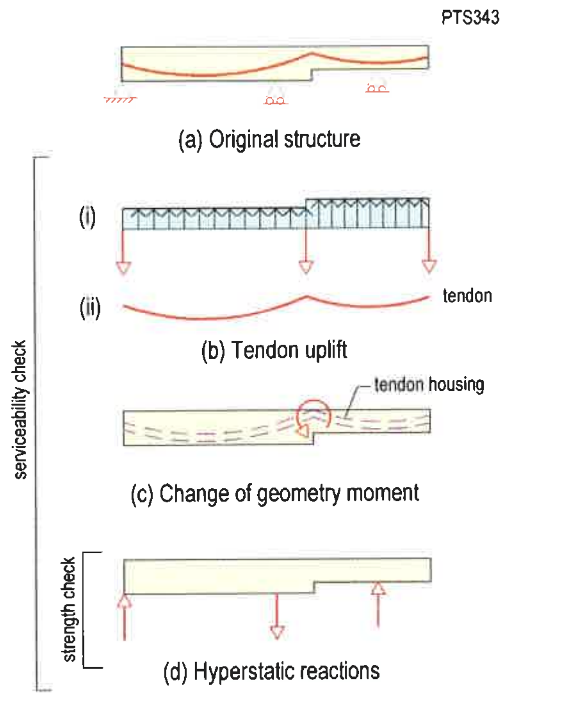


FIGURE 4.8.6.2E-1 Actions from Load Balancing and Their Participation in Design Process

application to a two-span post-tensioned member shown in part (a) of the figure.

Using simple load balancing [Lin, T.Y., 1963], the tendon is removed from its housing and its effects on the concrete member represented by an upward force determined from the tendon shape (part b).

Recognizing that the tendon is contained in concrete and the geometry of its container also impacts the bending caused by the tendon, moments are added at locations of change in centroidal axis of the member (part c) [Aalami, 1990]

The forces developed at the supports of the member due to member's tendency to deform under prestressing result in additional demand for strength. These are the hyperstatic actions and are shown in part (d) of the figure.

When investigating the serviceability of the structure, namely: deflection, stresses, formation of cracks and crack width, all the actions shown in parts b, c and d are considered. In addition, to those shown in parts b, c and d, at each section a uniform precompression caused by prestressing will be included. The precompression is not shown in the figure.

When dealing with the safety of the structure, only the actions shown in part (d) of the figure (hyperstatic action) enter the strength demand on each section.

The objective of the foregoing presentation was to introduce the concept of hyperstatic actions and their impact in the serviceability and safety of post-tensioned members in a simplified manner. The topic is re-visited and treated in greater detail in Section 4.11.2.1.

4.8.7 Constant Force and Variable Force Design

There are two schemes in common use for the presentation of post-tensioning on structural drawings. In North America, the prevailing practice is to express the post-tensioning by its profile and "effective force." The translation of the "effective force" to the number and layout of tendons is done by the post-tensioning supplier, who generates the fabrication drawings. Elsewhere, the practice is to show the actual number and position of each tendon on the structural drawings. The latter is referred to as "variable" force scheme. This section briefly explains the two options.

4.8.7.1 Effective and Variable Force Definitions: When stressed, the force in tendon drops along its length from the jacking point toward the tendon's far end. This is due to friction between the prestressing steel and its sheathing. Once seated, there will be additional stress losses in tendon due to time-dependent effects. The loss in stress diminishes with time, when the tendon reaches

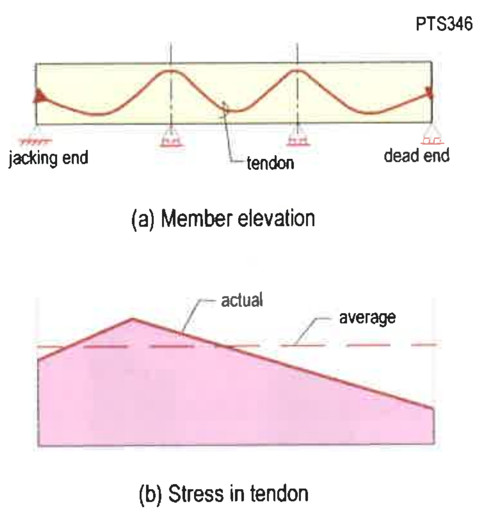
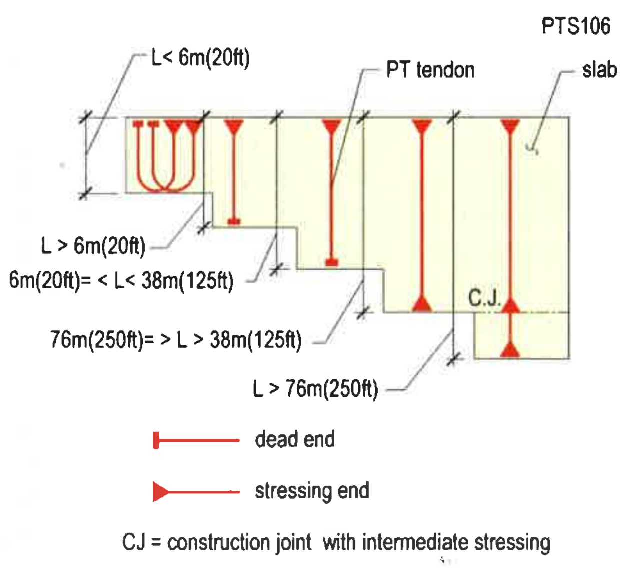


FIGURE 4.8.7.1-1 Distribution of Stress along a Post-Tensioned Tendon

its final steady-state stress value. The immediate and long-term losses of stress in prestressing steel are discussed in detail in Chapter 10. The focus of this Section is the application of prestressing force in design, after all stress losses in tendons have taken place.

Figure 4.8.7.1-1a shows the schematic of a post-tensioned member stressed at the left end. The distribution of the force in tendon, after all stress losses, is likely to look like the diagram in part (b) of the figure. The average of the stress distribution is also shown in the same figure.

The response of each section along a member to the applied loads is governed among other factors by the value of the prestressing force at that section. The variable force option provides the information necessary to evaluate a design section's response, using the local force of prestressing. This requires the knowledge of the force distribution along the length of a member. Designs based on the application of "local" prestressing actions to each section along a member use the variable force method. A simplification is to use the average force along a tendon for design of all the sections along a member's length. This approximation does away with the necessity of calculating the force profile.



Jacking Stress 80% of Guaranteed Ultimate Strength; Effective Force=120KN (27k) for 13 mm (0.5") Unbonded Tendons

FIGURE 4.8.7.1-2 Tendon Stressing Lengths

The practice is considered conservative, if the average stress assumed in design is not less than the actual value at critical locations of a member.

To arrive at an average stress that can reliably be used in design, the stressed length of a tendon should be somewhat short, since the loss of stress at far end of long tendons will be significant. The design based on the average stress of a long tendon is uneconomical, apart from approximations that may be excessive. For the geometry of common residential and commercial buildings, when using the effective force approach, the statistical works for common building construction lead to the recommendation illustrated in Fig. 4.8.7.1-2. The recommendations of the figure were derived for "unbonded" tendons, but are also used for "grouted" tendons, albeit with a smaller value for the effective force. Using the diagram, tendons that are longer than 125 ft (38 m), but shorter than 250 ft (75 m) are stressed at both ends.

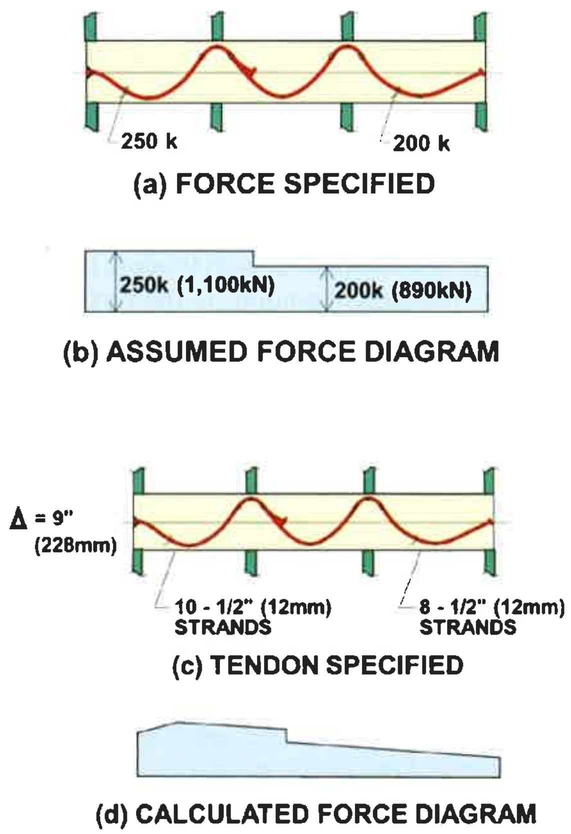


FIGURE 4.8.7.2-1 Illustration of Effective Force and Variable Force Presentations (P348; P546)

Using a jacking force equal to 80% of specified ultimate strength (270 ksi; 1860 MPa), for common unbonded tendons, greased and encased with plastic sheathing, and for grouted tendons in metal sheathing, the average effective stress commonly assumed in design is 1200 MPa (175 ksi) for unbonded tendons and 1100 MPa (160 ksi) for grouted tendons.

If the variable force option is used, in theory, there is no restriction on the stressing length of a tendon, since at each location along a member, the design is based on the force calculated at that location. A long stressing length, however, becomes uneconomical, due to high losses in stress.

4.8.7.2 Examples of Effective and Variable Force Designs: Structural drawings that are based on effective force, as practiced in North America, show the prestressing in a more symbolic manner, as illustrated in Fig. 4.8.7.2-1a. The alternative (part c of the figure) used widely elsewhere, shows on the drawing the number of strands and the associated elongation.

For projects that are based on the "effective force" option, the post-tensioning supplier will convert the force shown on the drawings to number of strands on fabrication drawings that include stressing details and elongations. The fabrication drawings also include information on the chair heights along the length of each tendon, in order to define the position of tendons in the vertical plane along the path shown on plan.

4.8.8 Tendon Layout

Recommendations for tendon layout for two-way floor systems serve several purposes: namely (i) improved performance of the floor in service and ultimate state, (ii) ease of construction, and (iii) efficient use of prestressing tendons.

The European code EC2 neither recommends tendon layouts, nor does it restrict specific practice. In this respect, the European code leaves the tendon layout to the design engineer and the contractor, as long as the design and construction meet the in-service and safety requirements of the code. Using EC2, it is permissible to deploy a single strand for the entire floor system, if the aggregate of the reinforcement provided meets the ultimate objectives of satisfactory in-service performance and safety requirements.

The premise of ACI 318-11 on the other hand is different from EC2. ACI 318, implicitly assumes that not everyone attempting to design a post-tensioned floor system may have adequate experience and knowledge to fully understand the concepts of prestressing, and be familiar with its practice to arrive at a serviceable and safe construction. Hence, ACI 318 prescribes specific guidelines that are proven by test and practice to result in satisfactory performance. This premise also extends to plan checkers, who are charged with review and approval of construction drawings, without having had the benefit of the underlying design experience, nor familiarity with the practice. In this respect, ACI 318 requirements such as minimum precompression or the maximum spacing of tendons are safeguards for serviceable and safe structures. The drawback to the ACI 318 is that its recommendations can become restrictive, when dealing with structures outside the norm, where it becomes necessary to rely on the objective of design, as opposed to the letter of the code. While ACI 318 has the provision of overriding the prescriptive code, this is rarely attempted for common structures.

A. Arrangement of Tendons in Two-Way Systems: At design stage, the structure is subdivided into design strips, in each of the principal directions (see Chapter 3). The designer determines the total amount of post-tensioning required for each of the design strips. For safety of the structure, the total of post-tensioning determined for each design strip should be placed within the width of that strip. Contractors generally prefer to group the tendons in one direction along the line of supports, and uniformly distribute them in the orthogonal direction. This is referred to as banded-distributed layout. This arrangement is less labor intensive, and can better achieve the tendon profiles shown on plans, since it minimizes tendon intersections, and the necessity of tendon weaving. The practice is permitted in ACI 318, as long as the tendon spacing in the distributed direction does not exceed either 8 times the slab thickness, or 5 ft (1,500 mm)[25] whichever is less.

[25] ACI 318-11, Section 18.12.4

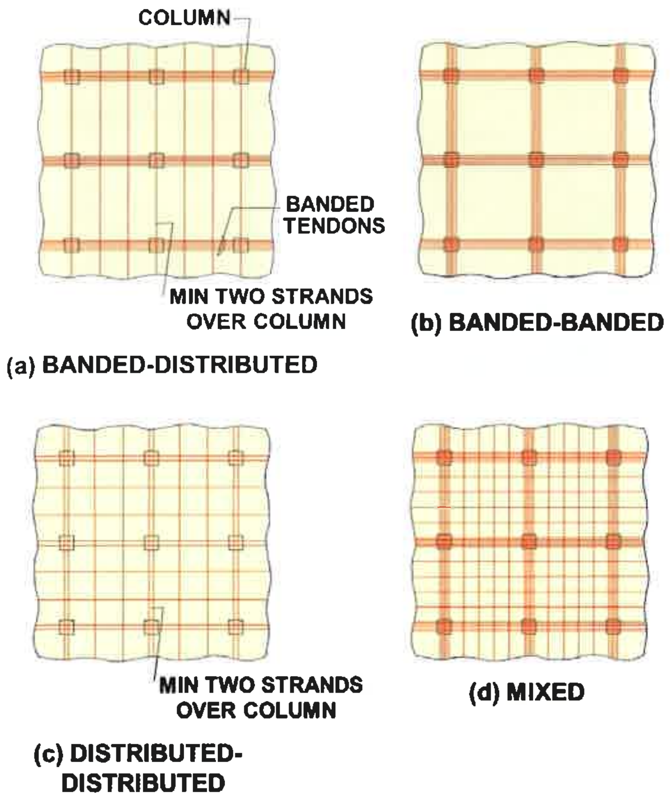


FIGURE 4.8.8A-1 Alternatives of Tendon Arrangement (P547)

Tests conducted at University of Texas, Austin, for different tendon layouts, using the same amount of post-tensioning, but different tendon arrangements (Fig, 4.8.8A-1) have concluded that the options shown in the figure all develop the required design strength [Burns, et al 1972; Smith et al, 1974], leading to the conclusion that the selection of preferred layout can be decided by ease of construction. Figures 4.8.8A-2 through 4.8.8A-5 illustrate several options of tendon layout. Figures 4.8.8A-2 through 5 illustrate several options of tendon layout.

In selecting the direction for grouping of tendons, preference is given to the direction, where columns do not line up (Fig. 4.8.8A-6, and 4.8.8A-7).

Where columns are on an array of uniformly spaced orthogonal lines (Fig. 4.8.8A-8), it is preferred to select the layout such as to minimize the number of triangular wedges between the tendon bands, where reinforcement in addition to prestressing needs to be added (part c) to meet the minimum requirements of reinforcement.

FIGURE 4.8.8A-2 View of a Banded-Distributed Tendon Layout, Using Unbonded Tendons (P328)

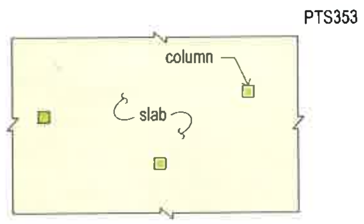
FIGURE 4.8.8A-3 View of Banded-Distributed Layout (Unbonded tendons; UK; P548)

FIGURE 4.8.8A-4 Tendon Layout, using Banded-Distributed Arrangement (Grouted Tendons Singapore) (P545)

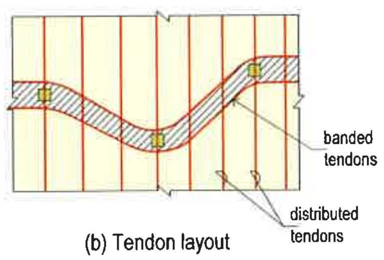
(a) Tendon Layout, using Distributed-Distributed Arrangement (Grouted Tendons; Thailand; P331)

(b) Tendon Layout; Distributed-Distributed Arrangement (Thailand; P546)

FIGURE 4.8.8A-5a,b Tendon Layout, using the Distributed-Distributed Arrangements



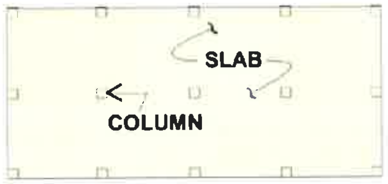
(a) Partial plan



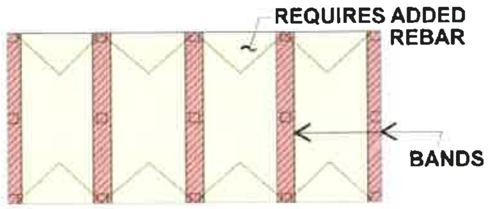
(b) Tendon layout

FIGURE 4.8.8A-6 Tendon Layout for Non-Aligned Columns

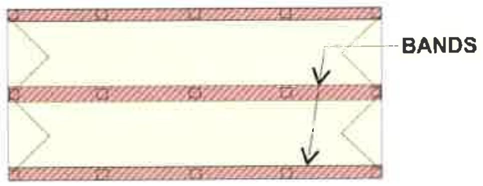
FIGURE 4.8.8A-7 Grouping of Tendons in Direction, where Supports do not Line Up (Indian Well, USA; P334)



(a) PLAN



(b) BANDS IN THE SHORT DIRECTION



(c) BANDS IN THE LONG DIRECTION

FIGURE 4.8.8A-8 Options for Arrangement of Grouped Tendons (Preferred arrangement is shown in Part c) (P667)

B. Tendons Over the Supports: ACI 318[26] requires that prestressing strands with total cross-sectional area not less than 0.3 in^2 (190 mm^2) pass over the column area between the longitudinal column reinforcement and be anchored beyond. The objective of these tendons – viewed as integrity steel, is to avoid catastrophic failure of the slab tributary to the column. These

[26] ACI 318-11; Sections 18.12.6 and 7

tendons shall extend through the span in each direction and be capable to sustain in tension the weight of the slab tributary to the column. Figure 4.8.8B-1 illustrates a column support during Northridge Earthquake (California), where the two strands passing over the column in each direction have been the primary mechanism in preventing the slab from falling down. The figure supports the concept of integrity steel, in preventing major collapse.

ACI 318-11 has relaxed this requirement and permits the strands be placed outside the column cross-section, but within the "effective support zone" – possibly a 45° truncated cone/pyramid extending from the face of support to slab mid-plane. It is not clear whether the new recommendations would have avoided a collapse of the type prevented by the strands through the cage of the column reinforcement shown in Fig. 4.8.8B-1.

Figures 4.8.8B-2 and 3 illustrate two examples of banded-distributed layout of unbonded tendons, where the requirement of tendons over the column support is observed.

Where due to congestion of column reinforcement, or size of tendons, such as for grouted tendons, it is impractical to pass the necessary amount of

FIGURE 4.8.8B-1 Collapse of Slab Prevented by Tendons Passing Over the Column (Northridge Earthquake; CA; P335)

FIGURE 4.8.8B-2 Tendon Layout with Two Strands over Column in Each Direction (P336)

FIGURE 4.8.8B-3 Tendon Layout with Two Strands over Column in Each Direction (P337)

FIGURE 4.8.8B-4 Congested Column Cage Prevents Passage of Tendon over Column (P338)

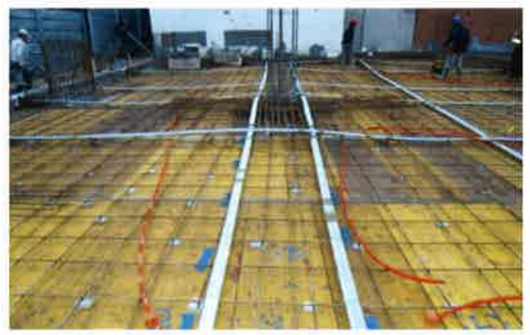


FIGURE 4.8.8C-1 Tendons Spaced More than 5 Times Slab Thickness (Morocco; P550)

tendons over the column support, such as shown in Fig. 4.8.8B-4, non-stressed integrity steel can be used. The area of non-stressed reinforcement shall not be less than 1.5 times that specified for minimum rebar for flexure of a conventionally reinforced concrete[27].

C. Tendon Spacing: ACI 318-11 requires that tendon spacing in one direction not to exceed eight times the slab thickness, nor 5 ft (1500 mm), whichever is less. In the banded-distributed layout, where tendons are grouped in one direction, the maximum tendon spacing does not apply to tendons in the banded direction.

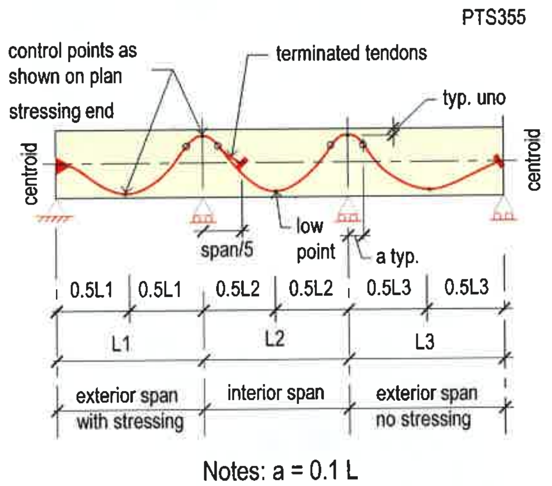


FIGURE 4.8.8D-1 Profile for Beams and Distributed Tendons

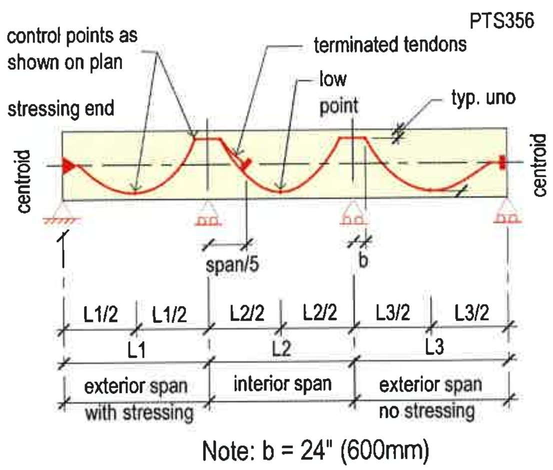


FIGURE 4.8.8D-2 Profile for Banded Slab Tendons

FIGURE 4.8.8D-3 Banded Tendon Pressed Against Top Reinforcement Grid (P339)

If the detailing of the location in question is such that the in-service and safety requirements of the code are satisfied, the maximum tendon spacing becomes moot. Figure 4.8.8C-1 illustrates a two-way slab construction, where the spacing of tendons exceeds 8 times slab thickness, as well as 5 ft (1500 mm), but through addition of rebar, the slab is detailed to meet the intended in-service and safety requirements.

D. Tendon Profiles: Tendon profiles are generally made up of parabolic segments and straight lines. Where tendons are spaced apart, and distributed parallel to one another, and in beams, the reversed parabola shown in Fig. 4.8.8D-1 is appropriate. For banded tendons, a partial parabola as illustrated in Fig. 4.8.8D-2 is commonly used. The latter choice is to accommodate the grid of top reinforcement over the column, where the tendons need to traverse, before being lowered into the span (Fig. 4.8.8D-3). In practice the sharp break shown for the banded tendon profile over the support is not achievable. The actual tendon profile follows a more gradual transition exiting the grid of top reinforcement over the column.

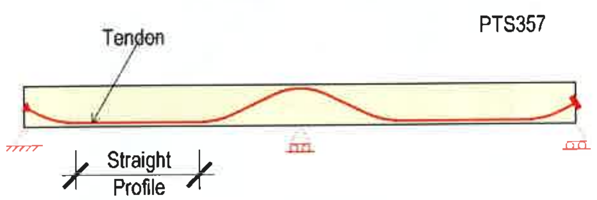
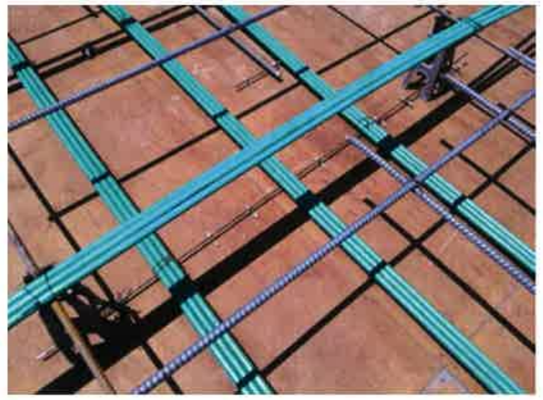


FIGURE 4.8.8D-4 Tendon Profile Under Self Weight

[27] ACI 318-11; Section18.12.7



(a) Slab reinforced with unbonded system (US; P519)

(b) Slab reinforced with grouted system (Freyssinet, UK; P704)

FIGURE 4.8.9-1 Views of Floor Systems Reinforced with Bonded and Unbonded Post-Tensioning Systems

The profiles shown in the preceding specify the low point of the tendon to be at middle of span. The selection of the mid-point is for ease of construction. At end spans, and where tendon profile is generally not symmetrical, a different low point can be more efficient. But, this is not commonly practiced in construction, for ease of layout that the selection of mid-point offers.

It is important to note that the selection of a reversed parabola profile is for ease of computation, when using the "load balancing" technique, as opposed to efficiency in tendon shape. Using a tendon made up of parabolic segments lends itself to a simple computation of the tendon forces for analysis.

At construction, rather than using multiple chairs to force a tendon in shape of a reverse parabola, it is simplest to fix the tendon at its high point over the support, and its low point in span, and allow the tendon to drop under gravity to its natural smooth shape, without additional chair supports. Ref [Wicke, M., 2005], justifiably argues that the construction effort in installing a tendon to follow a prescribed geometry is not warranted (Fig. 4.8.8D-4). The difference between the two profiles in performance of a tendon is insignificant, whereas from construction standpoint, the gravity profile is less effort consuming.

4.8.9 Post-Tensioning System Selection and Performance: Bonded/Unbonded

The two primary post-tensioning systems used in building construction are the "unbonded," and "bonded." The latter is also referred to as "grouted" system. The mechanical features of the two systems are discussed in Chapter 1. Figure 4.8.9-1 shows views of slab construction using each of the two systems.

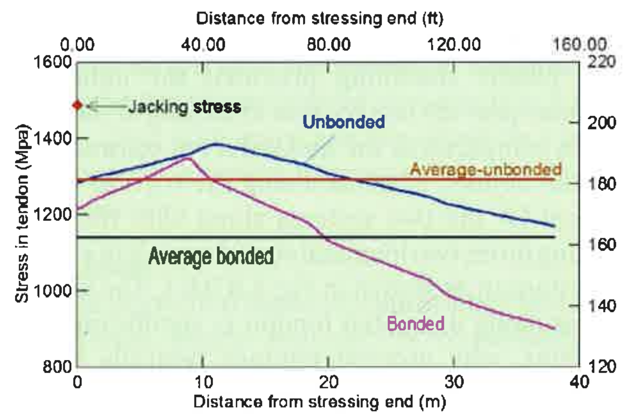


FIGURE 4.8.9A-1 Distribution of Stress Along Tendon Length (P668)

Much is discussed among the suppliers, practicing engineers, building authorities and owners regarding the merits and draw backs of each system. The application of the unbonded system was developed in the United States in mid 1950s. Over the past sixty years, thousands of buildings in different parts of the country have been built, using the unbonded system. The satisfactory in-service performance of these buildings is testimony of the integrity of the system. Elsewhere, there is a wide spectrum, ranging from outright use of grouted systems, to mixture of unbonded and grouted system.

TABLE 4.8.9A-1 Stressing Parameters (T155)

Coefficient of Friction	Bonded	Unbonded
Angular	0.25/rad	0.07/rad
Wobble	0.0001 rad/m (0.000033 rad/ft)	0.0046 rad/m (0.0014 rad/ft)
Seating loss	6mm (0.25 in)	6mm (0.25 in)

Background to friction loss calculation is given in Chapter 10.

Each of the two systems enjoys advantages and has its drawbacks. Each can be designed and constructed to meet or exceed the design requirements. The choice and preference of one over the other are matters of economics of construction and marketing – not the underlying technical merits. As part of a marketing scheme, frequently technical merits or relative disadvantages of one system are highlighted to support the selection of the preferred system over the other.

In the following we review several of the comparative aspects of each of the two systems.

A. Efficiency in Service Condition: The grease and plastic sheathing provided for unbonded systems provide less friction at jacking of tendons, when compared to the metal duct in common use for the bonded systems. Using the friction values typical for the two systems along with the same jacking force, two identical strands result in a stress distribution as shown in Fig. 4.8.9A-1. The drop of stress along a grouted tendon is significant. This explains, why grouted tendons typically longer than 35m (115 ft), where practical, are stressed at both ends.

The typical friction values (Table 4.8.9A-1) and tendon geometry Fig. 2.6.1B-3 are used for Fig. 4.8.9A-1. The jacking force is 80% of the specified ultimate strength. The average force provided for an unbonded strand is about 12% more than its corresponding grouted tendon for service condition[28]. The friction values assumed for the comparative figure are:

B. Efficiency in Providing Strength: Grouted tendons provide more strength to resist the design moments at ultimate limit state. Using ACI 318 provisions, for the example used in the preceding section a grouted strand provides 24% more force to resist the ultimate design moment (ULS). At the location of incipient failure, the strain in a grouted tendon corresponds closely to local strain in concrete. For an unbonded tendon the strain spreads over a longer length of strand, resulting in a smaller increase of tendon force at ULS, but a much larger deflection of the member.

Using EC2, for the same example, a grouted tendon provides 28% more strength at ULS, when compared to an unbonded tendon alternative. In most designs based on ACI 318 the determination of the required post-tensioning is governed by the service condition, where strict limits in the value of the hypothetical extreme fiber tensile stresses must be observed. The prestressing determined for service condition, along with other in-service minimum requirements of the code often satisfy the strength condition (ULS) of a member at the lower value of tendon force provided by unbonded tendons. In other words, the lower value of ultimate stress in an unbonded tendon does not necessarily lead to an increase in total reinforcement of a member, since in most cases the "provided" reinforcement to meet the service condition is in excess of that needed for ULS, when using ACI 318. The same is not true, when using EC2. Following EC2, it is permissible to use the full strength of a tendon for ULS, and satisfy the service condition by added non-prestressed reinforcement. As a result, EC2 affords full advantage to be taken in the greater strength that a grouted tendon can provide.

C. Crack Mitigation and Control: Unbonded tendons provide approximately 12% more precompression for the average tendon lengths (approx. 30 m; 100 ft stressing length). Added precompression helps in mitigating crack formation. However, once a crack is formed, the bond between the duct of a grouted tendon and concrete around it provides a more effective crack control. For this reason, the grouted tendons are lumped with non-prestressed reinforcement,

[28] The unbonded tendon profile for the same application can accommodate a larger drape, hence a greater angular change along its length.

when checking for the amount of reinforcement required for crack control. Unbonded tendons are excluded in the computation of area of reinforcement required for crack control.

D. Deflection Control: For the same member thickness and post-tensioning, unbonded tendons provide more uplift, due to their higher value of in-service force, and larger drape. For common building dimensions, such as the example of reference floor shown in Fig. 2.6.1B-1and 2.6.1B-2, unbonded tendons are between 10 to 15 percent more effective in reducing deflections from selfweight. This is due to the combination of larger drape and increased effective prestressing force.

E. Effectiveness in Mitigating the Consequence of Cracks due to Restraint of Supports: At the first, and to a lesser extent, the second and third floors above the foundation, cracks are likely to develop, if the supports provide excessive restraint to the free shortening of the floors. Loss of precompression to the restraining supports leads to a reduction in the moment capacity of a slab. At the ultimate strength limit, however, the consequence of support restraint in capacity reduction is different for the two post-tensioning systems.

At ultimate limit state, neither the total force developed in unbonded, nor grouted tendons will be fully available to resist the demand moment. In each case, the tendon force available to develop a section's resistance is reduced by the fraction of the prestressing forces that are diverted to the restraining supports.

However, due to the increase in strain of grouted tendons being "local," the grouted tendons are more effective in compensating the loss of force to the restraining supports. For unbonded tendons, an additional fraction of increase in tendon force will again be re-directed to the support restraints. The concept is explained in detail in references [Aalami, 2006].

F. Construction Considerations:

❖ Hardware

Anchor devices of grouted tendons (Fig. 2.4-7) are larger than those of unbonded tendons. Grouted tendon anchors do not fit readily where a wall or column is congested with reinforcement.

Where congestion of reinforcement does not allow unencumbered installation of anchors at the stressing edge of a member, tendons have to be terminated short of their full extension and stressed at the top of a member through specially constructed stressing pans (Fig. 2.5.2C-2). The stressing pans are generally located between 800 to 1000 mm (30 to 40 in) from the obstruction to allow for handling of the stressing jack.

Metal ducts of grouted tendons are more susceptible to mishandling and damage than sheathing of unbonded tendons in during construction. Collapse of a duct or kink cannot be repaired with the same ease, compared with damaged sheathing of an unbonded tendon. Duct buckling may lead to opening of seams, and possible increase in friction at stressing.

❖ **Layout**

Rigid flat metal ducts used in grouted systems typically do not allow more than 10 degrees of deviation on plan, and a curvature in elevation of about 2000 mm (6' – 6"), thus forcing the design to favor a layout with straight tendons in both directions (Fig 4.8.9F-4). The ability of unbonded tendons be grouped and swerve in one direction along the line of non-aligned supports (Fig. 4.8.9F-5), coupled with equal spacing of tendons in the orthogonal direction improves the efficiency in construction.

❖ **Labor**

Layout of grouted tendons is more labor intensive. Additional efforts required in construction of grouted tendons include: splicing the ducts; inserting the strands through the ducts, installing vents for grouting, and grouting.

❖ **Quality Control**

There is less that can go wrong in installation of unbonded tendons. Grouting of bonded tendons requires technicians with greater skill and stricter quality control than necessary for construction with unbonded tendons. Due to its importance in the overall integrity of the tendons, generally in the US, grouting is required to be performed by technicians who have received a formal training in grouting, have passed a qualifying exam, and are licensed to grout.

❖ **Cost and Availability of Hardware**

Anchorage devices for grouted tendons are generally propriety items carried by established post-tensioning suppliers. Anchorage castings for unbonded tendons are more readily available. For mono-strand construction, the cost of anchorage device per strand is generally higher for grouted than unbonded.

❖ **Tendon Failure**
A broken strand, or one severed unintentionally, loses the entire force along its length, if unbonded. However, a grouted strand responds in essentially the same manner as a non-prestressed reinforcement bar. When severed, the effectiveness is lost at the severed location and over the development length on each side of the cut. The remainder of the prestressing steel remains unchanged and effective. This difference is oftentimes leveled as a strong disadvantage against the application of unbonded systems. The concept of loss of force over the entire length of a tendon is no different than the loss of force in one of the stays of a cable stayed bridge - yet the latter is not viewed as negatively. It is the adequate redundancy in the system that provides the safeguard against catastrophic consequences. Moreover, the observation of satisfactory performance of thousands of post-tensioned buildings in the US constructed with unbonded tendons over one-half of century places this consideration in its proper perspective.

❖ **Repair and Retrofit**
Modifications and retrofit in a floor system reinforced with an unbonded system is readily possible, but requires experienced technicians capable to locate, expose, de-stress, replace if necessary, and re-stress the tendons at the repair/retrofit location. A damaged strand can be extracted, and re-threaded with a new one. Extensive changes in structural layout and

FIGURE 4.8.9F-4 Orthogonal Arrangement of Grouted Tendons in a Floor System (P235)

FIGURE 4.8.9F-5 Swerving of Unbonded Tendons to bypass an obstruction (Valencia Building, San Jose, CA; P232)

function of post-tensioned floor systems are not uncommon, but require diligence, beyond that necessary for grouted tendons, if the floor is reinforced with an unbonded system. Section 2.7 provides additional details.

❖ **Treatment in Building Codes**
ACI 318-11 differentiates between the design requirements of the bonded and unbonded systems. The impact of the ACI requirements. The impact of ACI requirements for typical construction is discussed in Chapter 2.

4.9 ANALYSIS OPTIONS

At this stage, referring to the previous Sections; (i) we have covered the guidelines for the choice of suitable dimensions; (ii) gone through the selection of material properties, and the post-tensioning system; (iii) identified a load path; (iv) and have decided on the connectivity of the members, namely: boundary conditions, and assignment of stiffness. The next step is to analyze the structure, and conclude with the deformations and the actions for the selected structural system. The following explains the options:

4.9.1 Underlying Assumptions
With respect to post-tensioning, major building codes concur on the underlying assumptions for the analysis. These are:

A. Gross Cross-Section: For gravity loads, the actions (moments, shears and axial forces) as well as deformation of a prestressed floor system are determined assuming the gross cross sectional dimensions of the floor's structural members.

In other words, in the computation of stiffness properties the entire cross section of each design strip can be considered effective. Notwithstanding the foregoing, as outlined in Section 4.6.7, prior to the initiation of analysis, designers have an option. In defining the "structural system," engineers may assign reduced stiffness values to the selected structural members, and/or specify non-rigid connectivity among the members that are otherwise integrally connected. Stiffness adjustment, and assumptions regarding the connectivity of the members have to take place prior to the analysis, however.

The outcome of the analysis is used directly for serviceability check, such as deflections and stresses. It is understood that under design loads, a floor may crack, leading to a change in the distribution of actions arrived at from the initial analysis. However, for design purposes and code compliance of post-tensioned members, the forces determined on the basis of gross cross-section, and designer assigned reduced stiffness are considered valid. These forces are used to compute the hypothetical stresses to be compared with the code allowable values or thresholds.

B. Linear Elastic Relationship: Linear elastic analysis is used to determine the deflection and actions in a floor system. Actions obtained from the elastic analysis are post-processed for the stress check and crack control, as well as the safety check of the floor system.

At strength limit state, recognizing the post-elastic properties of the structure, the distribution of actions will be different from those computed using an elastic analysis. However, the designs obtained based on linear elasticity will be safe, when used as entry values to determine the actions at the designated load path for ultimate strength - (i) the actions derived from the linear elastic solutions, and assigned to the strength load path will be in static equilibrium with the applied loads, (ii) the strength load path is designed to resist the assigned forces; and (iii) the load path will be provided with adequate ductility.

4.9.2 Analysis Methods
Typically, one of the three analysis methods outlined in Chapter 3 is used to determine the deformation and actions. These are: Simple Frame Method (SFM); Equivalent Frame Method (EFM); and Finite Element Method (FEM).

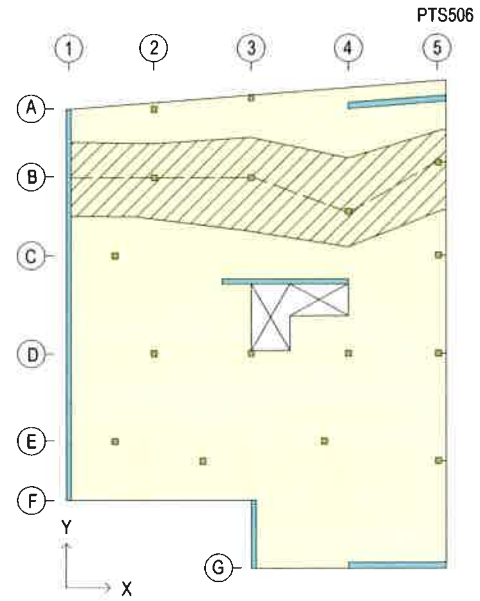


FIGURE 4.9.2-1 Delineation of a Design Strip in a Floor System

Briefly, expounding on the outline given in Chapter 3, SFM and EFM are based on extracting design strips (Fig. 4.9.2-1) along the line of supports. Each design strip will be assigned a width that is generally equal to the tributary of the associated support line. The extracted strip is analyzed in isolation with a number of simplifications.

4.9.3 Simple Frame Method (SFM)
The SFM models and analyzes the extracted structure as a slab frame, using strictly the gross cross-sectional geometry of its parts. Loads on the strip are assumed to act along the line of supports, irrespective of their offset from straight lines joining the adjacent supports.

4.9.4 Equivalent Frame Method (EFM)
Similar to SFM, the EFM is also based on subdividing a floor system into design strips as shown in Fig. 4.9.2-1 and the example shown in Fig. 4.9.4-1a. The extracted frame is idealized (Fig. 4.9.4-1 b, c) and squared. The geometry of the model shown in part (b) of the figure is used in SFM analysis. The EFM modifies the simple model shown in part (b) of the figure by considering the strips shown on each side of the support columns (part c of the figure) as torsional members. The torsion developed in the torsional members is resisted by the column through an assumed load path illustrated in Fig. 4.9.4-2.

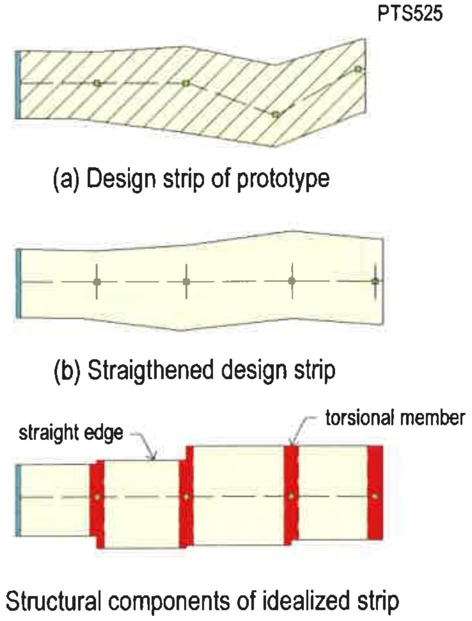


FIGURE 4.9.4-1 Isolated, and Idealized Design Strip showing the FEM Structural Components

In the EFM model (Fig. 4.9.4-2) the rotation of slab on the front and back of the column will twist the torsional members, which members will in turn rotate the column. The structural model for the load path from the slab to the column is illustrated in Fig. 4.9.4-2.

The inclusion of the torsional members along the load path from the slab to the column (Fig. 4.9.4-3b), in effect results in an equivalent column of less stiffness. The combined effect is illustrated as "equivalent column" shown in part (c) of the figure.

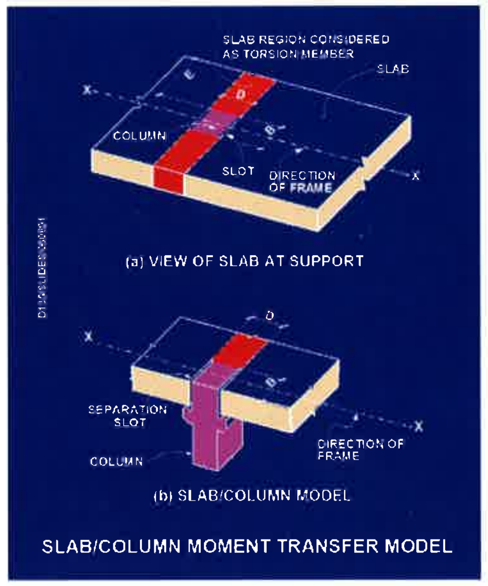


FIGURE 4.9.4-2 Torsional Member on Load Path to Column (P554)

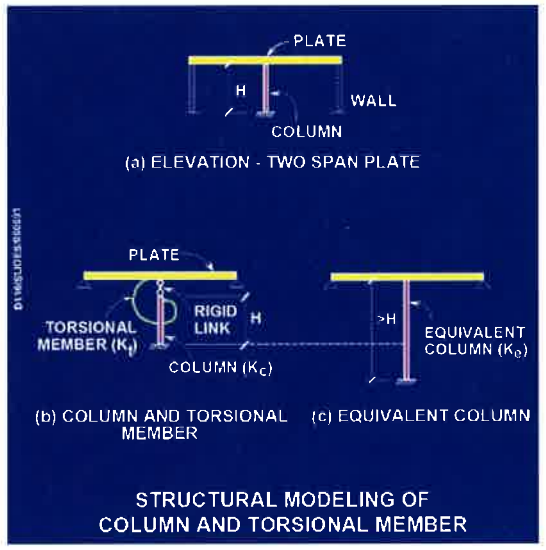


FIGURE 4.9.4-3 Column Modeling in EFM (P555)

The consequence of using the EFM, compared to SFM is lower column moments. However, both SFM and EFM methods deliver safe designs, since they both satisfy equilibrium, and each is designed to resist its respectively computed actions. EFM is believed to offer greater economy in design, in particular for non-uniform spans.

4.9.5 Finite Element Method (FEM)

FEM has emerged as the analysis method of choice for most of our structural designs. For many, FEM is still viewed as a black box. It provides the values we need for our designs. Indeed, FEM is a powerful and versatile tool. FEM — originally a structural engineering tool — has become the core of a diverse field of applications. While it is not critical for us, as structural engineers, to be familiar with the details of its core formulation, it is important to fully understand; correctly interpret, and have the right perspective regarding a FEM-based analysis. As structural engineers, we ought to use the results of a FEM analysis to fine tune our own understanding and prediction of a structure's response – not to find out the behavior of the structure and the range of its design values. In my opinion, we are qualified to use the black box, only if we can develop a valid design without it. We use FEM for expediency and greater accuracy; but not to supplement our expertise in understanding the structure, and its design process.

You will find, in application to the design of concrete structures, not all FEM tools deliver the same outcome. General-purpose-based FEM

tools may use formulations that can make their application to concrete floors, and in particular post-tensioning questionable. The objective of this Section is to explain in simple terms the underlying concepts of FEM when dealing with design of concrete floors. Chapter 11 extends the explanation to design of post-tensioned members. The explanation helps to better evaluate the outcome of a FEM-based analysis.

The following illustrates several of the basic concepts, with the premise that the reader has not had a course in FEM, nor is he/she interested in becoming involved in the intricacies of its formulation. Yet, he/she wants to know the strength and limitations of the method, when dealing with design of concrete floors.

4.9.5.1 A Word on FEM Concept: FEM subdivides a structure into smaller and simpler pieces (elements) whose behavior can be closely formulated. Each element is formulated to capture the local response of the structure at the element level. The local response of each element is based on such factors, such as material properties, geometry, location in the overall structure, and relationship with adjoining elements. The mathematical assemblage of these elements into the complete structure allows for the automated computation of the behavior of the entire structure in one step. The method has wide application and has proven to accurately recreate the response of real structures. The power of the method rests on the accurate formulation of the structural model, proper processing of the outcome, and its interpretation for structural engineering application.

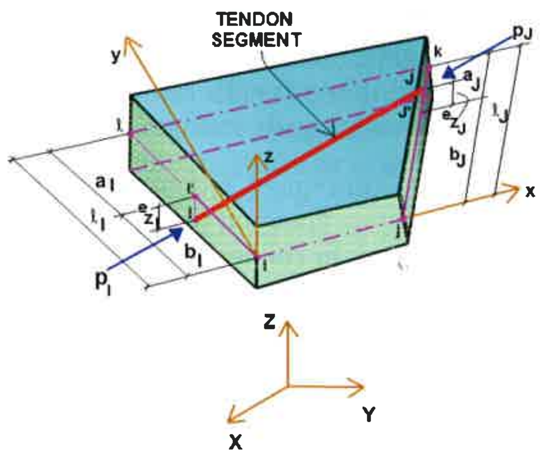


FIGURE 4.9.5.1-1 Representation of a Slab Element with Embedded Reinforcement (P556)

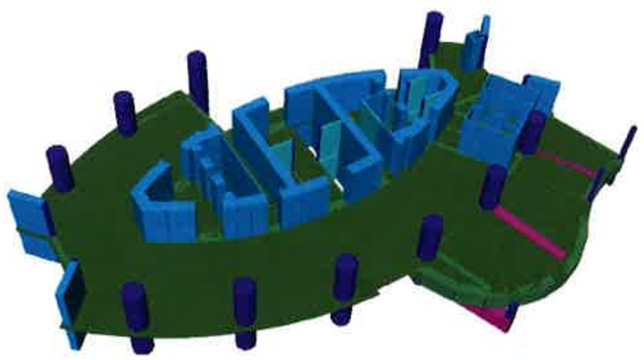
The current crop of finite element programs can be divided into two categories: programs that are formulated for wide application to many different structures, and programs that are tailored to handle a specific kind of structure. General purpose FEM programs afford the engineer the ability to analyze a wide variety of structures, without having to acquire and master multiple programs. However, general purpose programs tend to be cumbersome to apply to the specific behavior of concrete, and in particular to post-tensioning. Special purpose FEM programs are most often used when designing concrete structures, because they are optimized for the specific task, allowing the engineer to analyze and design a slab much more efficiently. The increased efficiency is especially pronounced when it can handle the three dimensional solid models of a building. When properly formulated, a specifically tailored finite element program can become part of the chain of BIM (Building Information Modeling) in providing a seamless design process, from the architectural inception of a project to its construction.

To analyze a floor, in particular if there are steps, beams, and/or post-tensioning, a FEM program must account for the effects of loads acting both normal to the slab and in its plane. Live and dead loads generally act normal to a slab; while the post-tensioning tendons apply both normal (transverse) and in-plane forces, where tendons are positioned eccentric to a member's centroid. Presence of beams and changes in elevation, also induce inplane forces in floor slabs under gravity loads. Other sources of inplane forces are column drops, drop panels, steps, and geometry features that cause a change in the elevation of a slab's centroidal axis.

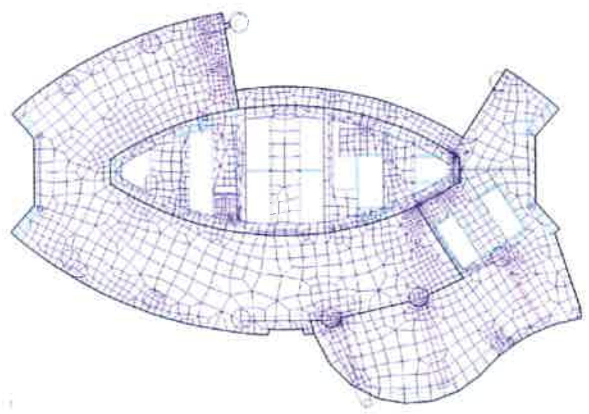
The loading on a floor is transferred to the supports by way of flexural and membrane actions of the slab and other structural members of a floor. Figure 4.6.1C-1 illustrates the actions for a concrete slab of uniform thickness. There will be forces from prestressing and reinforcement that may exist beyond those shown in the figure. The slab shown in the figure undergoes bending as well as displacements within its plane, namely membrane movement.

When dealing with reinforced concrete and pre- or post-tensioning, both the reinforcement and prestressing members participate in resisting the loads. It is common practice not to account for the added stiffness of reinforcement in a first analysis, aimed at uncracked response of a floor. However, in post-cracking regime, the amount, position and orientation of each bar or prestressing segment plays a central role. For this reason, and recognizing that real concrete structures crack, it is important that the elements used in a FEM analysis of concrete floors include the reinforcement and prestressing as discrete members. Figure 4.9.5.1-1 illustrates an element that features a prestressing segment. The concept and application is expounded in Chapter 11.

4.9.5.2 Discretization: A structure is subdivided into small elements, also referred to as "cells." In shape, the elements are generally either triangular or quadrilateral. Well proportioned quadrilateral elements generally provide a more accurate solution. For a reliable solution satisfying the equilibrium with applied loads, it is critical that the nodes of the elements coincide with one another.



(a) 3D model of a tower floor (P238a)



(b) Organic discretization of the floor system (P238b)

FIGURE 4.9.5.2-1 A Well-Discretized View of a Floor System (Rafal Tower, KSA)

4.9.5.3 Forces and Stresses: We first review a simple and familiar member to crystallize the concept, before dealing with the application of FEM to concrete structures. Refer to Fig. 4.9.5.3-1. The deflection of the beam, and the stresses that lead to the initiation of cracking are determined using the familiar beam equations, where plane sections are assumed to remain plane. The equations result in a distribution of shear and axial stress on a beam's section, such as section A-A (part b). To evaluate the safety of the beam we consider the demand actions M and V shown in part (b,iii). For safety consideration at section A-A, we envisage a resisting mechanism that invariably differs from the distribution of moment and shear shown at the face of the cut A-A (part b,ii). In most cases, we rely on a tensile force to be provided by reinforcement, and a compressive force from concrete. However, part (c) displays a different option; with the objective to emphasize that there can be more than one option in resisting the demand. Part (c) illustrates a strut-and-tie mechanism within the outline of the concrete member. It is valid and practical to determine the overall demand M and V using FEM-based elastic analysis, but establish the adequacy of section A-A for its safe load transfer using the mechanism shown in part (c).

Comparing the parts (b) and (c) of the figure, we conclude the following:

❖ It is valid and practical to use the distribution of actions derived from the service condition (part b) and apply it to a resisting mechanism (part c) that is different from the service condition. The practice is valid, since (i) the distribution of actions derived from the service condition satisfies equilibrium – so do the actions developed through the envisaged resisting path, and (ii) the resisting load path selected for strength condition will be provided with adequate strength and ductility.

To illustrate the concept of actions at a "design section" of a concrete floor and stresses at the face of the same section in connection with FEM, we now review the cantilever shown in Fig. 4.9.5.3-2 and 3. Part (a) of the first figure shows the cantilever under the action of an external load at the upper corner of its tip. Part (b) is discretization of the cantilever into a number of triangular finite elements. Selfweight is disregarded for simplicity.

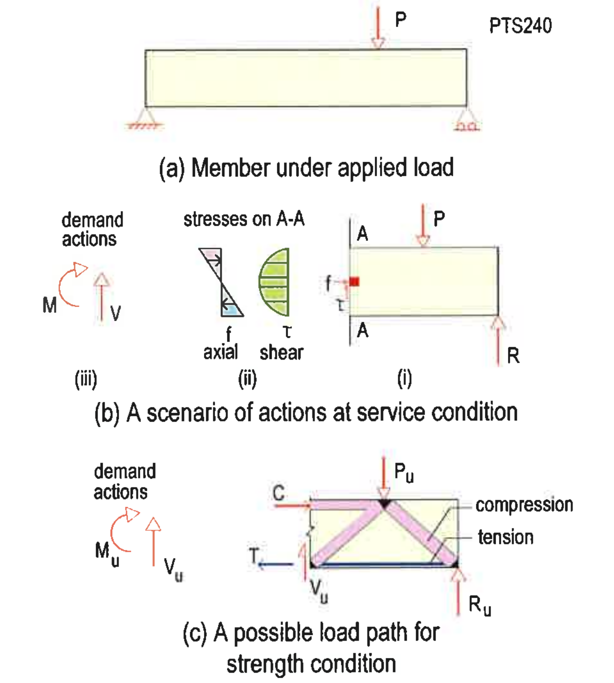


FIGURE 4.9.5.3-1 Service and Limit State Actions

In the mathematical formulation of the problem, the elements shown in the figure are connected together at their common nodes (vertices), where force is transferred from one element to the other.

Consider the triangular element that is isolated and illustrated in part (c) of Fig. 4.9.5.3-2. A condition imposed on the mathematical formulation of each element, such as the one shown, is that the total of the external forces that act on all the nodes of an element shall be in static equilibrium. However, the distribution of stress within the body of an element follows an "assumed" shape. The "ordinates" of the assumed shape are determined such as to satisfy the applicable energy theorems used in the element's formulation. A common assumption for triangular elements, such as the one shown, is to assume a constant distribution of stress (shown as f_1) within the entire body of the element. The magnitude of the constant stress f_1 is computed, such as to satisfy the energy equilibrium of the forces and displacements of the nodes with stresses and strains within the body of the element.

When dealing with concrete members, we generally make a cut at a selected location, determine the demand actions at the cut and check/provide for adequacy of resistance at the cut. Next, we do the same for the selected element (parts d and e).

The FEM solution to a problem will conclude with forces such as P_1, P_2, and P_3 at the nodes, along with a stress f_1, as shown (Fig. 4.9.5-3a). In the figure, for simplicity, the nodal forces in one direction are shown, namely P_1, P_2 *and* P_3. The FEM formulation ensures that these forces satisfy the static equilibrium.

$$P_1 + P_2 + P_3 = 0 \qquad \text{(Exp 4.9.5.3-1)}$$

Let the triangular geometry of the element selected be such that the vertex is one-third away from the base. Then, the forces at the nodes will be as shown in part (b) of the first and part (c) of the second figure.

If we make a section at 1-1, the distribution of stress at the cut section will be uniform with intensity f_1 Note that the shape of the stress distribution was based on an "assumption," while its value (f_1) was determined through computation. Two options are illustrated in parts (d) and (e) of the first figure to calculate the resultant of the actions at the cut section. Note that in concrete design, to determine the required reinforcement we need the resultant of the actions on a section, such as the total moment – not the distribution of stresses. The two options for calculating the demand value reported by FEM programs are outlined below:

A. Stress Integration: In this option, the resultant of the forces at the cut section is calculated by integrating the stresses over the area of the cut surface. Note that the integral of stresses will be a force F equal to $f_1 a$, assuming unit thickness for the element (part d of first figure, and part c of the second figure). This force acts at mid-length of the cut section. The calculated force is not in equilibrium with the externally applied force P, since for the diagram shown, F_1 neither is collinear with P, nor is it of the same magnitude. F_1 varies in magnitude with the length of the cut section, as opposed to be of a constant value equal to "P", irrespective of the location and length of the cut.

We conclude that, the integration of internal stresses does not result in forces that in general satisfy the equilibrium of the structure. The integration can underestimate, or overestimate the externally applied force, depending on the location of the cut. Where it underestimates the demand, it can lead to an un-conservative design. Prctically all geneal purpose FEM programs are based on "stress integration".

B. Nodal Integration: Consider part (d) of the second figure. It is argued that since the external forces to the element are applied at the nodes, the force (F_2) at the design section (1-1) must be in equilibrium with all the external forces acting on the cut piece. It is the application of the same concept in the simple beam of Fig. 4.9.5.3-1, where the actions on a design section are determined from the externally applied forces on the member.

$$F_2 = P \qquad \text{(Exp 4.9.5.3B-1)}$$

The force F_2 computed from the equilibrium of actions at the nodes is in equilibrium with the externally applied forces. The process of calculating the actions at a design section using the equilibrium of forces at the nodes is referred to as "nodal integration." There are several variations of nodal integration. The "extended nodal integration," is developed to yield the value of actions at a design section of arbitrary length and orientation irrespective of the location of the nodes and the meshing of the structure [ADAPT TN 302, 2009].

The calculation of design values using "nodal integration," is applicable to design of concrete structures, since in concrete design we use the "total" of forces for design, namely one moment; one shear force; and one axial force to design a beam section - not a distribution of stress at the face of a beam. The reason the general purpose programs do not generally feature the nodal integration scheme necessary for concrete structures is that in most other materials, such as glass, it is the stress at a "point" that determines the outcome of a design, as opposed to the integral (total) of the force.

Another important aspect of the FEM is that the forces at the common nodes of adjoining elements are in equilibrium with the externally applied forces at the same node. This combined with the foregoing fact that the forces at the nodes of a single element are also in equilibrium, leads to the conclusion that the design actions determined on the basis of nodal integration are in equilibrium with the externally applied loads on the structure. This is a critical conclusion, since "equilibrium" of forces used for design with the externally applied loads is a prerequisites for a safe design. The same is not necessarily true, when stress integration is used.

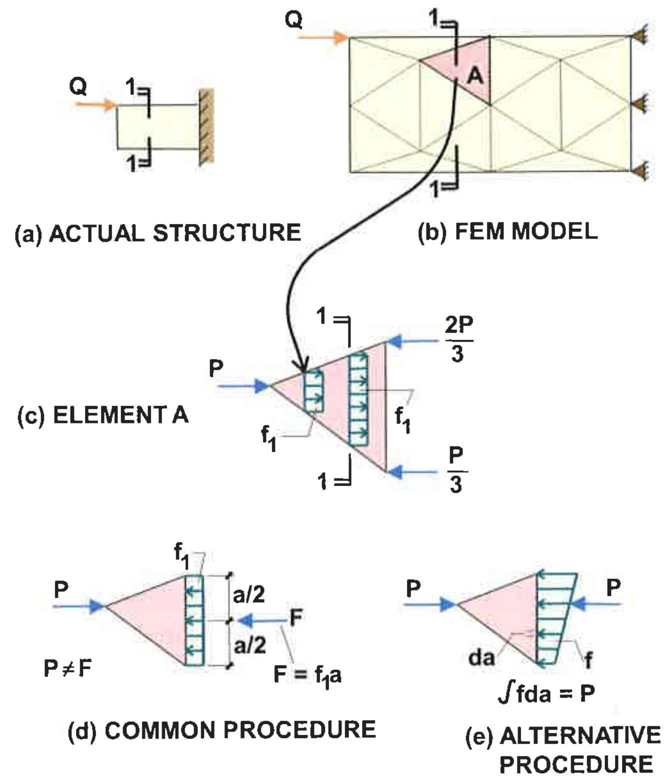


FIGURE 4.9.5.3-2 Discretization of a Cantilever (P558)

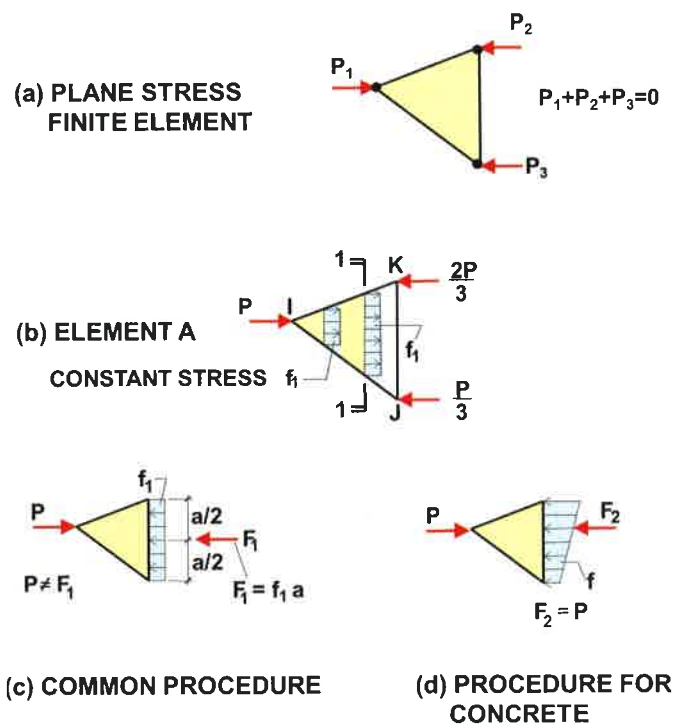
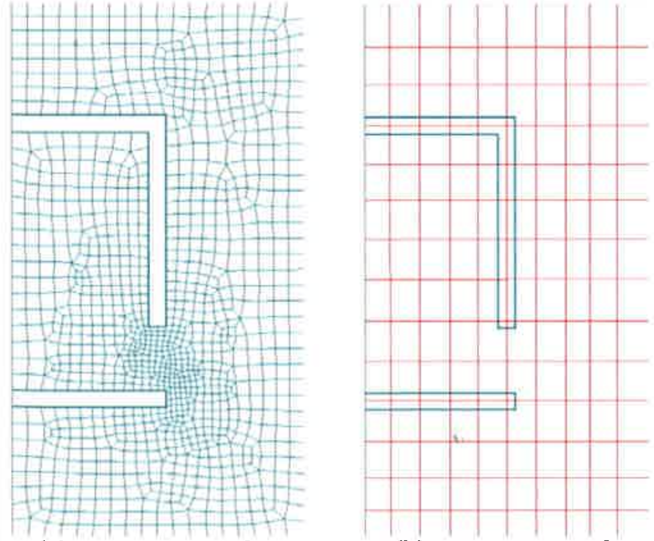


FIGURE 4.9.5.3-3 Forces and Stresses on a Finite Element (P559)

4.9.5.4 Mesh Density: Based on the foregoing discussion, the distribution of stress within an element follows an assumed shape. The ordinates of the shape are determined by analysis. Small-sized elements must be selected, in order for the computed stresses closely approximate the distribution of the stress in the prototype structure. That is why, for safety of the "glass panel," discussed in Section 3.3-A, for which the failure initiates from stress at a "point" small sized elements must be selected. A fine mesh can better capture the peak stress in a structure. For concrete structures, where the design sections are reinforced using the "total " of actions at a section, when using nodal integration, the fineness of the finite element mesh is no longer critical. This is illustrated in reference [ADAPT TN184, 2006].

Figure 4.9.5.4-1 shows two options of FEM meshing, one for use in design of concrete structures coupled with "nodal integration" scheme, and the other for general purpose, where the value of stress at a "point," governs the design, or where the extraction of design values is based on "stress integration." The course mesh shown in part (b) of the figure is adequate, when combined with "nodal integration."

4.9.5.5 Forces (Actions) on Design Sections: Subsequent to analysis, the demand actions at each design section are calculated. This is followed by checking the available resistance at the same



(a) Discretization for "point" controlled designs (P239a)

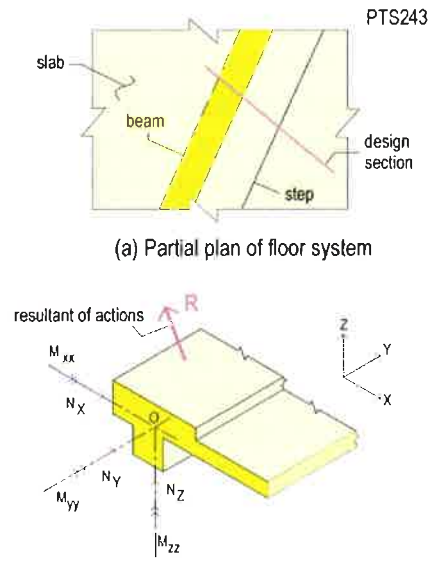
(b) Discretization for "section" controlled designs (P239b)

FIGURE 4.9.5.4-1 Alternative FEM Discretization Options

section. Reinforcement is added, if the existing reinforcement falls short of demand. Using FEM, coupled with nodal integration, actions at each design section, irrespective of length and orientation of the design section are evaluated using the following process.

Figure 4.9.5.5-1a shows partial plan of a floor system under applied loads. The plan also identifies a single design section. Part (b) of the figure shows the cross-sectional view of the design section identified in part (a). In the general case, the demand on any design section is a single force R (part b) acting in a direction and at a point determined by computation. However, the practice is to move this demand force to the centroid of the design section, and express it in its six components, with respect with local coordinates of the design section (part b). The components at the centroid are three forces and three moments.

The design process evaluates the adequacy of the section for each of the six components. In practice, the bending moment about the plane of the section (*Mxx*) and axial force (*Ny*) are considered jointly for the flexure of the design section about axis x-x. In common floor systems, the section is deemed adequate for in-plane shear (*Nx*)and moment about an axis normal to the plane of the floor (*Mzz*). The torsion about the centroid of a design section is due to the eccentricity of normal shear (*Nz*) with respect



(a) Partial plan of floor system

(b) Cross-sectional view of design section

FIGURE 4.9.5.5-1 Actions on a Design Section to the centroid of the section. There are several options to investigate the adequacy of a section to resist the torsion component. A number of engineers consider this to be the outcome of "twisting moments" in the slab and add the value of torsion to that of bending. The practice is referred to as Wood-Armer approximation. Its concept and application in the framework of design process based on designer assigned load path, and design sections of finite length is not applicable. Yet, the option is provided in a number of commercially available software[29], since it is conservative and conforms to the work of those who practice it.

4.10 SERVICEABILITY CHECK (SLS)

The general requirements for the in-service performance of a floor system were listed in Section 4.1.2. The following covers the specifics of each.

4.10.1 Load Combinations

The recommended load combinations simulate two scenarios, namely the "sustained," also referred to as "quasi-permanent," and the "total" also referred to as "frequent." The "sustained, or quasi permanent" condition is intended to reflect the load that a floor system is likely to be subjected to under normal service condition. This load combination is used to determine the long-term effects, such as creep deflection. The "total; or frequent" load combination is the "design load." It is the load that a floor system can possibly be subjected to over the course of its in-service life. The instantaneous adverse impact of the "design load" on cracking or malfunction of non-structural members that are attached to the floor system is checked.

ACI 318-11 defines "sustained load"[30] as any portion of the service live load "design live load" that will be present for a sufficient length of time to cause significant time-dependent deflections. ACI 318 leaves it to the designer to determine the fraction of the "design live load" that should be considered as "sustained" for each application. As a matter of fact, ACI 318 does not specify the load combination for "total load" either. The practice by design professionals is:

ACI 318-11; IBC

- Total load combination
  1.00 DL + 1.00 LL + 1.00 PT
- Sustained load combination
  1.00 DL + 0.30 LL + 1.00 PT

Where

DL is the sum of selfweight and superimposed dead load; and
LL is the "design" live load.

The European code EC2 recommends the load combination for service condition as:

European code EC2

- Total load (frequent)
  1.00 DL + ψ_0LL + 1.00 PT
- Sustained load (quasi-permanent)
  1.00 DL + ψ LL + 1.00 PT

Where, ψ is the recommended fraction of design live load to be considered as sustained (quasi permanent). The suggested values are given in Table 4.10.1-1 for sustained (quasi permanent) load combination. For serviceability check of post-tensioned members as outlined in Section 4.10.3, the frequent load combination uses a fraction of live load (ψ_0). The fraction depends on the occupancy of the area under consideration.

4.10.2 ACI 318 Crack Control – Stress Check – Non-Prestressed Rebar:

Crack control is a serviceability requirement. Depending on the exposure to corrosive elements, visual effects, and the procedure used for deflection calculation (cracked, or uncracked sections), the design targets to limit the width of probable cracks for service condition within an acceptable range. Crack control at design time is handled in two ways. In ACI 318, the formation and extent of cracking in post-tensioned floors is checked through the value of a hypothetical extreme fiber tensile stress. The European code EC2, uses an alternative method, where the probable crack width is computed and controlled through addition or arrangement of non-prestressed reinforcement. This Sub-Section describes the ACI procedure. EC2 procedure is outlined in the next Sub-Section.

In ACI 318-11, the crack control procedure depends on whether the member forms part of a "two-way" or "one-way" floor system.

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29 www.adaptsoft.com; ADAPT-Builder
30 ACI 318-11 R18.4.2

TABLE 4.10.1-1 Fractions of Design Live Load to be Considered as "Sustained; Quasi-Permanent."[31] (T113)

Occupancy	Fraction of design live load ψ
Dwellings and offices	0.3
Shopping; congested areas	0.6
Storage	0.8
Parking	0.6

A. Two-Way Systems: Under service load combinations, ACI 318 stipulates an upper limit for a hypothetically calculated extreme fiber tensile stress for limiting crack formation, and retaining the width of probable cracks to within acceptable values. The computation of the hypothetical tensile stress and its value are corroborated with test results, and observation of satisfactory performance of structures designed accordingly. The computation is based on the concept of design strips and actions associated with design sections. This is explained in detail in Chapter 3 and reference [Aalami, 2005]. It is summarized next using Fig. 4.10.2A-1.

(i) Computation of Hypothetical Stresses: The floor system is subdivided in design strips in two orthogonal directions (part a). Design actions are calculated at design sections that extend over the entire width of a design strip. Notwithstanding the fact that the forces vary along the length of each design section (part b) one representative value is calculated for each force distribution across each design section. The representative value is the resultant (integral) of the action being considered. For example, the moments acting on the design section shown are represented by their total value shown in the figure as "559" for the identified design section at the face of column. The design value obtained is applied to the entire cross-sectional geometry of the design section, using non-cracked gross cross-sectional properties. Thus, for each design section a single representative stress, such as average precompression, or maximum extreme fiber tensile stress is obtained. The representative values calculated are referred to as "computed," or "hypothetical." These values are not intended to reflect the status of stress at any given point on the design section. Rather, the hypothetical values are used as a means of controlling the stress-related overall response of a floor system, such as cracking, or creep.

(ii) Allowable Extreme Fiber Stresses: The allowable stresses in ACI 318 for two-way floor systems are[32] listed in Table 4.10.2A-1: The same table lists the allowable farthest fiber compressive stresses. Compressive stresses are controlled to mitigate excessive creep and thereby undesirable deflections. The method of computation of compressive stresses is the same as for the hypothetical tensile stresses explained above.

(iii) Minimum Non-prestressed Reinforcement: Non-prestressed bonded reinforcement in addition to post-tensioning helps to control cracking of a floor in service, and adds to ductility at strength limit state. The computation of the minimum amount of non-prestressed reinforcement, where necessary, is based on the concept of "design strip" as outlined in the previous Section. The following Flow Chart explains the details.

(1) The necessity and amount of non-prestressed reinforcement depends on the post-tensioning system used. The recommendations for unbonded and bonded systems are different.

(2) For unbonded systems, the necessity and amount of reinforcement depends on whether the location is regarded as support, or field (span). The delineation of how far away from the face of support "span" begins is left to the judgment of designer. The common practice is to consider the central six-tenth portion of the clear distance between the faces of the adjacent supports as "span" when checking for minimum rebar.

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31 Eurocode 1, Part 2.1 (ENV 1991-2-1)
32 ACI-318-11, Sections 18.3 and 18.4

TABLE 4.10.2A-1 Allowable In-Service Hypothetical Stresses ACI-318 (T114)

Load combination	Tensile stress psi (MPa)	Compressive stress
Sustained	$6\sqrt{f'_c}\left(0.5\sqrt{f'_c}\right)$	$0.45f'_c$
Total	$6\sqrt{f'_c}\left(0.5\sqrt{f'_c}\right)$	$0.60f'_c$

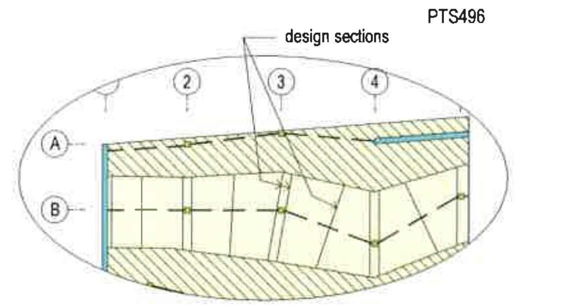
(3) Rebar over the supports: Regardless of the design values and geometry, there is a minimum amount of bonded reinforcement required over each support of all two-way floor systems reinforced with "unbonded" tendons. The amount of reinforcement is based on the cross-sectional area of the design strip in direction of analysis and the strip normal to it[33] as given below.

$$A_s = 0.00075A_{cf} \qquad \text{(Exp 4.10.2A-1)}$$

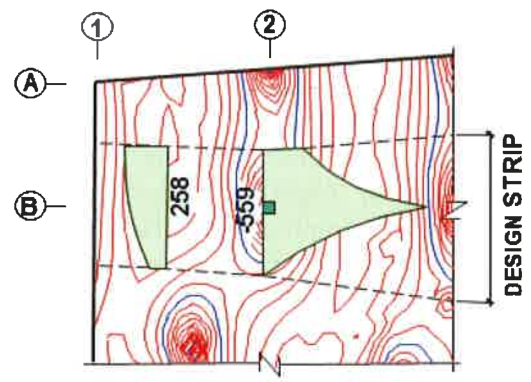
Where,

A_s = area of bonded reinforcement; and

A_{cf}= larger cross-sectional area of the design strips of the two orthogonal directions intersecting at the support under consideration (Fig. 4.10.2A-2).



(a) Design strip identifying the principal design sections



(b) View of distribution of action along a design section and display of its integral value (P560)

FIGURE 4.10.2A-1 Identification of Design Strip, Design Section, and Design Value

The area A_{cf} is calculated for each of the two orthogonal design strips, one in direction of analysis and the other normal to it. The larger of the two values is selected. Consequently, the area of minimum bonded reinforcement over a support will be the same in both directions. For the application of the suggested formula to irregular scenarios refer to reference [ADAPT TN 365, 2001].

(4) Calculation of hypothetical tensile stress (f_t) in the span : The necessity of a minimum amount of non-prestressed reinforcement in a span depends on the value of the hypothetical extreme fiber tensile stress. The computation of the hypothetical stresses is discussed in Section 4.10.2A-(i).

(5) Comparison of hypothetical tension stress (f_t) with the threshold for necessity of bonded reinforcement: The threshold stress is expressed next.

(6) Stress not exceeding the threshold: If the hypothetical tensile stress in span does not exceed the value specified in the following expression, no bonded reinforcement is required in the respective span.

$$f_t \le 2\sqrt{f'_c} \text{ (psi) } [\ f_t \le 0.17\sqrt{f'_c} \text{ MPa}]$$
(Exp 4.10.2A-2)

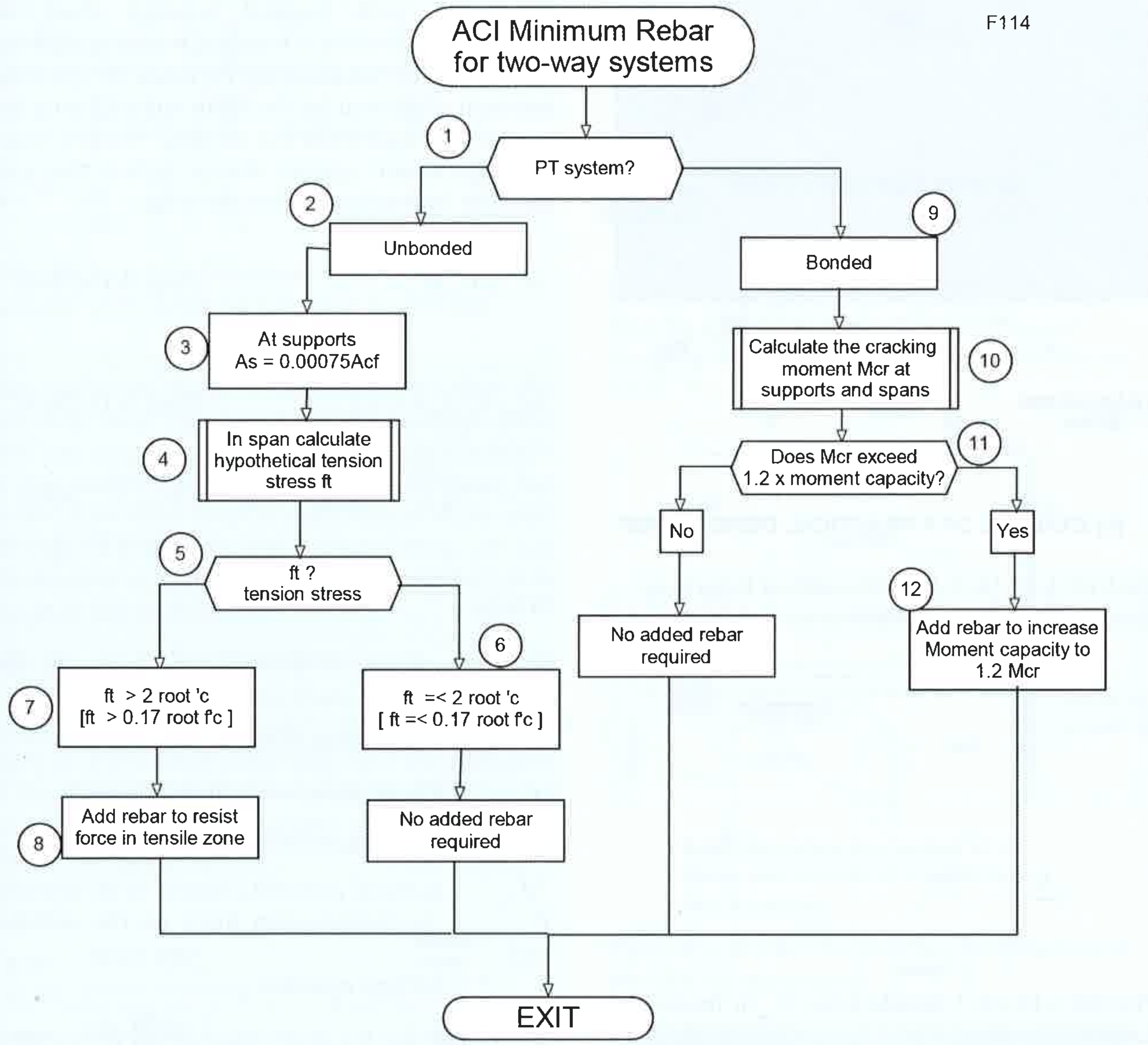
(7) Stress exceeding threshold: If the hypothetical tensile stress in span exceeds the value specified in the following expression, non-prestressed bonded reinforcement is required in the respective span.

$$f_t > 2\sqrt{f'_c} \text{ (psi) } [\ f_t > 0.17\sqrt{f'_c} \text{ MPa}]$$
(Exp 4.10.2A-3)

(8) Minimum nonprestressed Reinforcement: The minimum area of bonded reinforcement is given by:

[33] ACI 318-11, Section 18.9.3.3

FLOW CHART 4.10.2-1 Bonded -Reinforcement for Two-Way Floor Systems using ACI 318-11



$$A_s = \frac{N_c}{0.5f_y} \qquad \text{(Exp 4.10.2A-4)}$$

Where,

A_s = area of bonded reinforcement;

f_y = specified yield strength of reinforcement, not to be considered in excess of 60,000 psi (420MPa); and

N_c = tension force in the tension zone of concrete at the design section under consideration.

The computation of the tension force N_c shown in Fig. 4.10.2A-3 is given below:

$$N_c = 0.5(h-c)f_t \times b \qquad \text{(Exp 4.10.2A-5)}$$

$$h-c = \left[\frac{f_t}{(f_t+f_c)}\right]h \qquad \text{(Exp 4.10.2A-6)}$$

$$N_c = 0.5\left(\frac{f_t^2}{f_t+f_c}\right)bh \qquad \text{(Exp 4.10.2A-7)}$$

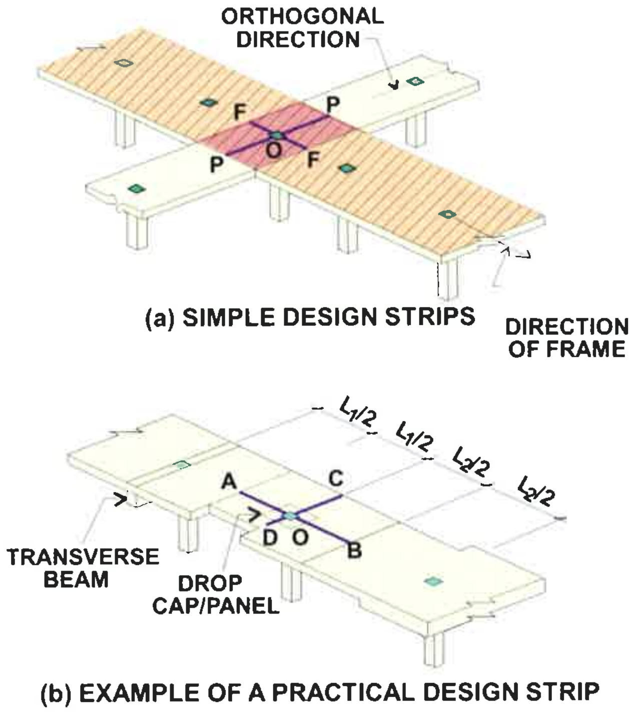


FIGURE 4.10.2A-2 Identification of Reference Sections for Bonded Reinforcement (P563)

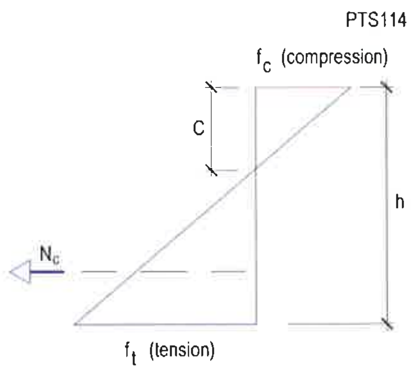


FIGURE 4.10.2A-3 Tensile force N_c in Tension Zone of a Section

Where,

b = member width.

(9) Bonded systems : The minimum non-prestressed bonded reinforcement in two-way slabs post-tensioned with bonded systems is based on their nominal design moment[34] capacity and the cracking moment at the same location, as detailed below:

(10) Cracking moment calculation: The necessity of added non-prestressed reinforcement depends on the value of the cracking moment along the length of a member. Hence cracking moments (M_{cr}) shall be calculated at all supports and spans.

(11) The total amount of prestressed and nonprestressed reinforcement at any section reinforced with bonded tendons shall be adequate to develop a nominal moment capacity at that section not less than 1.2 times the cracking moment computed for the same section, using the modulus of rupture of the section. The provision is a precaution against abrupt failure that can develop immediately after cracking.

$$M_n \geq 1.2 M_{cr} \qquad \text{(Exp 4.10.2A-8)}$$

$$M_{cr} = \left(f_r + \frac{P}{A} \right) S \qquad \text{(Exp 4.10.2A-9)}$$

$$f_r = 7.5\sqrt{f_c'} \quad \text{(psi)} \qquad [\ f_r = 0.62\sqrt{f_c'} \quad \text{MPa}] \qquad \text{(Exp 4.10.2A-10)}$$

Where,

A = gross cross-sectional area of the member;

f_r = modulus of rupture stress;

f_c' = 28-day concrete cylinder strength

M_{cr} = cracking moment of the section;

M_n = nominal moment capacity of the section;

P = precompression force on the section; and

S = section modulus.

An example for the application of this provision is given in Chapter 6.

(12) Add reinforcement: Provide additional bonded reinforcement, either non-prestressed, or prestressed, or a combination of both to satisfy the strength relationship given in step (11) above.

(iv) Length and Position of Minimum Reinforcement: Top bars intended for the minimum requirement of the code should extend not less than one-sixth of the clear span on each side of a support[35] (Fig. 4.10.2A-4).

[34] ACI 318-11, Section 18.8.2
[35] ACI 318-11, Section 18.9.4.2

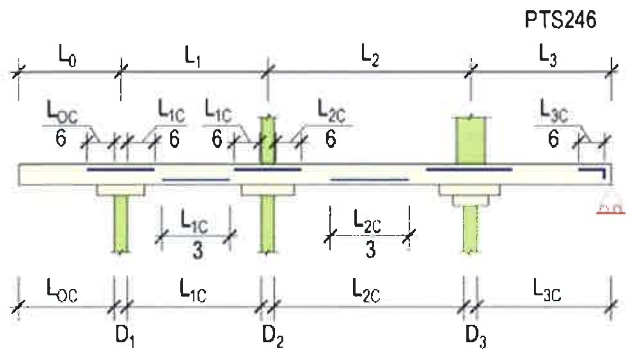


FIGURE 4.10.2A-4 Minimum Reinforcement Lengths and Layout for Common Conditions

The minimum bonded reinforcement over the supports must be placed within a narrow band over the support, which band is equal to the width of the support plus one and one-half times the depth of the slab/drop on each side as illustrated in Fig. 4.10.2A-5[36]. The arrangement of top reinforcement over the support is shown for a slab layout in Fig. 4.10.2A-6.

The **length** of the bonded minimum rebar for a span shall not be less than one-third of the respective clear span length[37]. This reinforcement shall be distributed uniformly over the tributary of the design strip as close to the tensile fiber as practical. Stagger the bonded reinforcement as shown in Fig. 4.10.2A-7.

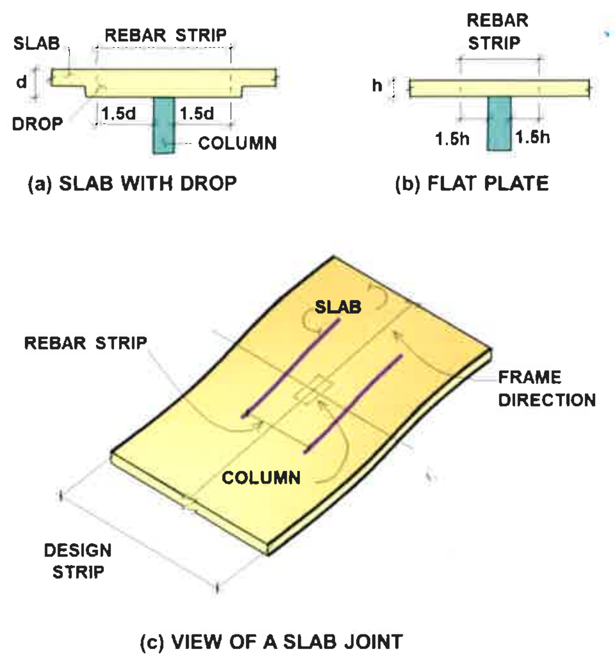


FIGURE 4.10.2A-5 Strip Identification for Placing of Minimum Bonded Reinforcement (P561)

FIGURE 4.10.2A-6 View of Tendon Layout Showing the Position of Tendons Over Column (P564)

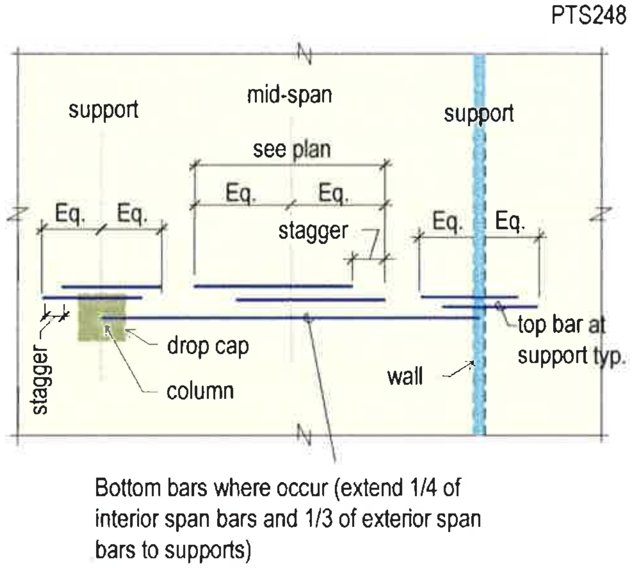


FIGURE 4.10.2A-7 Partial Plan; Distribution of Bonded Reinforcement

FIGURE 4.10.2A-8 View of Bottom Bars, Staggered by 12" (300 mm) (P241)

[36] ACI 318-11, Section 18.9.3.3
[37] ACI 318-11, Section 18.9.4.1

A good practice is to stagger the bars by a minimum of 12" (300mm), as illustrated in Fig. 4.10.2A-8.

B. One- Way Slabs and Beams: In ACI 318, the necessity, location and amount of non-prestressed bonded reinforcement in one-way slabs and beams are independent from the state of stress in the member. The value of the computed tensile stress in a member dictates the procedure to be used for estimating a member's deflection – namely Uncracked, Transition, or Cracked section analysis. For this reason, the allowable stresses for one-way slabs and beams are covered under the section for "deflection" calculation. The minimum non-prestressed reinforcement depends entirely on the geometry of a member's cross-section and the post-tensioning system used. The flow chart 4.10.2B-1 summarizes the stipulations

(1) Select the post-tensioning system used – unbonded or grouted.

(2) Unbonded systems: The minimum bonded reinforcement in one-way slabs and beams is a function of the cross-sectional geometry of the member. It is independent from the loading, the magnitude of service stresses, and span length. Unlike two-way systems reinforced with unbonded tendons, where it is permissible to design a floor system with no bottom rebar, ACI 318 requires bonded reinforcement over the supports and the bottom of all one-way slabs and beams reinforced with unbonded tendons, irrespective of the intensities of loads and stresses.

(3) Minimum non-prestressed reinforcement over the supports: The required area A_s is equal to:

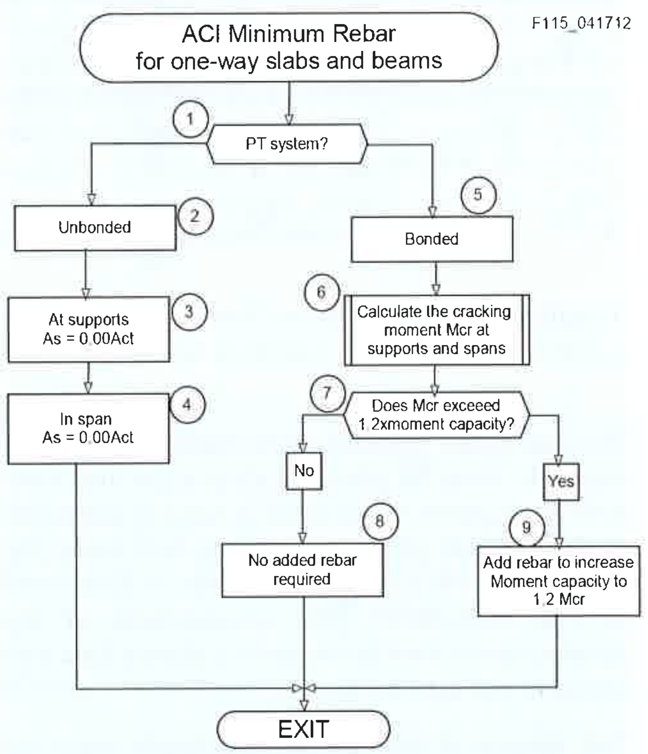
$$A_s = 0.004 A_{ct} \qquad \text{(Exp 4.10.2B-1)}$$

Where,

$_{ct}$ = area of the part of cross-section between the flexural tension face and the centroid of the cross-section.

The definition of area "A_{ct}" for the purpose of minimum rebar calculation is illustrated in Fig. 4.10.2B-1. Typically in the field of a span, the hatched area shown in the figure below the centroid of the section associates with the

FLOW CHART 4.10.2B-1 Bonded Reinforcement for One-Way Floor Systems and Beams (F115)



tension zone. Over the support region, it is the non-hatched area that is commonly subjected to tension.

The minimum reinforcement calculated shall be distributed uniformly over the width of the member, as close to the tension side as practical.

For beams, the reference area "A_{ct}" is defined in Fig. 4.10.2B-2. The width (b + $16t$) shown in part (a) of the figure is not stipulated in the code. It is an arbitrary upper bound for the computation of minimum rebar using the width of flange that is commonly considered in designs of conventionally reinforced beams based on simple beam theory. It is subject to engineering judgment.

(4) Minimum bonded reinforcement in span: The relationship used for supports applies also to spans, with the difference that in this case the tensile zone is likely to be at the bottom. Again, the computation of the location of the section's centroid is subject to engineering judgment. The practice is to use the same width of flange as used in the computation of rebar at support.

$$A_s = 0.004 A_{ct} \qquad \text{(Exp 4.10.2B-2)}$$

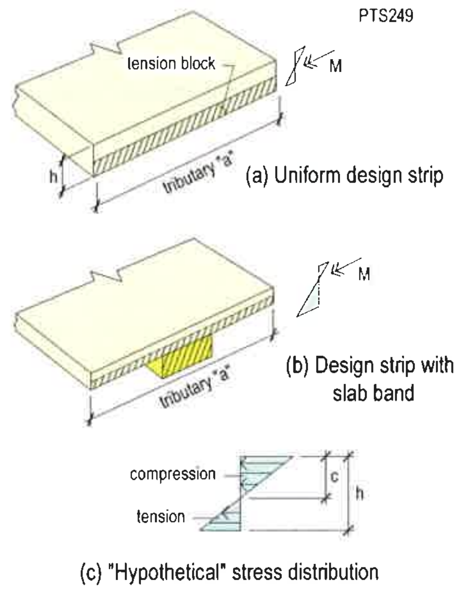


FIGURE 4.10.2B-1 Hypothetical Stress Distribution and Tension Block

(5) Bonded systems: The minimum non-prestressed bonded reinforcement in one-way slabs and beams post-tensioned with bonded systems is based on their nominal moment[38] capacity. It is the same provision that is required for two-way systems reinforced with bonded post-tensioning and outlined in Section 4.10.2A.

(6) Calculate the cracking moment at supports and in spans, using the modulus of rupture of the section's concrete. The computation is outlined in item 11 of the two-way systems. A numerical example is given in Chapter 6 and one in Chapter 7. The relationship from item 11 is repeated below for ease of reference.

$$M_{cr} = \left(f_r + \frac{P}{A} \right) S \qquad \text{(Exp 4.10.2A-9)}$$

(7) Calculate the nominal moment capacity (M_n) at each support and at spans: Compare the nominal moment capacity with 1.2 times the cracking moment (M_{cr}).

(8) Case of 1.2 times cracking moment not exceeding the nominal moment capacity: No added rebar is necessary.

$$M_n \geq 1.2 M_{cr} \qquad \text{(Exp 4.10.2B-4)}$$

(9) Add Reinforcement to increase the nominal moment capacity to a value not less than $1.2 M_{cr}$.

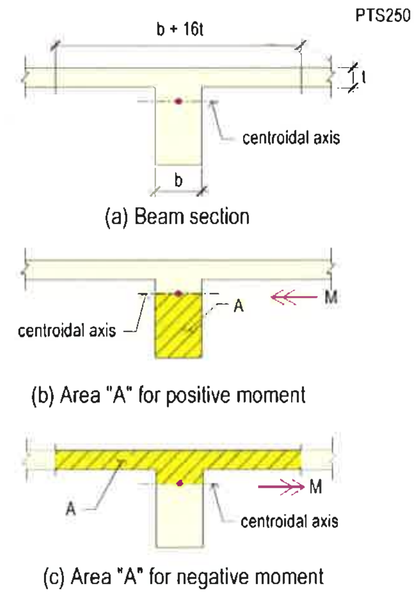


FIGURE 4.10.2B-2 Definition of Parameter "A" for the Minimum Rebar of One-Way Unbonded Systems

Length and Position of Minimum Reinforcement: The length of the minimum bonded reinforcement is the same as stipulated for two-way systems.

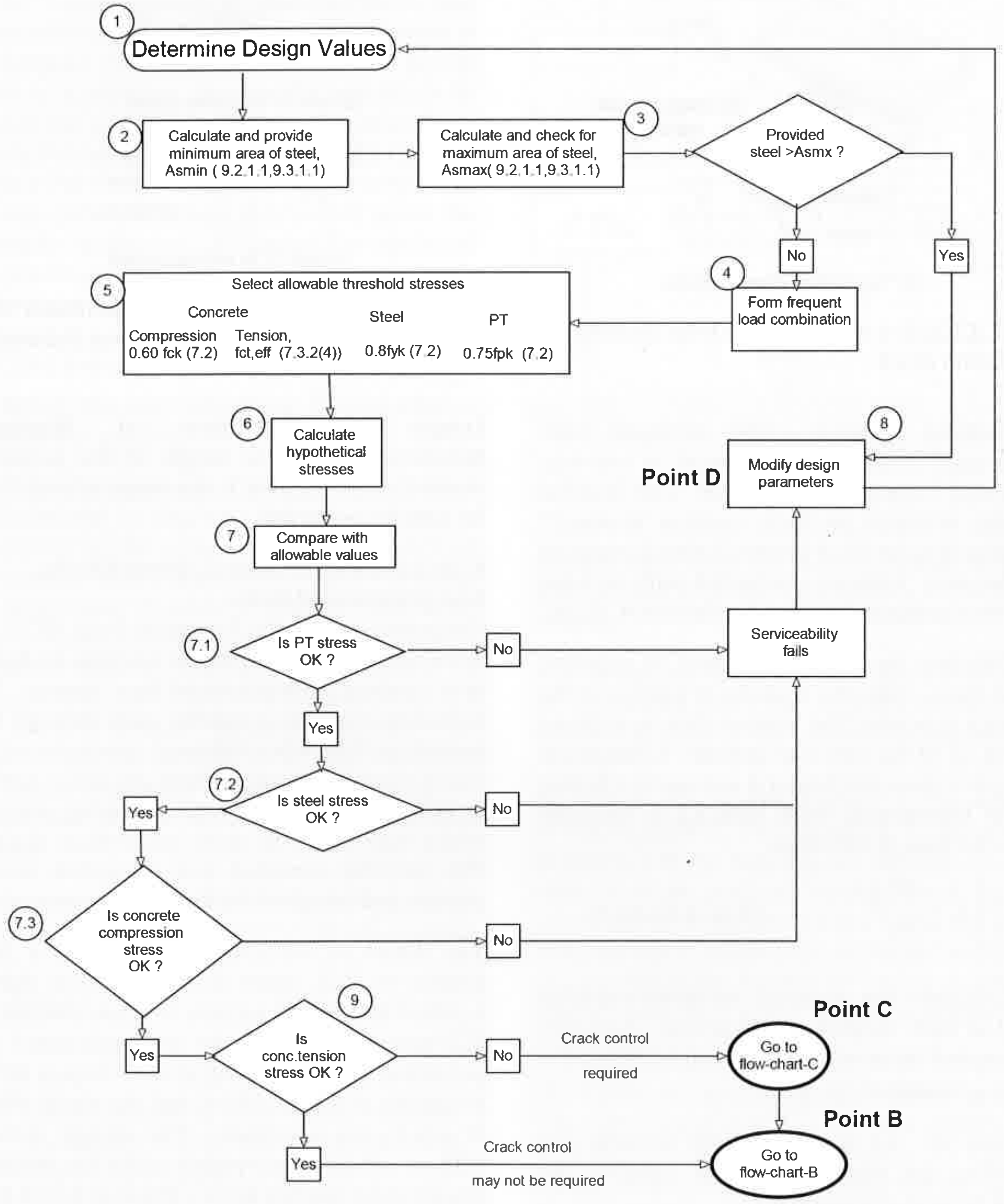
4.10.3 EC2 Crack Control; Stress Check; Non-prestressed Rebar

The provisions of the European Code EC2[39] are more complex than ACI 318 to navigate for design of a common post-tensioned floor system. The following presents a specific path through EC2 provisions that, when followed, concludes with a code-compliant design. There are other options in EC2 than included in the following, some of which may result in more economical designs. The material compiled and presented next is concise and complete for its objective, however.

The thrust of serviceability check of a floor system in EC2, apart from deflection that is handled in another section, is crack control. In EC2-based designs, cracks are anticipated and permitted to form, as long as their impact on the durability of the structure, and the visual effects of cracks are acceptable. The design initiates with an assumed acceptable width for probable cracks under service loads – this is referred to as the "design crack width." The values are generally

[38] ACI 318-11, Section 18.8.2
[39] EN 1992-1-1:2004(E)),Section 7

FLOW CHART 4.10.3-1 EC2 Serviceability Check for One-Way and Two-Way Floor Systems
Flow Chart A (F117)



between 0.1 to 0.3 mm (0.004 to 0.012 in.). It is also permissible to forego crack width control, if the impact of crack width is not critical for the specific application. The recommendations for crack width selection are in Section 7.3.1 of EC2. The serviceability check is performed for both sustained "quasi-permanent" and total "frequent" load combinations.

A detailed numerical design based on the application of EC2 to a column supported two-way system is given in Chapter 6. Chapter 7 offers a design example for a beam frame, using EC2.

Using EC2, the following flow charts outline the sequence of steps to follow for serviceability check of a post-tensioned floor system. The numerically marked paragraphs that follow the flow chart offer added explanations for the steps of the flow chart that bear the same number.

The relationships given in this Section for the serviceability check of EC2 use SI units (N, mm).

(1) Determine Design Values
At this stage, it is assumed that the analysis of the floor system, including its post-tensioning is complete; design strips and design sections have been identified; the cross-sectional geometry of each design section is known; and the design actions for each section are calculated. The design actions generally include moments, shears and axial loads. The serviceability check reviews the status of each design section and determines whether the width of the probable cracks are within the acceptable range, and whether additional reinforcement beyond what is available at the section is necessary.

(2) Minimum Overall Reinforcement
Each design section shall be checked to have a total amount of prestressed and non-prestressed reinforcement not less than a specified minimum amount, and not more than a specified maximum value. This step determines the minimum reinforcement associated with each design section. At this stage the layout and amount of prestressing at each design section are known. Also, a designer may have selected a certain amount of non-prestressed base-rebar, such as a top or bottom mesh, or a given amount of bars at specific locations. If the total amount of provided reinforcement is less than the minimum required, the shortfall is added. The minimum reinforcement depends both on the cross-sectional geometry of the design section and in the case of beams its moment capacity, if the beam is reinforced with unbonded tendons.

A. Based on Cross-Sectional Area: The minimum amount of reinforcement A_{smin}, is given in EC2 Section 9.2.1.1 for beams, and Section 9.3.1.1 for slabs. Both sections use the following relationship.

$$A_{smin} \geq \frac{0.26 b_t d f_{ctm}}{f_{yk}} \geq 0.0013 b_t d \quad \text{(Exp 4.10.3(2)-1)}$$

Where, in SI units

d = depth to the centroid of non-prestressed steel. The distance (d) refers to where non-prestressed steel is either located or would be positioned, where needed;

b_t = mean width of the tension zone;

f_{ctm} = mean axial tensile strength according to Table 3.1[40] of the code; and

f_{yk} = yield stress of non-prestressed reinforcement; f_{pk} is used in lieu of f_{yk}, where section is prestressed.

If both prestressed and non-prestressed steel are present in a design section, weighted average of their characteristic strengths is used. The intent of this provision is crack control arising from shrinkage and temperature changes.

The provided reinforcement to be compared with the minimum required is given by:

$$A_{sprov} = A_s + A_{ps} \times \frac{f_{pk}}{f_{yk}} \quad \text{(Exp 4.10.3(2)-2)}$$

The reason the area of prestressed steel (A_{ps}) is enhanced by the ratio given above is to indirectly recognize the precompression it provides. In theory, precompression alone could be adequate to mitigate shrinkage and temperature cracking. Otherwise, based on tensile stress that would develop due to shrinkage, the adjustment in area of prestressing steel would not have been justified.

[40] EN 1992-1-12004(E), Table 3.1

If the provided (A_{sprov}) reinforcement is less than the minimum determined (A_{smin}), increase the reinforcement to satisfy the minimum value.

B. Based on Moment Capacity: The total amount of prestressed and non-prestressed reinforcement at any section of a beam reinforced with unbonded tendons shall be adequate to develop a moment capacity at that section not less than 1.15 times the cracking moment computed for the same section[41].

> ***Comment:***
> *It is interesting to note that ACI 318 has a similar provision for the post-cracking strength of members reinforced with "grouted" tendons, whereas in EC2 the requirement applies to members reinforced with "unbonded" tendons.*

(3) Maximum Overall Reinforcement
Each design section is checked not to contain more than a maximum total reinforcement (A_{smax}), using EC2 code Section 9.2.1.1 for beams and 9.3.1.1 for slabs. Both sections use the following relationship.

$$A_{smax} = 0.04 A_c \qquad \text{(Exp 4.10.3(3)-1)}$$

Where, A_c is the gross cross-sectional area of the design section.

If both prestressed and nonprestressed steel are present, weighted average of their characteristic strengths is used.

$$A_{sprov} = A_s + A_{ps} \times \frac{f_{pk}}{f_{yk}} \qquad \text{(Exp 4.10.3(3)-2)}$$

If the provided steel (A_{sprov}) is more than the maximum allowable A_{smax}, the design has to be revised as indicated in item (8) of the flow chart.

(4) Frequent (Total) Load Combination[42]
Serviceability check is performed for both the frequent and quasi-permanent load combinations. We start with forming the frequent load combination by selecting the appropriate value of ψ, based on the occupancy of the floor system. See Section 4.10.1 for suggested values.

For residential and office buildings, the load combination is:

1.00 Selfweight + 1.00 DL + ψ LL + 1.00 PT

(5) Select the threshold stress values associated with "frequent" load combination for the materials used. These are:

- Concrete
  - Maximum allowable compressive fiber stress[43][0.60 f_{ck}].
  - Maximum allowable hypothetical tensile stress in concrete ($f_{ct,eff}$)[44] Where calculated values exceed this threshold, crack control reinforcement may have to be added as noted farther in the flow chart.
- Nonprestressed reinforcement
  - The maximum allowable stress is (0.80 f_{yk})[45].
- Prestressing steel
  - The maximum allowable stress under service condition is (0.75 f_{pk})[16].

(6) Calculate the Hypothetical Extreme Fiber Stresses
In slab construction the local stress at the surface of concrete varies significantly from one point to the next. Stresses are high over the supports and drop rapidly with distance away from the support. In practice, rather than focusing on or calculating stresses at a point, "hypothetical" stresses associated with selected "design sections" are calculated and used as values for crack control. Computed stress at a "point" bears no design significance for in-service response of a floor slab, since there is a poor correlation between computed local stresses and the actual values in prototype construction. Hypothetical stresses based on design sections of finite length are used to evaluate the likelihood of crack formation and their overall control. The selection of design sections and the computation of hypothetical stresses are outlined in Chapter 4, Sections 4.10.2A and 4.10.4A.

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41 EN 1992-1-12004(E) Section 9.2.1.1(4)
42 Design Aids for Eurocode 2, part 1 [ENV 1992-1-1], Section 4.1
43 EN 1992-1-1:2004(E), Section 7.2(2)
44 EN 1992-1-1:2004(E), Section 7.1(2) and 7.3.2(4)
45 EN 1992-1-1:2004(E), Section 7.2(5)
46 EN 1992-1-1:2004(E), Section 7.2(5)

(7) Compare Computed and Allowable Stress Values

(7.1) Stress in prestressing tendons:
This is to control the tendon stress in service not exceed the allowable value in (5). Through friction loss and long-term stress losses, this provision is generally satisfied, since tendon stress in service is generally about *0.60* f_{pk}. The provision is noted here for completeness, but it does not generally apply. It is automatically satisfied in common construction.

(7.2) Stress in non-prestressed reinforcement:
This is a requirement, but in practice it is not generally carried out, since in common construction, the stress in non-prestressed reinforcement is generally much less than the code threshold. The provision is included for completeness, and unusual conditions.

(7.3) Concrete compression stress check:
If the maximum hypothetical compressive stress in concrete is more than the allowable, the design of the section has to be revised. Selection of a larger section, or possibly reduction of prestressing is two of the possible remedial choices.

(8) Modify Design
If the calculated stresses in either the prestressing steel or non-prestressed steel or compressive stresses in concrete exceed the respective allowable values, the original parameters of design have to be modified to lower the exceeding stresses. The structure has to be re-analyzed to determine the new design values, and stresses re-checked.

(9) Compare Hypothetical Concrete Tension Stress with Threshold Limit
The hypothetically calculated concrete extreme fiber tension stress of the design section is compared with the threshold value stated in step 5. The outcome determines whether the section has to be checked for crack control, and if so, what measures have to be followed.

If the hypothetical stresses do not exceed the stated threshold, no crack control reinforcement will be needed for the "frequent" serviceability limit condition. For this case, the serviceability check will be continued following Flow Chart B that covers the "quasi permanent" load combination.

If the hypothetical tensile stresses exceed the stated threshold, probability of crack formation exists. Non-prestressed bonded reinforcement should be added to control the formation and width of probable cracks. Go to Flow Chart C.

(10) Flow chart C deals with the computation of the minimum non-prestressed bonded reinforcement

(A_{scrack}) for crack control. Details are in the following steps.

(11) The minimum area of bonded non-prestressed reinforcement for crack control (A_{scrack}) depends on the post-tensioning system used. Select "bonded" or "unbonded" system. As it is reflected further on, more reinforcement may be necessary than the minimum $_{scrack}$

(12) Crack Control Reinforcement for Bonded Systems ($_{scrack}$).
Since the hypothetical tension stress at the farthest fiber exceeded a lower allowable threshold, control of potential cracks becomes necessary for the affected design sections.

The requirement of minimum reinforcement for crack control is met by providing bonded reinforcement in the tensile zone of the concrete section. The minimum amount of the bonded reinforcement necessary under this provision will be increased, if crack width controls that are in the other steps of the flow chart conclude with additional reinforcement.

The requirement will be first expressed in its simplified format applicable to floor systems. Then, it will be presented in the EC2 code format for the general application.

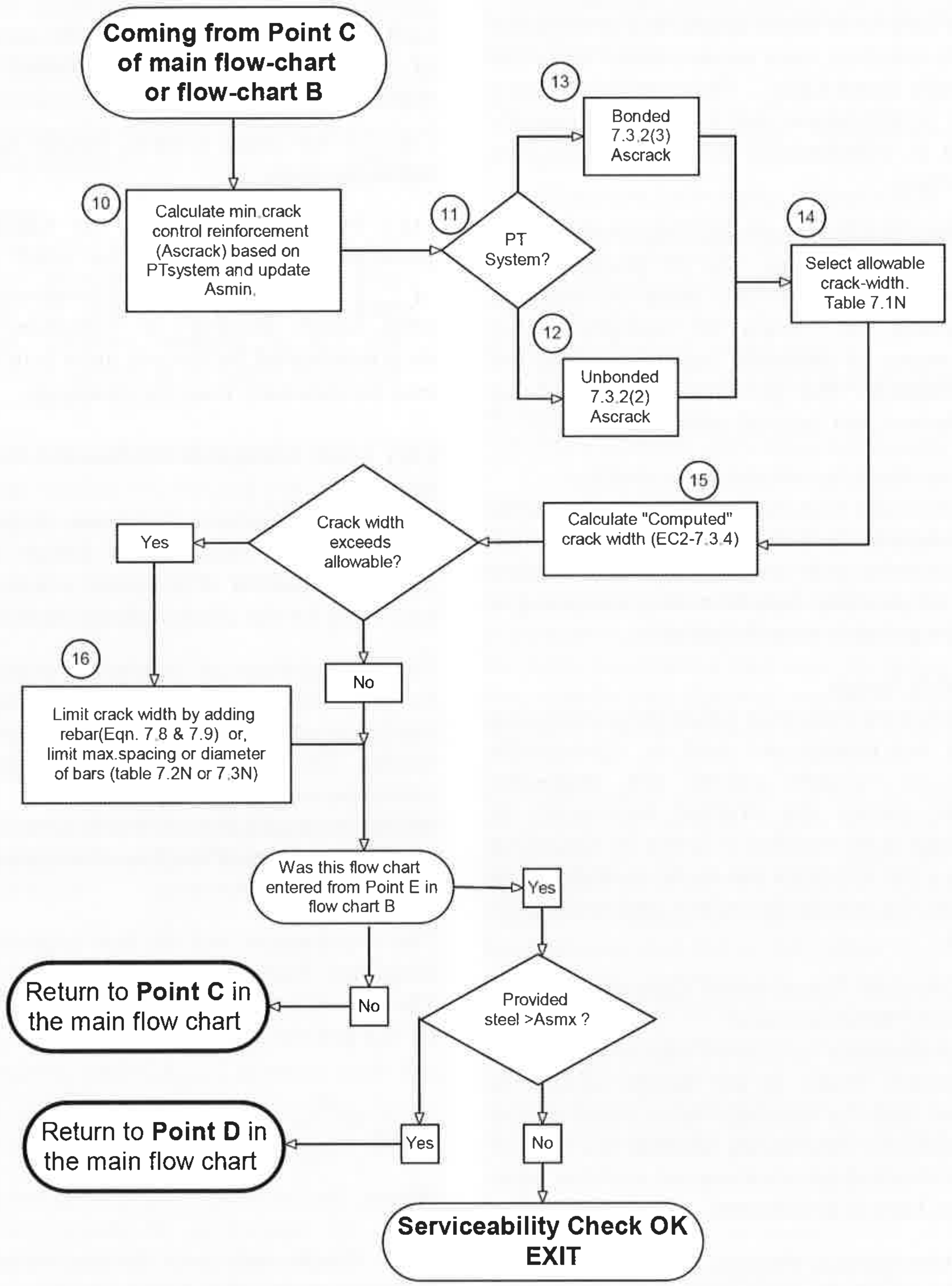
$$A_{scrack} = \frac{N_c}{f_{yk}} \qquad \text{(Exp 4.10.3(12)-1)}$$

Where,

N_c = Tensile force over the tension zone of the concrete section (Fig. 4.10.3-1); and

f_{yk} = yield stress of the reinforcement used for crack control.

FLOW CHART C (F118)



Note that f_{yk} is much higher than the value suggested for a similar condition in ACI 318. However, its recommendation in EC2 is justified, since the probability of crack formation, and crack width are addressed separately and accounted for – a step that is not detailed in ACI 318.

The format of the preceding expression, as given in EC2 is more involved, since it is intended to cover the general scenario. The express is[47]:

$$A_{scrack} = \frac{k_c \times k \times f_{ct,eff} \times A_{ct}}{\sigma_s} \quad \text{(Exp 4.10.3(12)-2)}$$

Where,

A_{ct} = area of concrete within tensile zone. The tensile zone is that part of the section which is calculated to be in tension just before formation of the first crack;

k = 1.0, for webs with h<= 300 mm or flanges with widths less than 300 mm;
= 0.65, for webs with h>= 800 mm or flanges with widths greater than 800 mm;
for other members, linear interpolation is used.

k_c = determined based on the maximum fiber stresses as follows:

- For pure tension k_c = 1.0
- For bending or bending combined with axial forces:

$$k_c = 0.4\left[1 - \frac{\sigma_c}{k_1 (h/h^*) f_{ct,eff}}\right] \le 1 \quad \text{(Exp 4.10.3(12)-3)}$$

For flanges of T-sections:

$$k_c = 0.9 \frac{F_{cr}}{A_{ct} \times f_{ct,eff}} \ge 0.5 \quad \text{(Exp 4.10.3(12)-4)}$$

Where,

$$\sigma_c = \frac{N_{ED}}{bh}; \text{ average precompression} \quad \text{(Exp 4.10.3(12)-5)}$$

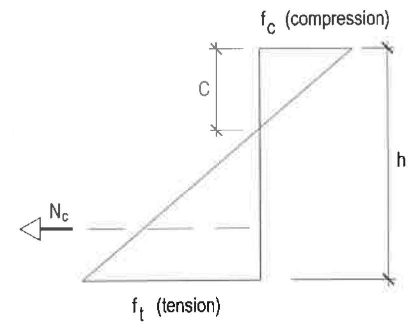


FIGURE 4.10.3-1 Computed Distribution of Stress over Concrete Section

N_{ED} = axial force at the serviceability limit state;
h^* = h for h <1.0 m
= 1.0m for h ≥1.0 m
k_1 = 1.5 if N_{ED} is a compressive force
$= \frac{2h^*}{3h}$ if N_{ED} is a tensile force

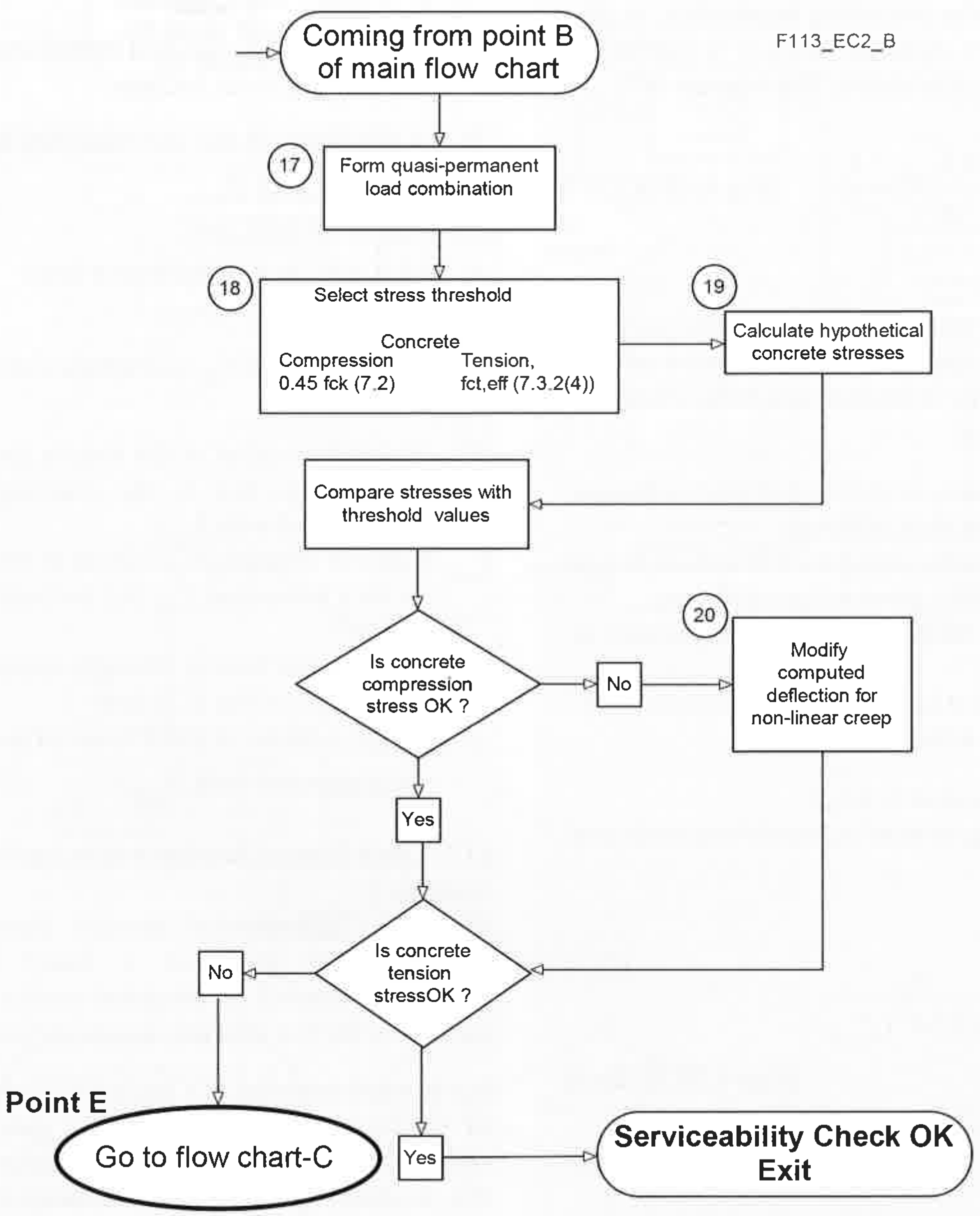
F_{cr} = absolute value of the tensile force within the flange due to the cracking moment calculated with $f_{ct,eff}$;
$f_{ct,eff}$ = tensile strength of concrete at time of crack formation, f_{ctm}, but not less than 3 MPa[48];
f_{ctm} = mean axial tensile strength according to Table 3.1 of the EC2; and
σ_s = may be taken as yield stress of non-prestressed steel, f_{cyk}.

(13) Crack Control Reinforcement for Bonded Systems ($_{scrack}$).
Since the hypothetical tension stress at the farthest fiber exceeded a lower allowable threshold, control of potential cracks becomes necessary for the affected design sections.

For bonded systems, the required minimum area of reinforcement is the same as described for the unbonded systems, with the difference that the available area of grouted tendons will be accounted for their contribution to crack control. In this respect, EC2 specifies a maximum spacing (300 mm[49]) between the reinforcement that is considered to be effective for crack control. For column-supported slab construction, this

[47] EN 1992-1-1:2004(E), Section 7.3.2(2)
[48] Design Aids for Eurocode 2, Part 1[ENV 1992-1.1], page 76
[49] EC2[EN 1992-1-1:2004(E)], Section 7.3.2(3)

FLOW CHART B



is viewed to serve the same objective as in ACI 318 provision for the spacing over the support intended for crack control and ductility - ACI 318[50] specifies not more than 12 " (305 mm). For the common case of column supported floor slabs, the EC2 provision translates to the following:

$$A_{scrack} = \frac{N_c}{f_{yk}} - A_{ps} \qquad \text{(Exp 4.10.3(13)-1)}$$

Where,

A_{ps} = Cross-sectional area of bonded tendons deemed to be available at location of probable crack formation. For column supported floor construction this is considered to be the width of column support extended on each side by 1.5 times the member thickness.

As it is outlined below in the detailed expression, EC2 has a restriction on the fraction of the cross-sectional area of the bonded tendons that can be included in the preceding relationship. However, the restriction does not apply to floor systems, on the premise that the effective stress in tendons under service is generally between 60 to 65 % of tendon's ultimate strength, or less. And, the difference between the effective stress of tendon in service, and tendon's ultimate strength is in excess of the yield stress of non-prestressed reinforcement used for crack control. In other words, stress in a tendon in service can be raised by the amount of the yield strength of an adjacent non-prestressed reinforcement, without failing the tendon.

For bonded systems, the minimum area of reinforcement (A_{scrack}) required for crack control is given by the following[51] :

$$A_{scrack} = \frac{k_c k f_{ct,eff} A_{ct} - \xi_1 A_p' \Delta\sigma_p}{\sigma_s} \qquad \text{(Exp 4.10.3(13)-2)}$$

Where, the definitions of the new symbols are:

ξ_1 = the adjusted ratio of bond strength taking into account the different diameters of prestressing and reinforcing steel.

$$= \sqrt{\xi \frac{\phi_s}{\phi_p}} \qquad \text{(Exp 4.10.3(13)-3)}$$

ξ = ratio of bond strength of prestressing and steel according to Table 6.2[52] of EC2;
Φ_s = largest bar diameter of reinforcing steel;
Φ_p = equivalent diameter of tendon according to EC2 6.8.2;
= $1.6\sqrt{A_p}$ for bundles, where A_p is the area of prestressing tendon or tendons
= $1.75\ \Phi_{wire}$ for single 7 wire strands where Φ_{wire} is the wire diameter

If only prestressing steel is used to control cracking, $\xi_1 = \sqrt{\xi}$

A_p' = area of tendons within $A_{c,eff}$;
$A_{c,eff}$ = effective area of concrete in tension surrounding the reinforcement or prestressing tendons of depth $h_{c,ef}$;
$h_{c,ef}$ = lesser of $2.5(h\text{-}d)$, $(h\text{-}x)/3$ or $h/2$;
h = depth of the member; and
$\Delta\sigma_p$ = stress variation in prestressing tendons from the state of zero strain of the concrete at the same level.

(14) Select Allowable Crack Width
The allowable crack width for each floor system depends on the anticipated exposure of the floor to corrosive elements, or the aesthetic impact of probable cracks. The exposure classifications[53] and the recommended values are given in Table 7.1N of the EC2. The values range from 0.2 to 0.4 mm. For members reinforced with unbonded tendons, the most common selection is 0.3 mm. For members reinforced with bonded systems, the suggested value for most exposures is 0.2 mm.

(15) Calculate "Computed" Crack Width
Using the procedure described below[54], and the provision of bonded non-prestressed reinforcement from the previous steps, the serviceability check continues with the computation of the probable crack width (w_k)

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50 Give aci bare spacing
51 EC2[EN 1992-1-1:2004(E)], Section 7.3.2(3)
52 EN 1992-1-1:2004(E), Sections 7.3.2(3) and 6.8.2
53 EN 1992-1-1:2004(E), Section 4.2, Table 4.1
54 EN 1992-1-1:2004(E), Section 7.3.4.

for each design section. The calculation of the probable crack width (w_k) is explained below. A numerical example of it is given in Chapter 6.

Computed probable crack width,

$$w_k = s_{r,max}\left(\varepsilon_{sm} - \varepsilon_{cm}\right) \qquad \text{(Exp 4.10.3(15)-1)}$$

Where,

$s_{r,max}$ = maximum crack spacing;

ε_{sm} = mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening; and

ε_{cm} = mean strain in the concrete between cracks.

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}\left(1+\alpha_e \rho_{p,eff}\right)}{E_s} \geq 0.6\frac{\sigma_s}{E_s} \qquad \text{(Exp 4.10.3(15)-2)}$$

Where,

σ_s = the stress in the tension reinforcement calculated on the basis of a cracked section [f_{yk}];

α_e = E_s/E_{cm}

$$\rho_{p,eff} = \frac{\left(A_s + \xi_1^2 \times A_p'\right)}{A_{c,eff}} \qquad \text{(Exp4.10.3(15)-3)}$$

$A_{p'}$ = area of tendons within $A_{c,eff}$

$A_{c,eff}$ = effective area of concrete in tension surrounding the reinforcement or prestressing tendons of depth $h_{c,ef}$;

$h_{c,ef}$ = lesser of $2.5(h\text{-}d)$, $(h\text{-}x)/3$ or $h/2$;

h = depth of the member;

x = depth of neutral axis from the compression fiber;

$$\xi_1 = \sqrt{\xi \frac{\phi_s}{\phi_p}} \qquad \text{(Exp 4.10.3(12)-5)}$$

ξ = ratio of bond strength of prestressing and steel according to Table 6.2 of EC2;

Φ_s = largest bar diameter of reinforcing steel;

Φ_p = equivalent diameter of tendon according to 6.8.2 of EC2;

k_t = factor dependent on the duration of the load;

= 0.6 for short term loading

= 0.4 for long term loading

E_s = modulus of elasticity of steel; and

$s_{r,max} = 1.3(h\text{-}x)$ (Exp 4.10.3(15)-4)

(16) Limit Crack Width
If the computed probable crack width from the previous step exceeds the value selected for design, the code provides two remedial options. Either add reinforcement using the following relationship to limit the probable crack width (w_k), or select non-prestressed bar diameter and spacing according to Table 7.2 N or 7.3 N of EC2. Adding reinforcement will increase $\rho_{p,eff}$ in the following.

$$w_k = s_{r,max}\left(\frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{p,eff}}\left(1+\alpha_e \rho_{p,eff}\right)}{E_s}\right)$$

(Exp 4.10.3(16)-1)

Where,

$$\rho_{p,eff} = \frac{\left(A_s + \xi_1^2 \times A_p'\right)}{A_{c,eff}} \qquad \text{(Exp 4.10.3(15)-3)}$$

(17) Quasi-Permanent (sustained) Load Combination[55]
The general load combination is:

1.00 Selfweight + 1.00 DL + ψ LL + 1.00 PT

The recommended values for ψ are given in Section 4.10.1. For common residential and commercial buildings, the recommended value is 0.3.

(18) Select Stress Thresholds
Stress limitations used for the "quasi-permanent" load combination are as follows:

[55] Design Aids for Eurocode 2, part 1 [ENV 1992-1-1], Section 4.1

- Concrete
  - Maximum threshold for compressive stress is $(0.45f_{ck})$[56].
  - Maximum threshold for hypothetical tensile stress is $(f_{ct,eff})$[57].
- Nonprestressed reinforcement
  - No additional stress check is required.
- Prestressing steel
  - No additional stress check is required.

(19) Calculate the Hypothetical Concrete Stresses
Extreme fiber concrete stresses for each design section are calculated using the same procedure outlined in the main flow chart.

(20) Modify Calculated Deflection due to Creep
If the hypothetical farthest fiber compressive stress of concrete in a design section exceeds the threshold for this load combination, the structure is anticipated to undergo greater deformation due to increased creep in concrete. The code recommends using a non-linear procedure for the creep component of the deformation in the structure when calculating the design deflections.

4.10.4 TR43 Crack Control; Stress Check; Non-prestressed Rebar
TR43 [TR43, 2005] is a report generated by the Concrete Society in the UK[58]. It includes practical recommendations for design of post-tensioned floor systems in building construction. TR43 is not a building code. However, it is included herein on account of its usefulness to design engineers. TR43, similar to ACI 318 treats the serviceability checks of two-way slab systems and one-way slab/beams separately. The following, is an excerpt from TR43 that covers the serviceability design of two-way and one-way floor slabs and beams. The excerpt is complete in the sense that when followed, the design will meet the recommendations of TR43. The following is not intended to be a summary of the Report.

A. Two-Way Systems: Like ACI 318, TR43 follows the concept of design strips and design sections in addressing the stress check, and the minimum requirements for bonded non-prestressed reinforcement. The computation of the service stresses to be used for design follows the same procedure as described for ACI 318 in Chapter 4, Section 4.10.2. In this procedure, for each design section a single hypothetical stress is computed.

This stress, when tensile, is used as an index to predict the formation of probable cracks. The compressive value of the hypothetical stress relates to the extent of the anticipated creep, and its impact on the long-term deflections. The hypothetical stress of each design section is matched against an allowable value, in order to determine the compliance with the Report's recommendations, and the proper design action to follow.

(i) Computation of Hypothetical Stresses: TR43 offers two options for stress check at the face of a support. Recognizing that the distribution of stress peaks at column locations, in one option the Report recommends a design section narrower than the full tributary of a design strip. This is considered to better capture the peak in stress (Fig. 4.10.4A-1a). The hypothetical stress evaluated from this narrow section is compared with an allowable value intended for narrow design sections.

The second option considers a design section that extends through the full tributary of a design strip – part (b) of the figure. The hypothetical stress calculated for the full tributary would generally be lower than the former option. This stress calculated from the full tributary is compared with an allowable stress that is less than the recommended value for the narrow design section. The second option is referred to as full tributary option.

Either of the two options can be used for design. The two options are deemed to serve the same serviceability objective.

The safety check of the design strip follows the full tributary design sections, as illustrated in part (c) of the figure.

Two considerations favor the selection of the "full panel" for design.

First, in practical design, where imaginative architectural layouts may feature irregular design strips and loads, it is not always guaranteed

[56] EN 1992-1-1:2004(E), Section 7.2(3)
[57] EN 1992-1-1:2004(E), Sections 7.1(2) and 7.3.2(4)
[58] The Concrete Society, Post-Tensioned Concrete Floors: Design Handbook, No. 43; pp. 110; 2005

that the face-of-support will provide the largest hypothetical stress. In regular layouts too, where multiple design sections should be checked (part c of the figure) to account for openings, steps and other irregularities of a slab panel, it is not always apparent at what location away from the face of support the narrow strip should change to the full tributary, or how the width of the design section should transition from short to wide, since at mid-span the full tributary is recommended.

Second, since for strength check at the ultimate limit state (ULS) the design values used are those of the entire tributary (part c of the figure), it is expedient to opt for a design section that covers both cases of SLS and ULS – namely the full tributary. The option for selection of full tributary supersedes the question transition from one width to the next and treatment of irregularities in a panel.

The following outlines the procedure for serviceability compliance recommended in TR43 using the second option, namely the "full panel."

(ii) Flow Chart for Serviceability Check: The following flow chart covers the steps recommended in TR43 for serviceability check of

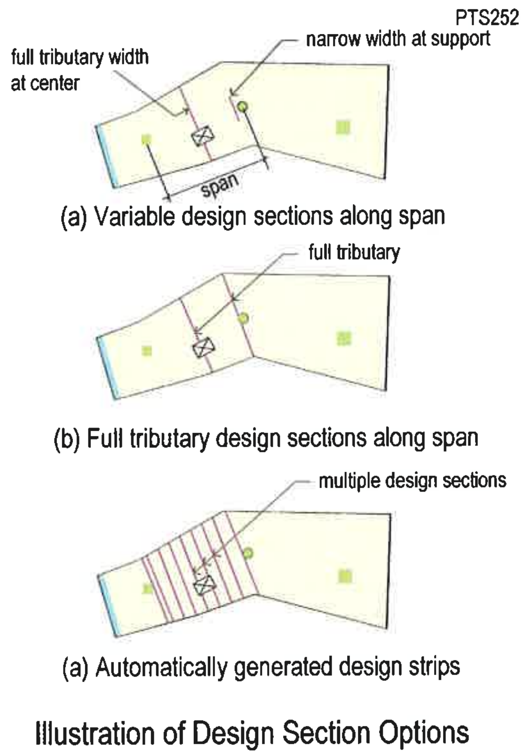


Figure 4.10.4A-1 Illustration of Options for Design Sections

two-way floor systems. The paragraph numbers of the descriptions that follow the flow chart refer to the numbered items in the flow chart.

(1) Select load combination
Serviceability check is performed for "frequent" load combination. The factors for the frequent load combinations are those recommended in the European code EC2 and discussed in Chapter 4, Section 4.10.3-4.

(2) Calculate the Hypothetical Tensile Stresses
For each design section, calculate the extreme fiber hypothetical tensile stresses as outlined in Chapter 4, Section 4.10.2

(3) Compare the Hypothetical Stresses with the Allowable Values
The allowable stress values for full tributary are given in table 4.10.4A-1.

In the Table, for the stress check, the "support" region is considered to extend from the face-of-support into span a distance equal to $0.2L_c$, where L_c is the clear span. The remainder of the unsupported span length ($0.6L_c$) is considered "span" for stress check.

The required amount of the bonded non-prestressed reinforcement associated with the higher value of allowable stress in the table is explained later in the flow chart.

(4) Check/Provide Minimum Nonprestressed Bonded Rebar over Support
This is based on geometry of the floor system[59]. The minimum area of non-prestressed reinforcement over each support, irrespective of the post-tensioning system used is given by:

$$A_{smin} = 0.00075 A_c \qquad \text{(Exp 4.10.4A(4)-1)}$$

Where A_c is the average of the cross-sectional areas of the design strips on the forward and rear for the interior columns, and the mid-span cross-sectional area of the design strip for end spans. The above provision applies to both bonded and unbonded post-tensioning systems.

[59] TR43- 5.8.8

FLOW CHART 4.10.4A-1 TR43 Serviceability Check for Two-Way Floor Systems (F120)

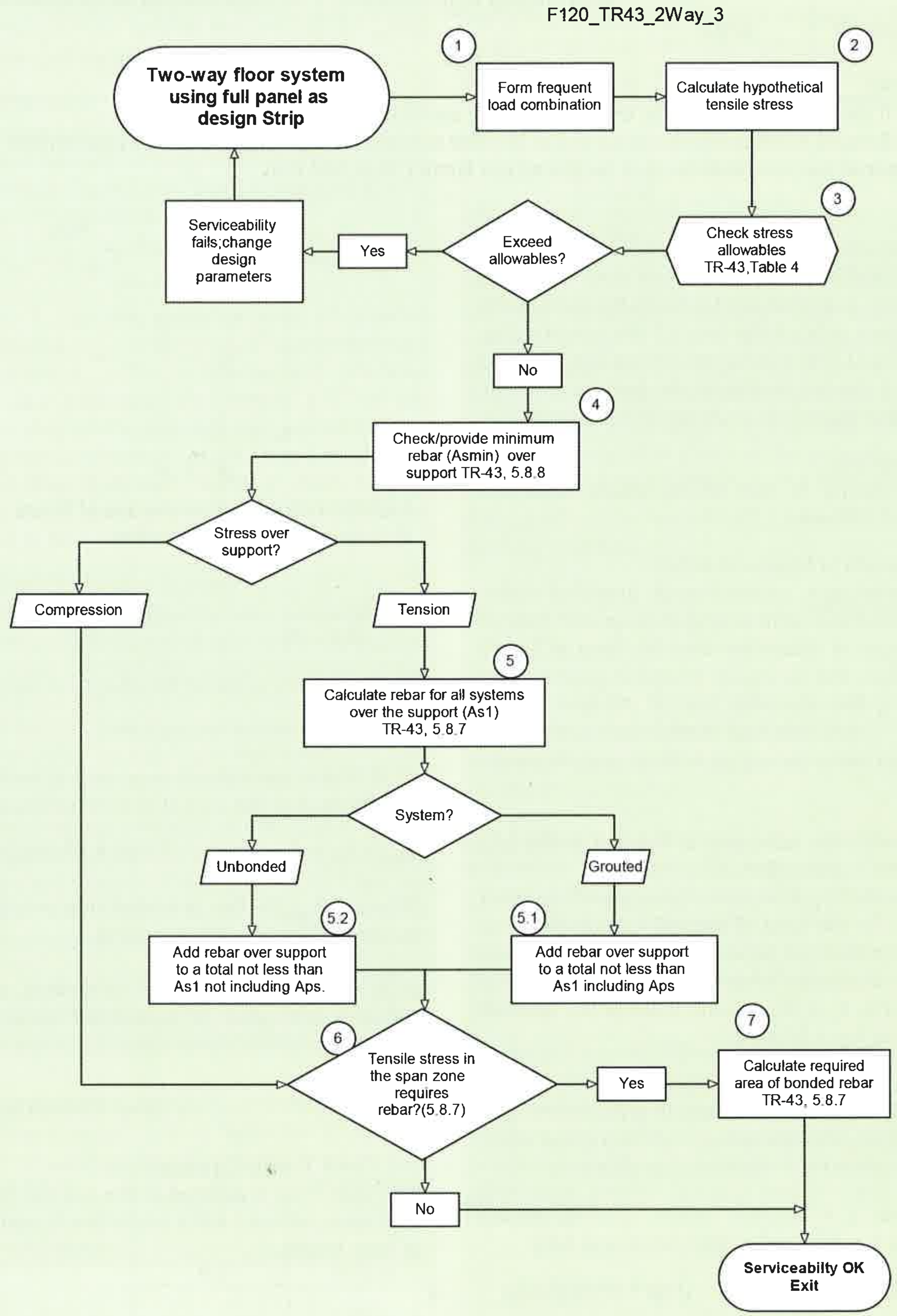


TABLE 4.10.4A-1 Allowable Hypothetical Extreme Fiber Stresses[60] (T115)

Location	In Compression	In Tension	
		With bonded reinforcement[2]	Without bonded reinforcement
Support	$0.3f_{ck}$[1]	$0.9f_{ctm}$	$0.3f_{ctm}$
Span	$0.4f_{ck}$[1]		

1 - If ductility check is made, this limit may be exceeded.
2 - Bonded reinforcement can be either bonded tendons, or non-prestressed reinforcement. Where required, bars or tendons shall be placed not farther than 500 mm.

❖ Placement of Minimum Rebar
The calculated minimum rebar shall be placed between lines that are 1.5 times the floor depth on either side of the face of the column (Fig. 4.10.2A-5). This reinforcement shall be placed as close to the top surface of the floor as practical, with due regard for cover and tendon location.

❖ Spacing
The spacing of this reinforcement shall not exceed 300 mm

❖ Length of Minimum Rebar
The minimum reinforcement provided under this provision shall extend *0.2L* on each side of the support centerline into the span of length *L*, unless due to highly irregular geometry or loading the top fiber tensile stresses exceed $0.3f_{ctm}$, in which case extend the bars beyond *0.2L* to cover the region with stresses in excess of $0.3f_{ctm}$[61].

(5) Check the Adequacy of Bonded Rebar (A_s) over the Support Based on Stresses
Where the hypothetical stress at top of a support is tensile, the total of bonded reinforcement at support shall not be less than A_{s1}[62]. The amount of this minimum rebar is based on the value of the force F_t in the tensile zone of the section. Refer to Fig. 4.10.4A-2.

For a simple design section with rectangular geometry, the relationship is given below. For design sections including slab bands and other irregularities the values will be different.

Legend: f_t = tension stress; f_c=compression stress; h = depth of section; x=neutral axis;

$$F_t = b \times f_t \times x / 2 \qquad \text{(Exp 4.10.4A(5)-1)}$$

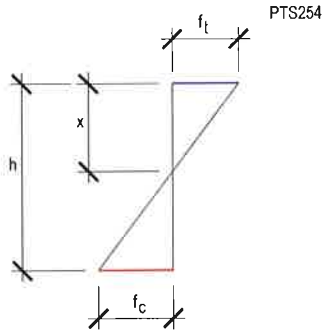


FIGURE 4.10.4A-2 Distribution of Stress through the Depth of a Section

$$A_s = \frac{F_t}{(5f_y/8)} \qquad \text{(Exp 4.10.4A(5)-2)}$$

Where, *b* is the width of the design section

Two conditions are encountered:

(5.1) Where bonded system is used, allow for the contribution of the area of bonded tendons;

$$A_{s1} = A_{ps} + A_{s,prov} \qquad \text{(Exp 4.10.4A(5.1)-1)}$$

Where, $A_{s,prov}$ is the provided non-prestressed bonded reinforcement, including A_{smin}

(5.2) Where unbonded system is used, do not consider the area of unbonded tendons to contribute:

$$A_{s1} = A_{sprov} \qquad \text{(Exp 4.10.4A(5.2)-1)}$$

(6) Check Tensile Stress in Span Zone
The span zone is defined as the central *0.6 L* of each span, where *L* is the centerline to centerline of span length.

[60] TR43-second edition, Table 4
[61] TR43- Second edition, Section 5.8.7
[62] TR43- Second Edition, Section 5.8.7

The necessity of placing reinforcement in span zone is based on the value of the hypothetical tensile stress at the farthest fiber. The threshold for members with bonded tendons is *1.2* f_{ctm} and for unbonded tendons *0.3* f_{ctm}[63].

(7) Calculate the Amount of Bonded Reinforcement[64]. If the threshold value is exceeded, bonded reinforcement shall be provided according to Fig. 4.10.4A-2 and its associated area of steel computations.

$$A_s = A_{ps} + A_{s,prov} \qquad \text{(Exp 4.10.4A (5.7)-1)}$$

Where, A_{ps} is the available area of bonded tendons and $A_{s,prov}$ is the area of non-prestressed reinforcement. The reinforcement provided under this provision shall extend 0.6*L* on the tension side of the member and extend beyond 0.6*L* by the development length of the bar selected.
B. One-Way Systems: The flow chart for the serviceability check of one-way slabs and beams is given in flow chart 4.10.1B-1:

(1) Calculate Tensile Stresses
The farthest fiber tensile stresses are calculated for each design section. In this case, it is assumed that the stresses are uniform across the width of the design strip, beam or slab. The load combination factors for frequent load combinations are those recommended in the European code EC2 and discussed in Chapter 4, Section 4.10.3-4.

(2) Check Stresses with Allowable Values
The allowable values for one-way slab and beam systems are given in Table 3 of TR43. They are reproduced in Table 4.10.4B-1. The values are different for bonded and unbonded systems:

(3) For unbonded systems there are up to three requirements for the minimum amount of non-prestressed tension reinforcement. These are listed in this step and the following steps. The first is based on the gross cross sectional area of the design section as given in EC2-9.2.1.1 and explained in detail in Chapter 4, Section 4.10.3, item 2 of the flow chart, and reproduced below for ease of reference. We refer to this as $A_{smin,EC2}$ to be compared with values to be calculated in other steps.

$$A_{smin,EC2} \geq \frac{0.26 b_t d f_{ctm}}{f_{yk}} \geq 0.0013 b_t d \qquad \text{(Exp4.10.4B(3)-1)}$$

(4) For sections reinforced with unbonded systems the area of non-prestressed reinforcement in the tensile zone shall not be less than the value associated with the tensile force of concrete in tension zone of the section[66] . This is explained in detail in step 5 of the preceding flow chart for two-way system as A_s. We refer to this as $A_{smin,tensile\ zone}$ to be compared with other values for final selection.

$$A_{smin,tensile\ zone} = \frac{F_t}{(5f_y/8)} \qquad \text{(Exp4.10.4A(5)-2)}$$

(5) Compare the two preceding minimum areas for non-prestressed reinforcement, and select the larger of $A_{smin,EC2}$ and $A_{smin,tensile\ zone}$ to be specified for the design section.

(6) Compare the extreme fiber tensile stress computed in step (1) with the threshold recommended in Table 3 of TR43 given in step (2) herein. If the computed stress does not exceed the threshold from Table 3, the section meets the serviceability requirements. Exit the computation (step 10)

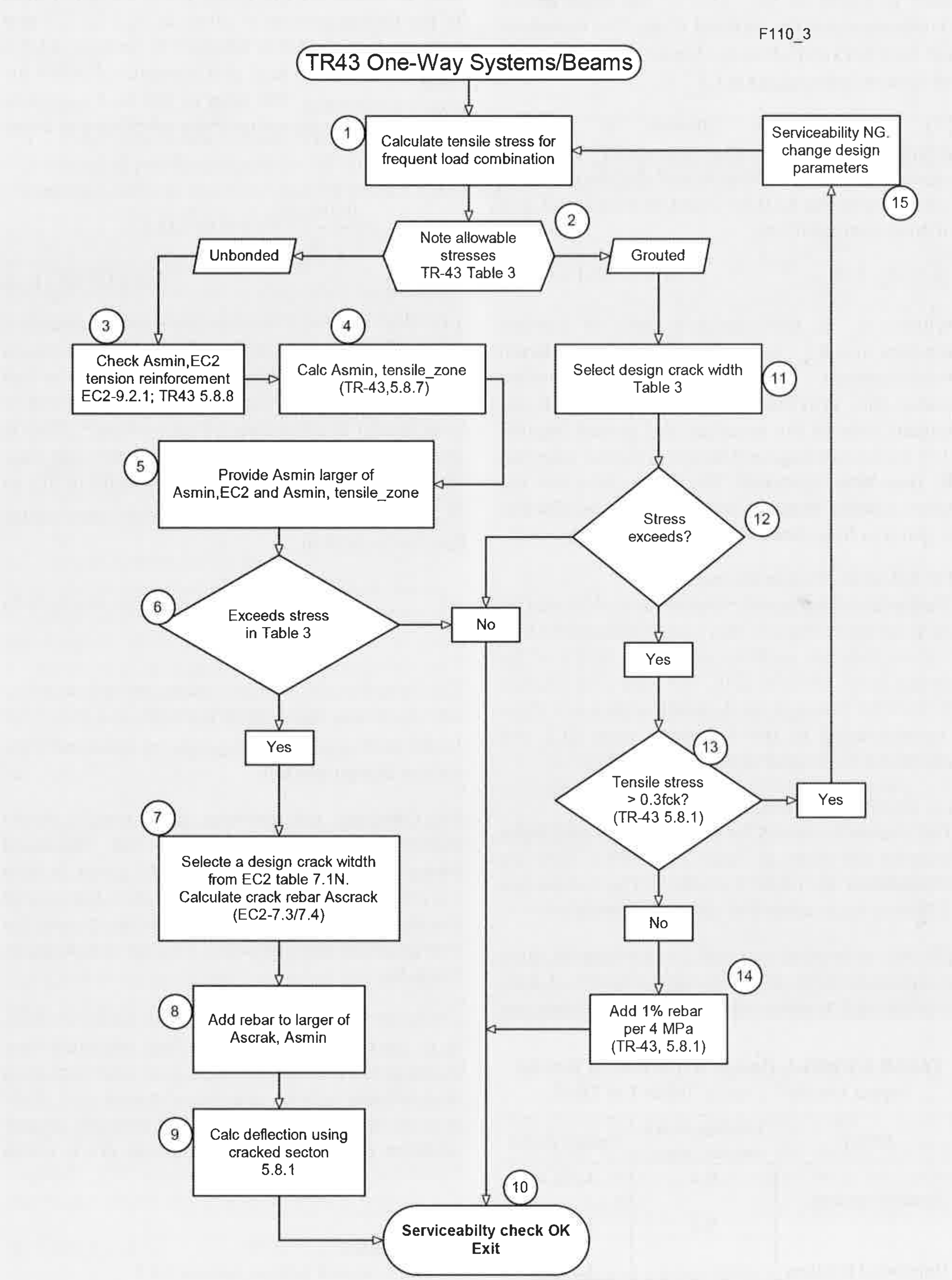
(7) For sections reinforced with unbonded tendons, it is permissible to exceed the recommended extreme fiber tensile stresses, provided formation of probable cracks are investigated and their computed widths are controlled through proper addition of rebar. Select a design crack width

TABLE 4.10.4B-1 Design Hypothetical Tensile Stress Limits[65] (T116). Table 3 of TR43

Group	Limiting crack width (mm)	Design stress
Bonded tendons	0.1	$1.35f_{ctm}$
	0.2	$1.65f_{ctm}$
Unbonded tendons	----------	$1.35f_{ctm}$

[63] TR43-5.8.7
[64] TR43- Second Edition, Section 5.8.7
[65] TR43, Table 3; section 5.8.1
[66] TR43-5.8.7.

FLOW CHART 4.10.4B-1 TR43 Serviceability Check Requirements for One-Way Slabs and Beams (F110)



and compute the probable crack width for the design section under consideration according to EC2 -7.3 and 7.4. For sections reinforced with unbonded tendons, EC2-7.3.1 recommends design crack widths equal to 0.3 and 0.4 mm, or larger, depending on the exposure of the member. Using the design crack width selected, calculate the required bonded reinforcement in the tensile zone using the same EC2 section. The procedure is detailed in steps 15 and 16 of the flow chart for EC2 (Chapter 4; Section 4.10.3). A numerical example is included in Chapter 6. We refer to the area of reinforcement needed for this provision as $A_{s,crack}$.

(8) Compare $A_{s,crack}$ calculated in the previous step with the reinforcement determined in step (5) above. Update the reinforcement in the section to the larger of the two.

(9) Since the section is deemed cracked when in service, calculate the service deflection of the member using cracked section properties. Obviously, this will take place after all design sections of the floor system, or the member under consideration are checked and the required reinforcement for all the sections is calculated and specified, before a cracked deflection analysis is performed.

(10) Satisfactory conclusion of the serviceability check for the current design section. Move to the next design section.

(11) For members reinforced with grouted tendons, select a design crack width from TR43 Table 3 (step 2 above) and note the associated recommended extreme fiber tensile stress.

(12) If the computed extreme fiber tensile stress does not exceed the selected value in step (11), the serviceability check is deemed satisfied. Exit to step (10) and move to the next design section. No additional non-prestressed reinforcement is required for members reinforced with grouted tendons.

(13) (14) For members reinforced with bonded post-tensioning systems, it is permissible to exceed the allowable stresses given in the preceding step up to $0.3f_{ck}$, by providing additional bonded reinforcement within the tension zone as close to the tension faces of the concrete as practical. The amount of the additional bonded reinforcement is proportional to the cross-sectional area of concrete in tension zone. For 1% of additional reinforcement, the allowable stresses may be increased by 4 MPa.

4.10.5 Significance of Allowable Stresses and Guidelines for Code Compliance

4.10.5.1 Background: Initiation of cracking and its control is achieved through "computed" tensile stresses at the extreme fiber of a member. For code compliance, the extreme fiber tensile stress is calculated using the gross cross-sectional parameters of the member, even when a section is deemed to have cracked. In common practice, the presence of reinforcement in sharing the actions on the concrete section, and thereby reducing the tensile stresses is generally not accounted for. Simply, the extreme fiber stress is computed, considering the entire cross-sectional area covered by concrete. The following relationship is used.

$$f = \left(\frac{P}{A} + \frac{Mc}{I} \right) \qquad \text{(Exp 4.10.5.1-1)}$$

Where,
f = computed extreme fiber stress;
P = axial force;
A = gross cross-sectional area of concrete section;
M = applied moment;
I = second moment of area; and
c = distance from the centroid of the section to the extreme tension fiber.

The computed stress neither represents the value at the tip of a crack, nor is it the average stress on the section. It is referred to as "hypothetical" stress, used as an indicator for expediency and the type of remedial measure, if any.

4.10.5.2 Extreme Fiber Stress and Cracking: For sections similar to those shown in Fig. 4.10.5.2 -1, cracking initiates when the extreme fiber stress (f_t) exceeds the cracking stress of concrete. Once cracked, the computed crack width is generally determined based on several parameters of the section, such as stress in reinforcement, distance of crack location to the next bar, bar diameter. The relationships commonly used are detailed in Section 4.10.3. A numerical example for crack-width computation is given in Chapter 7. The focus of this Section is the practical significance and application of the hypothetical tensile stresses. Figure 4.10.5.2-1 identifies two of the parameters, namely cover to the reinforcement, and distance of a point on the member soffit to the nearest reinforcing bar.

4.10.5.3 Significance of Computed Crack Width: The reliability of computed crack width depends on (i) the accuracy with which the tensile stress (f_t) at the "point" of interest is estimated, and (ii) among other parameters, the "distance" between the point selected for evaluation and the nearest reinforcement (Fig.4.10.5.2-1).

When dealing with a beam stem, as shown in Fig. 4.10.5.2-1c, the parameters for crack control can be predicted fairly well. The bending moment in

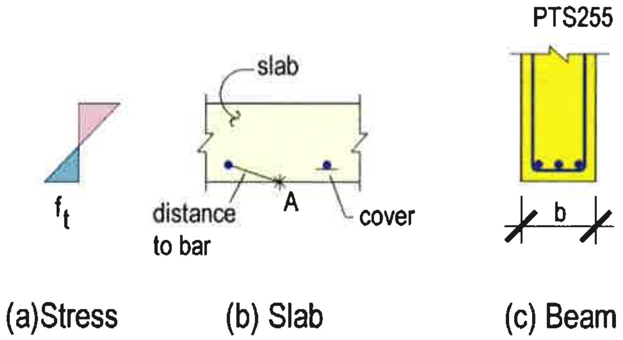


FIGURE 4.10.5.2 -1 Several of the Parameters that Affect Crack Width

a beam results in a fairly uniform tensile stress across the bottom fiber of the stem. Further, the known value of a beam's width, along with the number of bars in the beam stem, leads to a reasonably good estimate of the farthest distance of a point on the soffit to the nearest rebar. The same is not true for post-tensioned floor slabs. In common construction, neither the stress at a "point," nor the distance of a point on slab surface to the first reinforcement can be reliably predicted. The following explains.

A. Distribution of Stress in Slabs: Figure 4.10.5.3A-1 illustrates a panel from a column-supported floor system and the associated distribution of bending stress across a section at the face of a support. Observe that the stress varies significantly over the tributary of each column support. The degree of variation depends on the geometry of construction and the distribution of

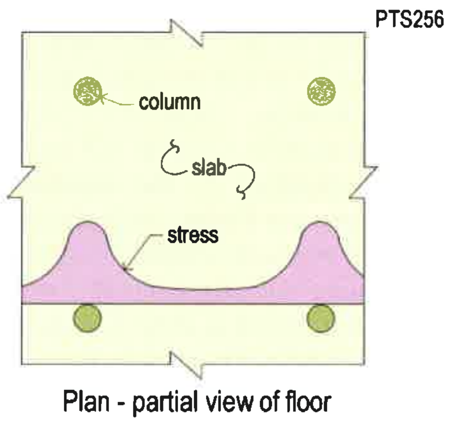


FIGURE 4.10.5.3A-1 General Distribution of Flexural Stress across a Section at Face of Support

loads.

Schematics of Fig. 4.10.5.3A-2 illustrate qualitatively the distribution of bending stress across a design section for different support widths. The figure includes both the variation of the actual stress and the associated hypothetical value for code compliance.

Note that in part (c) of the figure, where the support line extends over the entire width of

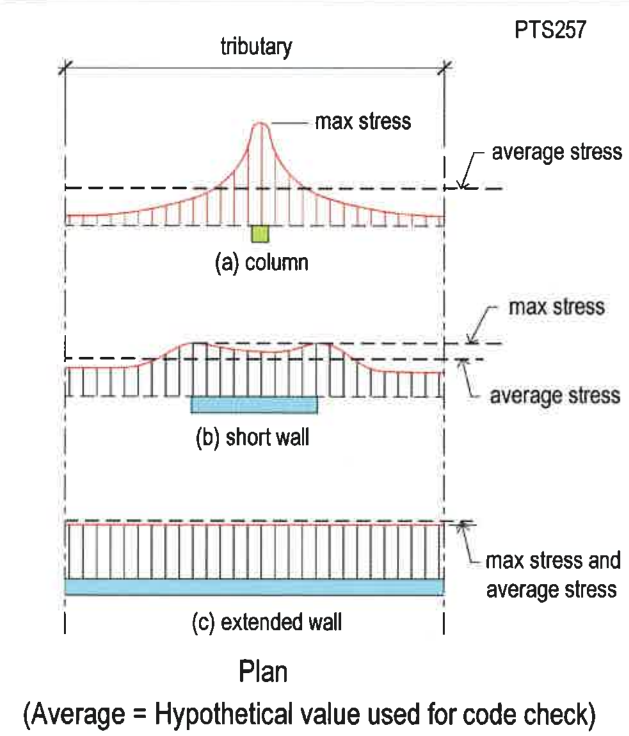
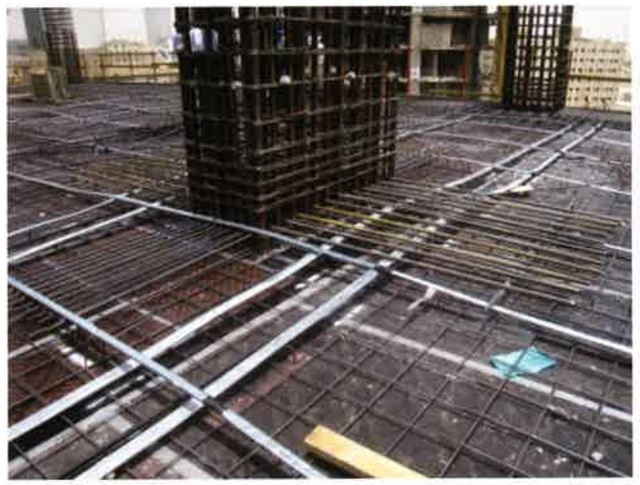


FIGURE 4.10.5.3A-2 Distribution of Maximum Bending Stress in a Design Strip

(a) Unbonded slab construction (P242)

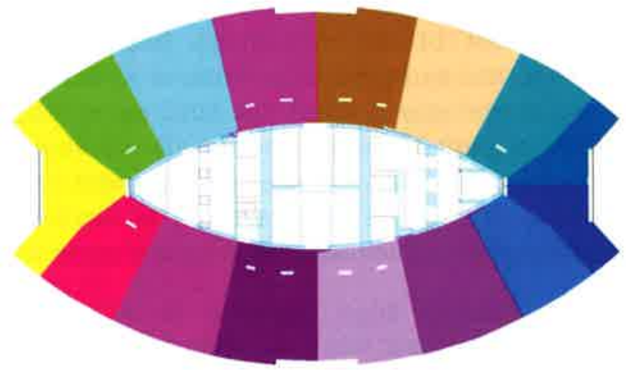


(b) Grouted slab construction (P243)

FIGURE 4.10.5.3B-1 Views of an Unbonded and Grouted Construction. Bars are not Uniformly Spaced

FIGURE 4.10.5.3B-2 Construction of a One-Way System, where Bars are More Uniformly Spaced (P244)

(a) Floor view (P245)



(b) Radial design strips (P246)

FIGURE 4.10.5.4 -1 Floor System of a Multi-Story Building and its Radial Design Strips (Burj Rafal KSA)

The distribution of extreme fiber stress for the selected radial design strip is shown in 4.10.5.4-2.

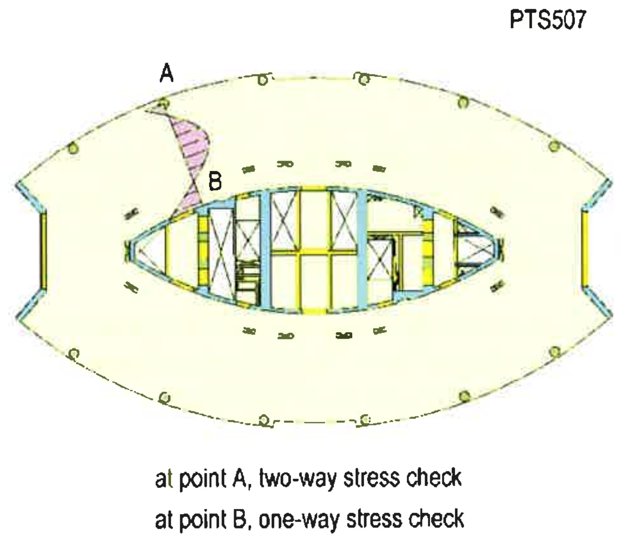


FIGURE 4.10.5.4-2 Distribution of Extreme Fiber Stress along a Typical Radial Design Strip of a Mixed Structural System (Rafal Tower, KSA)

the design strip, the distribution of stress at the face of support is uniform. For this condition, the hypothetical and the actual stresses closely agree. This reflects the scenario of a "one-way" system. On the contrary, where width of support is small (part a), the maximum stress occurs at a "point" and is much higher than the hypothetical value of the design section. For dimensions common in construction, the maximum stress can exceed more than 2.5 times the hypothetical stress calculated for code compliance. This scenario is representative of the "two-way" system of construction.

On account of large difference between the hypothetical and the actual value of stress at a point in two-way systems, the outcome of crack width computations for two-way floors should be evaluated with engineering judgment. The same is not true for one-way systems, since the hypothetical and actual stresses closely agree.

The preceding adds credence to ACI 318's recommendations for stipulating a higher allowable hypothetical tensile stress for one-way systems, coupled with provision to exceed the threshold, provided crack formation and its impact are recognized and allowed for.

B. Distribution of Reinforcement: In column-supported floor systems, the distance from a probable point of crack formation on a slab surface to the nearest reinforcement is highly unpredictable, since the reinforcement is often not uniformly distributed over a design strip. Figure 4.10.5.3B-1 illustrates construction views of two-way floor systems. The figures confirm that at design stage, it is not practical to arrive at a realistic value of the distance between the "point" of maximum stress, and its "distance" to the nearest bar.

For a one-way floor system, as illustrated in the one-way beam and slab construction of Fig. 4.10.5.3B-2, bars are more uniformly spaced.

4.10.5.4 Application of Allowable Stresses for Code Compliance: Figure 4.10.5.4-1 illustrates a typical upper level of a building tower featuring a core wall at center and column supports at perimeter. Design strips in the radial direction are shown in part (b) of the figure.

The distribution of extreme fiber stress for the selected radial design strip is shown in 4.10.5.4-2.

Referring to the diagram of stress distribution for different support widths (Fig. 4.10.5.3A-2) and the application of the allowable stress values; it is evident that at connection to the core wall (point B), the one-way allowable stress values apply; whereas over the column support (point A), the two-way values should be used. This is an instance where, within the same design strip, conditions of both one-way and two-way systems are present, for compliance with allowable stress values. Using ACI 318, there will be no upper limit for the value of computed stress at the core wall if deflections are calculated and evaluated using cracked sections. However, using the two-way criterion, there will be a maximum threshold value at the face of the column.

4.10.6 Deflections

Deflection control is a central consideration in serviceability of floor systems. This Section reviews the range of acceptable deflections and the currently available methods for their calculation. The Section concludes with a review of the merits and applicability of each method.

4.10.6.1 Overview: The primary reasons for deflection control are:

- Deflections should not be noticeable by occupants. Noticeable deflections can convey a sense of structural inadequacy, safety concerns or discomfort.
- Large deflections can damage non-structural elements.
- Large deflections can impair a floor's functions, such as drainage, and ponding.

4.10.6.2 Limits for Acceptable Deflection

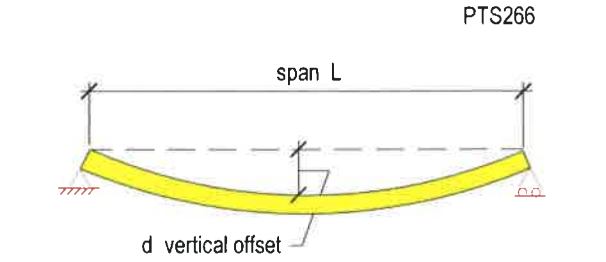
A. Deflection Index (d/L): With respect to allowable values, deflection of a member is expressed in terms of deflection "d" to span "L" ratio, as illustrated in Fig. 4.10.6.2A -1 for a simply supported member.

B. Measurement of Deflection: Using modern schemes of analysis and automated design, the deflection compliance of a floor system is based on "notional spans (L)," and the associated vertical offset (d) from a datum line. Figure 4.10.6.2B -1 illustrates the traditional definition of (L) and (d).

The traditional approach requires the recognition of "span" as is common in the structural engineering practice. The traditional identification of "span," however, is neither a straight forward operation in automated designs, such as those based on FEM, nor the best parameter for expressing the adverse effects of a floor's deformation. The impact of a floor's displacement on the elements that it supports is governed by the deformation contour of the floor surface, as opposed to distance between its supports below the floor. For this reason, in automated design a generalized definition as illustrated in Fig. 4.10.6.2B -2 is used.

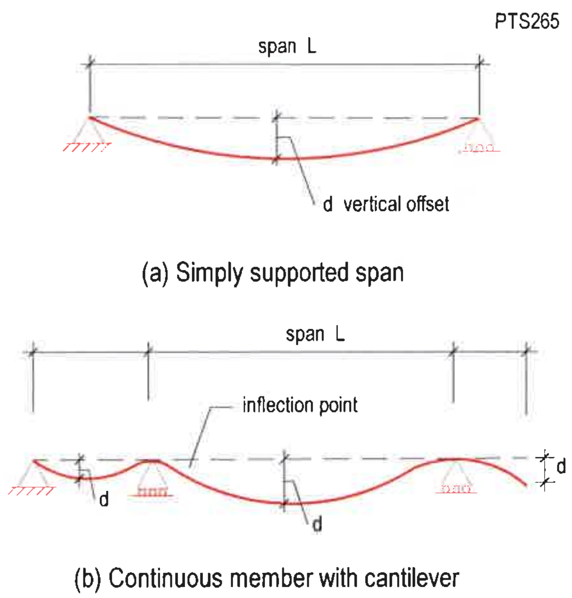
For the interior region of a floor system (Fig. 4.10.6.2B -2a), the notional span is defined as distance between two adjacent crests (points of zero slope) on the floor. The points do not necessarily coincide with the location of supports below the floor. For slab edges, the notional span is defined as the distance from a point on the slab edge to a crest at interior of the floor (Fig. 4.10.6.2B -2b).

As an example, consider the floor slab of Fig. 4.10.6.2B-3. The figure shows the deflection contour of the slab along with two cuts, each showing the vertical deflection from an interior column to the free edge. One cut shows the deflection profile to the closest point on the slab edge (point A), and the next shows the profile to the point of maximum deflection of overhang (point B). In this example, the deflection index (d/L) is larger for point A, indicating a more



Deflection Index Expressed in Terms of L/d

FIGURE 4.10.6.2A -1

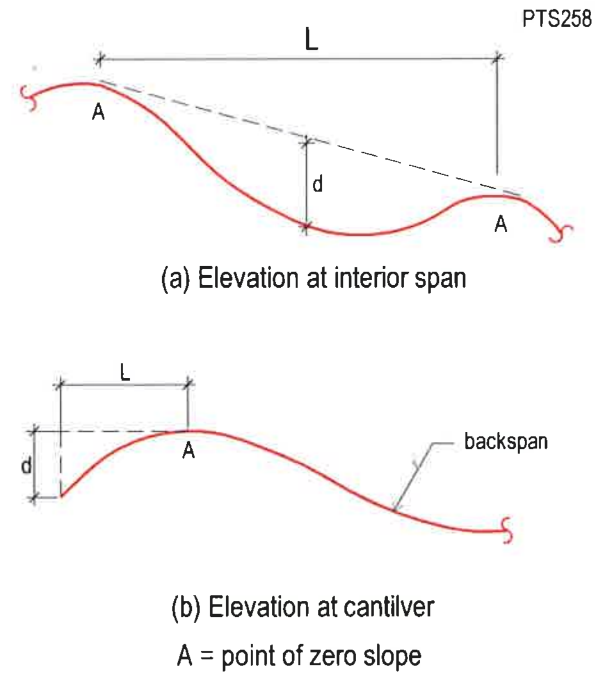


(a) Simply supported span

(b) Continuous member with cantilever

Traditional Parameters for Deflection Evaluation

FIGURE 4.10.6.2B -1



(a) Elevation at interior span

(b) Elevation at cantilver

A = point of zero slope

FIGURE 4.10.6.2B -2

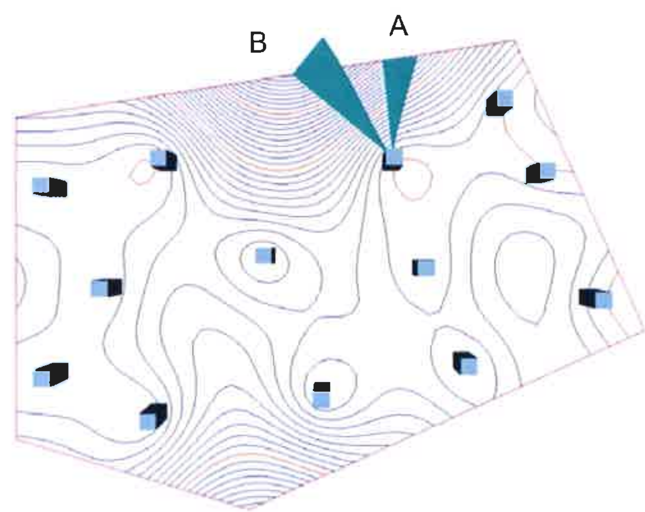
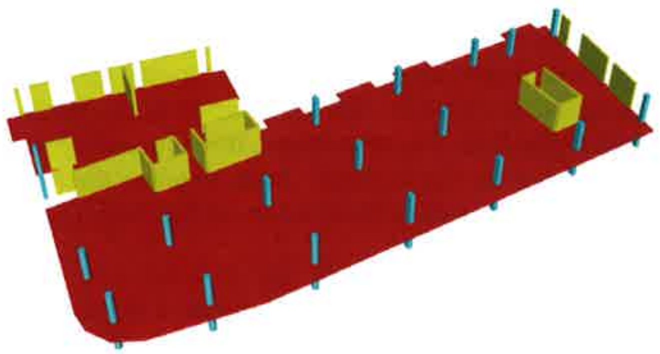
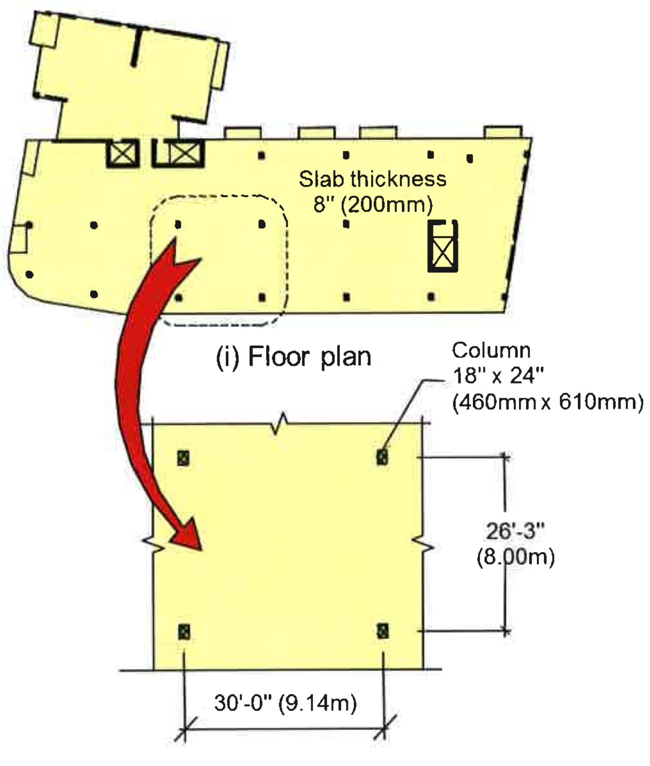


FIGURE 4.10.6.2B-3 Deflection Contour of a Slab with Two Cuts Showing Deflected Profile (P248)

FIGURE 4.10.6.3A-1 Noticeable Out-of-Level Displacement of a Floor (KSA; P249)



(a) 3D View of typical floor system (P250)



(b)

FIGURE 4.10.6.4-1. Typical Floor Highlighting the Span under Consideration (P565b)

adverse impact than the point with the maximum deflection. The objective of this illustration is to reiterate that the adversity of deflection is not necessarily related to its maximum absolute value.

4.10.6.3 Allowable Deflections: Allowable deflection depends on the purpose of its evaluation, and the stage at which deflection is measured. The following explains the common conditions.

A. Aesthetics and Sense of Comfort: For aesthetics and sense of comfort, the important criterion is the out-of-level condition of a finished floor, as opposed to its stiffness. Sensitive individuals, walking over, or viewing a floor in elevation, are claimed to perceive a floor's sag where the vertical out-of-level to span ratio is in excess of 1/250, and for cantilevers in excess of 1/125. The out-of-level condition of a floor system can be controlled through camber at time of construction. ACI 318-11[66] recommends 1/240 and for comfort. EC2[67] recommends 1/250. An instance of clearly noticeable deflection for the floor of the uppermost level of a building is shown in Fig. 4.10.6.3A-1.

B. Deflection Limits to Mitigate Damage to Non-structural Construction: Increase in displacement of a floor from the time a non-structural element likely to be damaged is installed shall not cause harm to the installed element. The recommended limits in the two major building codes are similar. ACI 318-11 recommends 1/480 for damage mitigation. EC2[68] recommends 1/500 for the same. Specifically, ACI 318-11 permits the recommended values to be exceeded, if for the specific condition the computed deflection is acceptable.

C. Deflection Limits to Mitigate Malfunction: The limit depends on the specific condition. For drainage, ponding, proper operation of doors or shelving 1/250 is generally assumed.

4.10.6.4 Methods of Deflection Calculation and their Comparison: Under otherwise unchanged conditions, the deformation of a loaded concrete member continues to increase, albeit at a reduced

[66] ACI 318-11 Table 9.5(b)
[67] EC2 (EN 1992 -1-1:2004), Section 7.4.1(4)
[68] EC2 (EN 1992 -1-1:2004)

TABLE 4.10.6.4 A-1 Deflection Coefficients k for Two-Way Slabs (PTS617)

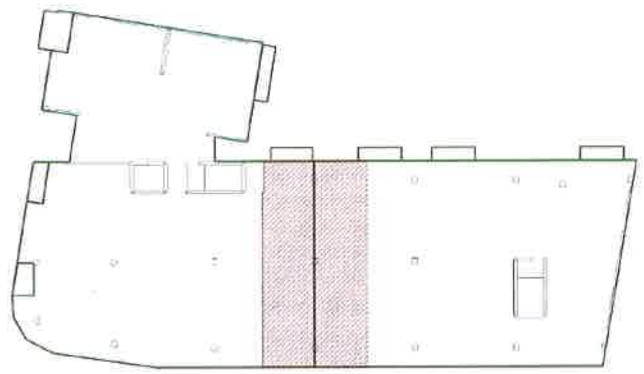
γ	1	2	3	4
1	0.0457	0.0143	0.0653	0.0491
1.1	0.0373	0.0116	0.0548	0.0446
1.2	0.0306	0.0094	0.0481	0.0422
1.3	0.0251	0.0075	0.0436	0.0403
1.4	0.0206	0.0061	0.0403	0.0387
1.5	0.0171	0.0049	0.0379	0.0369
2.0	0.0071	0.0018	0.0328	0.0326

rate with time. The increase is due to creep under applied load, and shrinkage from loss of moisture and chemical reactions. While rigorous methods of computation for time-dependent long-term deflections are available[69], the common engineering practice is to determine the instantaneous response of a structure under the applied load, and modify the instantaneous displacement to account for the time-dependent factors of creep and shrinkage. The following explains the common practice.

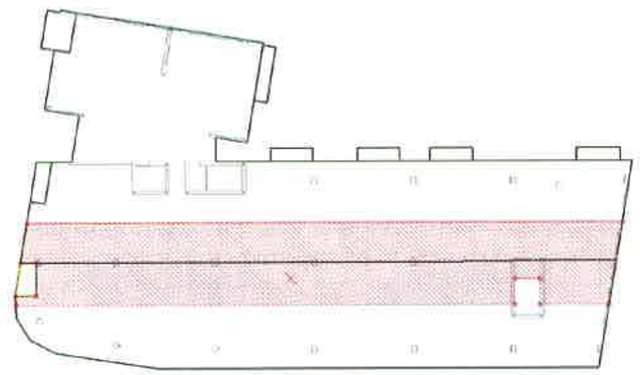
Instantaneous deflection is mostly calculated using concrete's modulus of elasticity at 28 days, linear elastic theory, and in most cases the gross-cross sectional area. Detailed calculations can account for cracking of member and loss of stiffness. The common methods are;

1. Closed form formulas or tables, available primarily for uncracked sections;
2. Strip method, using (i) linear elastic response; and (ii) no cracking;
3. Strip method, using (i) linear elastic response,(ii) allowance for cracking, and (iii) an average equivalent moment of inertia (I_e) due to cracking;
4. Strip method, using (i) linear elastic response; (ii) allowance for cracking, and (iii) local equivalent moment of inertia (I_e) combined with numerical integration;
5. Modeling of (i) the entire floor system, using a Finite Element Method (FEM) based program, (ii) linear elastic response; and (iii) no cracking; and
6. Modeling of the (i) entire floor system, using FEM; (ii) linear elastic response, and (iii) allowance for cracking.

The listed procedures are all in use and have been considered valid for estimating deflection values and establishing code compliance. For the same structure and conditions, the listed procedures report different results. Recent developments in computational technology tend to use FEM technology, with due allowance for cracking, where cracks are likely to develop. Reference [ADAPT-TN292, 2008] gives a detailed account of each of the above procedures, followed by the application of each method to the structure described below. This Section provides a short description of each method and the outcome of the numerical example to illustrate the range of values obtained from different methods for the same structure. The presentation concludes with a summary of the results from the different methods and their evaluation.



(a) Strip in up-down direction (P251)



(b) Strip in left-right direction (P252)

FIGURE 4.10.6.4B-1 Subdivision of the Floor into Design Strips

[69] Computer program ADAPT-ABI; www.adaptsoft.com

Consider the floor system shown in Fig. 4.10.6.4-1. Estimate the deflection of the panel identified in part (b) of the figure for the following condition.

Given:

Span length along X-X direction	=	30' (9.14 m)
Span length along Y-Y direction	=	26.25' (8.0 m)
Slab thickness	=	8 in (203 mm)
Column size	=	18x24" (460x610 mm)
Floor to floor height	=	10'-0" (3.05 m)
f'_c (28 day cylinder strength)	=	5000 psi (34.47 MPa)
W_c (unit weight)	=	150 pcf (2403 kg/m^3)
E_c (modulus of elasticity)	=	4.287×10^6 psi (29,558 MPa)
Non-prestressed reinforcement (f_y)	=	60 ksi (413.69 MPa)

Superimposed dead load = 25 psf (1.2 kN/m^2)
Including allowance for partitions
Live load = 40 psf (1.9 kN/m^2)

Required:
Immediate deflection at the middle of the panel identified in part (b) for the following load combination

1.00DL + 1.00LL

A. Method 1: Closed Form Formulas: Closed form formulas are readily available for beams and one-way slabs. The large number of variables that describe the geometry of a panel within a two-way floor system makes it impractical to compile a meaningful set of tables to cover them all. Some degree of approximation is inevitable. For non-cracked sections, compilations such as the one listed in Table 4.10.6.4 A-1 are available in the literature [Bares, 1971].

In the application of tables, engineering judgment is required for selection of support conditions.

Notes and Legend
Poisson's ratio assumed 0.25
$\gamma = a/b$ (aspect ratio)
Boundary conditions
1 = rigid supports; rotationally free;
2 = rigid supports; rotationally fixed;
3 = central panel from an array of identical panels supported on columns; deflection at center; and
4 = similar to case 3, but deflection at center of long span at support line

$$w = k\left(\frac{a^4 \times q}{E \times h^3}\right) \qquad \text{(Exp 4.10.6.4A-1)}$$

Where,

w = deflection normal to slab;
a = span along X-direction;
q = total service load;
E = Modulus of elasticity; and
h = slab thickness.

Method 1 Example

Required:
Immediate deflection of the panel shown in Fig. 4.10.6.4-1 for combined actions of dead and live load specified.
Aspect ratio $\gamma = 30/26.25 = 1.14$

Total service load,

$q = \left[(25+40)+(150\times 8/12)\right]/144$
$= 1.146$ lb/in^2 (7.9×10^{-3} N/mm^2)

From Table 4.10.6.4A-1

$\left[(a^4 \times q)/(E \times h^3)\right] =$

$\left[(30\times 12)^4 \times 1.146/(4.287\times 10^6 \times 8^3)\right]$

$= 8.77$ in (222.76 mm)

For mid-panel deflection, consider case 3 from Table 4.10.6.4A-1

k = 0.0548 (value for aspect ratio 1 used approximately)

Deflection, $w = k\left[(a^4 \times q)/(E \times h^3)\right]$

$= 0.0548 \times 8.77 = 0.48$ in (12.20 mm)
For deflection at midpoint of column lines in X-direction, from Table 4.10.6.4A-1
k = 0.0446

Deflection, $w = k\left[(a^4 \times q)/(E \times h^3)\right]$

$= 0.0446 \times 8.77 = 0.39$ in (9.91 mm)

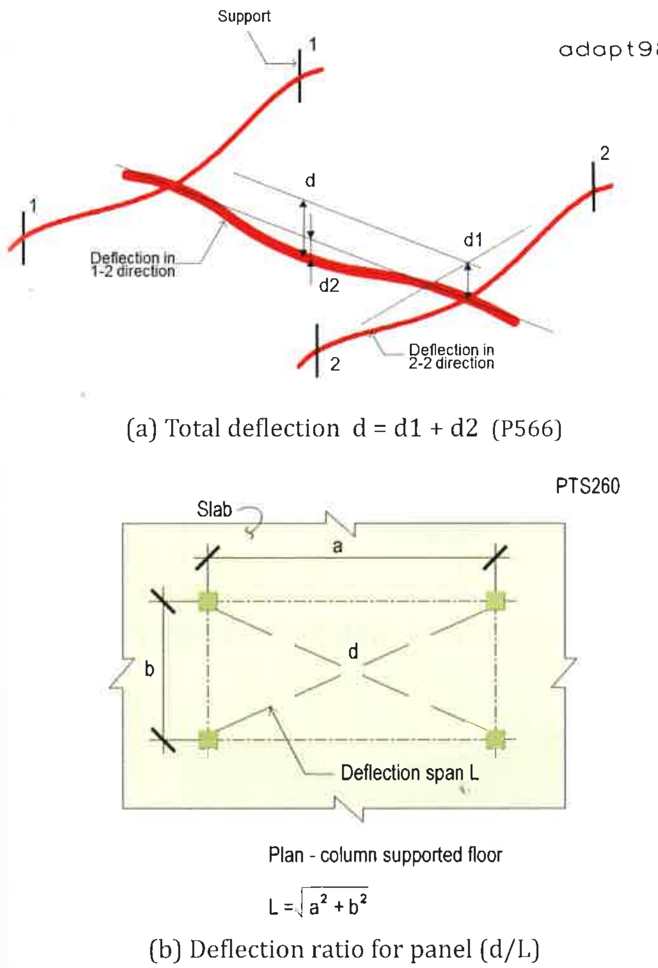
B. Method 2: Strip Method, (i) Linear Elastic, and (ii) No Cracking : This method has been widely in use, since the advent of personal computers, and the availability of software based

on "design strips." In this method the floor is subdivided into strips along the line of columns, each covering in width the tributary of a line of supports. Each strip is extracted and analyzed in isolation. Figure 4.10.6.4B-1 illustrates the strips that are applicable for the panel of this numerical example.

When calculating the deflections from the strips in orthogonal directions, the deflection (d) at center of a panel is taken to be the sum of the deflections obtained from each of the orthogonal strips (d_1, and d_2) (Fig. 4.10.6.4B-2a).

For the evaluation of displacement at center of panel (d), the notional span (L) being the diagonal length of the panel is used to form the (d/L) ratio (Fig. 4.10.6.4B-2b).

Using the parameters described in the preceding, along with a computer program[70] the strip in the up-down direction was analyzed. Its maximum



(a) Total deflection d = d1 + d2 (P566)

(b) Deflection ratio for panel (d/L)

FIGURE 4.10.6.4B-2 Deflections Determined from Sum of Values of Orthogonal Strips

linear elastic deflection, with no allowance for cracking, was determined as 0.231 in. (5.9 mm). A similar analysis must be performed for the strip in the orthogonal direction to determine the associated deflection. However, for expediency, and recognizing that the panel is almost square, the deflection at center of the panel is estimated to be twice the value calculated from one of the strips.

Deflection at center of panel = 0.231 * 2 = 0.462 in. (11.7 mm)

C. Method 3: Strip Method, (i) Linear Elastic; (ii) Allowance for Cracking, and (iii) Average Equivalent Moment of Inertia (I_e): This method, also referred to as simplified ACI318 method, is based on a reduced equivalent moment of inertia (I_e) to account for cracking. The following describes briefly the reduction in moment of inertia due to cracking, since the same concept is followed in FEM procedures.
Cracking forms, where the applied moment

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(M_a) at a section exceeds the cracking moment (M_{cr}) at the same section. The consequence of cracking is reduction in the second moment of

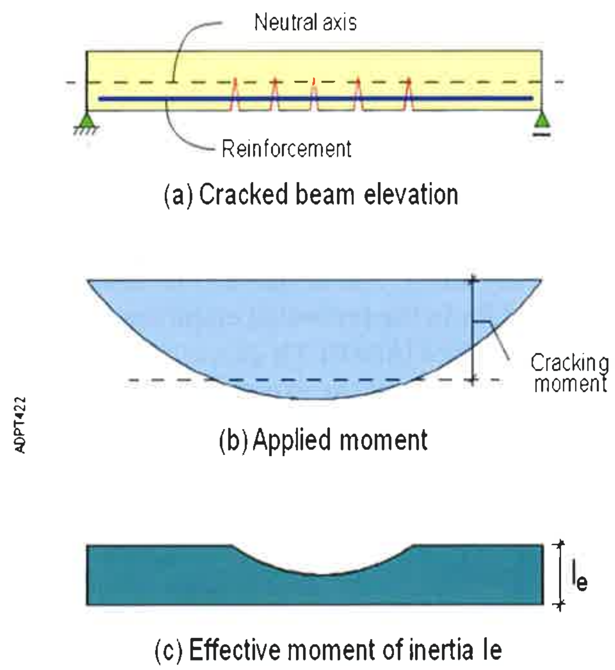


FIGURE 4.10.6.4C-1 Illustration of Effective Moment of Inertia in Partially Cracked Slab (P567)

[70] ADAPT-RC

area of the section at the location of crack. Figure 4.10.6.4C-1a illustrates the formation of crack in a member and the associated reduction in its second moment of area, referred to in common practice as "moment of inertia I." The reduced moment of inertia is called "equivalent moment of inertia, I_e."

In this method, the original moment of inertia of a member, referred to as "gross moment of inertia I_g" is substituted by a reduced value (I_e) to account for the loss of local stiffness. Among the available expressions put forward to estimate the reduced value of moment of inertia due to cracking, ACI 318-11[71] recommends the following:

$$I_e = (M_{cr} / M_a)^3 \times I_g + \left[1 - (M_{cr} / M_a)^3\right] \times I_{cr} \le I_g$$

(Exp 4.10.6.4C-1)

Where,

I_g = gross moment of inertia;

I_{cr} = moment of inertia of cracked section;

I_e = effective moment of inertia;

M_a = applied moment at location of crack; and

M_{cr} = cracking moment.

The applied moment, M_a, is calculated using elastic theory and the gross moment of inertia (I_g) for the uncracked section. The change in distribution of moment in indeterminate structures resulting from cracking in concrete is generally small, and is deemed to have been accounted for in the preceding empirical formula for I_e. Reference [ADAPT TN 293, 2008] provides a detailed account of the expressions for I_e along with numerical examples.

In the simplified method, an average value of I_e is used for the entire span. For each span, the average value is calculated.

$$I_{e,avg} = 0.5\left[\left(I_{e,left\ support} + I_{e,right\ support}\right)/2 + I_{e,midspan}\right]$$

(Exp 4.10.6.4C -2)

For cantilevers, the equivalent moment of inertia at the support is used for the entire span.

Using this method, the same design strips that were identified in preceding section are used. The stiffness of the strips is modified to account for the reduction of moment of inertia from cracking. The reduction was implemented in the computation by way of reducing the member thickness in the affected spans. The outcome of the analysis resulted in a deflection at the center of panel equal to 0.53" (13.4 mm).

D. Method 4: Strip method; (i) linear elastic; (ii) cracking; and (iii) numerical integration: This method provides increased accuracy by recognizing the change in stiffness along the length of a member, through numerical integration. The procedure is detailed in reference [ADAPT TN294, 2008]. In this method, each span, such as the example illustrated in Fig. 4.10.6.4D-1 is subdivided into a number of divisions. The moment (M_a) of each division is applied to the cross-sectional geometry and reinforcement of the same division to determine the associated

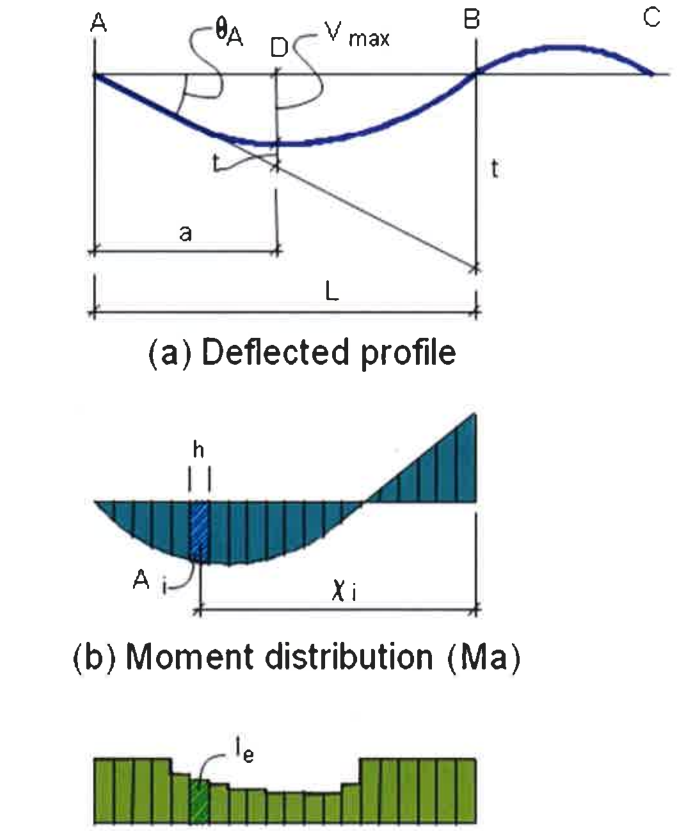


FIGURE 4.10.6.4D-1 Variable Moment of Inertia along a Cracked Member (P568)

[71] ACI 318-11 Section 9.5.2.3

reduction in moment of inertia (I_e). From this method, the total deflection is estimated as 0.47 in. (11.9 mm).

E. Method 5: (i) Entire Floor Modeling; (ii) FEM Analysis; (iii) Linear Elastic; and (iv) No Cracking: Using Finite Element Method (FEM), the salient features of the geometry and loading that are otherwise approximated in the aforementioned methods is faithfully modeled. The 3D modeling of the entire floor results in a more representative estimate of slab deflection. Figure 4.10.6.4E-1 illustrates the FEM discretization of the floor system used in the previous examples. The deflection values are shown in Fig. 4.10.6.4E-2. In this case, unlike the previous methods, where the deflection from design strip in one direction was added to that of the orthogonal direction, the deflection at the center of panel can be read directly from the results. The value obtained is 0.54 in (13.7 mm)[72].

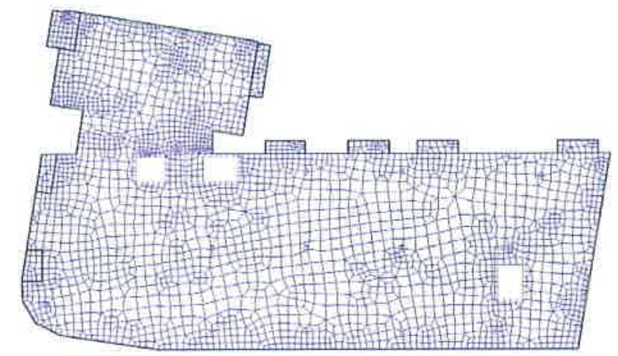


FIGURE 4.10.6.4E-1 Discretization of the Typical Floor Slab for Finite Element Analysis (Floor Pro) (P255)

FIGURE 4.10.6.4E-2 Deflection Contour of the Floor System under Combined Actions of Dead and Live Loads (P256a, P256b)

F. Method 6: (i) Entire Floor Modeling; (ii) FEM Analysis; (iii) Elastic Response; and (iv) Allowance for Cracking: Formulation of finite elements with allowance for cracking is somewhat complex. The complexity arises from the fact that cracking and reduction in stiffness depend on the presence, amount and orientation of reinforcement, including prestressing tendons, if any. Before a cracked solution is obtained, the detailing of reinforcement of a floor system must be fully known. The same is true, when using the previously explained methods for the computation of the equivalent moment of inertia, with the difference that in the previous methods, the reinforcement is assumed to be parallel to each design strip.

The following briefly describes the steps for a finite element deflection calculation, with allowance for cracking, featuring commercially available software[73].

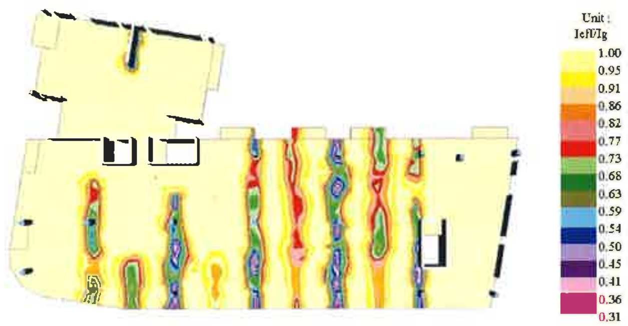


FIGURE 4.10.6.4F-1 Extent of Cracking Shown Through Reduction in Effective Moment of Inertia Ie About Y-Y Axis (P253)

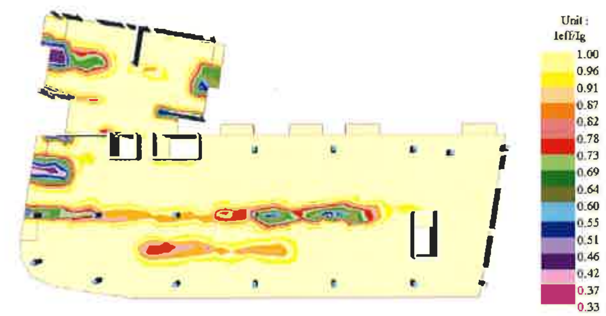


FIGURE 4.10.6.4F-2 Extent of Cracking Shown Through Reduction in Effective Moment of Inertia Ie About X-X Axis (P254)

[72] ADAPT-Floor Pro www.adaptsoft.com
[73] ADAPT-Floor Pro, www.adaptsoft.com

1. Obtain a solution based on gross moment of inertia (I_g), and determine the distribution of moments (M_a).

2. Design the entire structure and add reinforcement. At any location, the available reinforcement to be used for crack evaluation can consist of:

a - User defined top and bottom mesh reinforcement, if any;

b - User defined grouped or distributed reinforcement bars at top and bottom of columns, slab and beams;

c - Reinforcement calculated and reported by the program for minimum requirements of the code, strength check, initial condition, or other code related criteria; and

d - Post-tensioning tendons defined by the user, each with its own location and force.

3. Compare applied moment (M_a) of each finite element cell along with the cracking moment (M_{cr}) of the same cell. Where (M_a) exceeds (M_{cr}) cracking occurs. Determine local reduction in stiffness.

4. Re-generate the stiffness matrix of each cell with reduced stiffness.

5. Re-analyze. It is recommended to terminate the computations at this point after re-analysis.

Using the above procedure, the cracked deflection of the floor was calculated. The value at the center of the panel under consideration is 0.70 in. (17.8 mm) compared to 0.54 in. (13.7 mm) for the uncracked slab. The cracked deflection can be reduced by adding reinforcement at the locations of crack formation, in addition to the minimum requirements of the code already included in the analysis.

The locations of crack formation and the extent of cracking are illustrated in Figs. 4.10.6.4F-1 and 4.10.6.4F-2. At each location, the reduction in effective moment of inertia is based on the calculated moment at that location and the amount, position and orientation of reinforcement at the same location. The largest loss of stiffness occurs around the columns and the support lines joining the columns. The maximum loss of stiffness is 69% reducing the effective moment of inertia to 31% of its uncracked value.

4.10.6.5 Comparison of Methods of Deflection Calculation: Table 4.10.6.5 -1 lists the outcome of the various methods reviewed in the preceding for estimating the deflection at center of a specific panel. Note that for the typical floor system selected, the difference between the various methods can be as much as 30%. Finite Element Method with allowance for crack formation gives the largest deflection. The strip method with no allowance for cracking produces the smallest value.

TABLE **4.10.6.5**-1 Deflection Values at Center of Panel of the Numerical Example (T123)

	Calculation method	Deflection in(mm)	Normalized deflection
1	Closed form formulas	0.48 (12.2)	69 %
2	Strip method (uncracked) ACI 318	0.46 (11.7)	66 %
3	Strip method (cracked) ACI 318 – averaging *Ie*	0.53 (13.5)	75 %
4	Strip method (cracked) ACI 318 – numerical integration	0.47 (11.9)	67 %
5	Finite Element Method (FEM) No allowance for cracking	0.54 (13.7)	77 %
6	Finite Element Method (FEM) With allowance for cracking	0.70 (17.8)	100 %

4.10.6.6 Deflection of Post-Tensioned Floors: Two-way post-tensioned floor systems designed to ACI 318 provisions either do not crack under service condition, or crack to an extent that does not invalidate calculations based on gross cross-sectional geometry and linear elastic theory. Unlike EC2, the allowable tensile stresses in ACI318 are relatively low, resulting in minor cracking.

4.10.6.7 Live Load Deflection: For members that are not cracked, using linear elastic response, the principle of superposition applies. Hence, deflection to live load can simply be obtained by prorating deflections from similar loads, such as selfweight. However, as indicated in Fig. 4.10.6.7-1, for cracked members, the increment of deflection due to live load depends on the existing loads on the member and the extent of cracking.

The load combination application for live load deflection can be summarized as follows:

❖ Uncracked member

A single solution with the following load combination

1.0 × LL

❖ Cracked member

Two solutions are required, with the net live load deflection being the difference between the two, as indicated below

First solution indicated by U1 with load combination (DL + LL)

Second solution U2 using load combination (DL)

Live load deflection = U1 – U2

4.10.6.8 Long-Term Deflections: With lapse of time, concrete members continue to deform due to creep and shrinkage. Shrinkage is due to loss of moisture, and hydration. Creep is increase in displacement under stress. Under constant loading, such as selfweight, the effect of creep diminishes with time. Likewise, under normal conditions, with loss of moisture, the effect of deformation due to shrinkage diminishes. Restraint of supports to free shortening of a slab due to shrinkage or creep can lead to cracking of slabs and possibly an increase in deflection.

While the methodology and tools to determine the increase in instantaneous deflection of a

PTS263

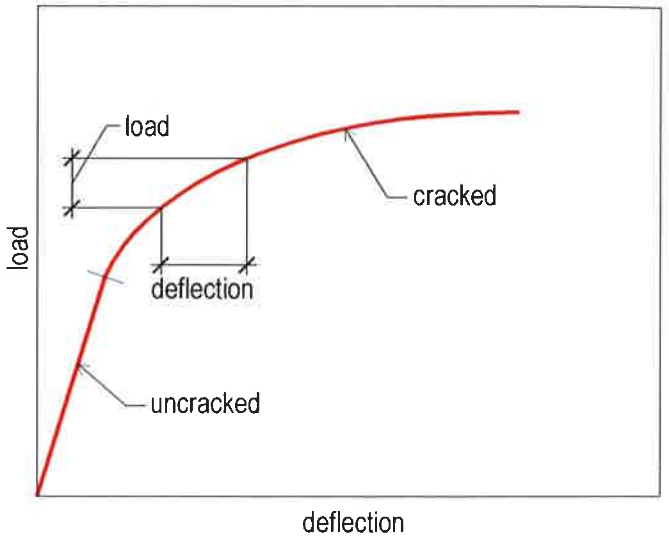


FIGURE 4.10.6.7-1 Load-Deflection with Allowance for Cracking

floor system due to creep and shrinkage over any time interval are readily available, the common practice for residential and commercial buildings is to estimate the long-term deflection through magnification of the instantaneous values. The magnification is done using multiplier factors.

A. Multiplier Factors for Long-Term Deflections: For design purposes, the long-term deflection of a floor system due to creep and shrinkage can be approximated as a multiplier to its instantaneous deflection.

Long-term deflection due to sustained load:

$\Delta_l = C\,\Delta_i$ (Exp 4.10.6.8A-1)

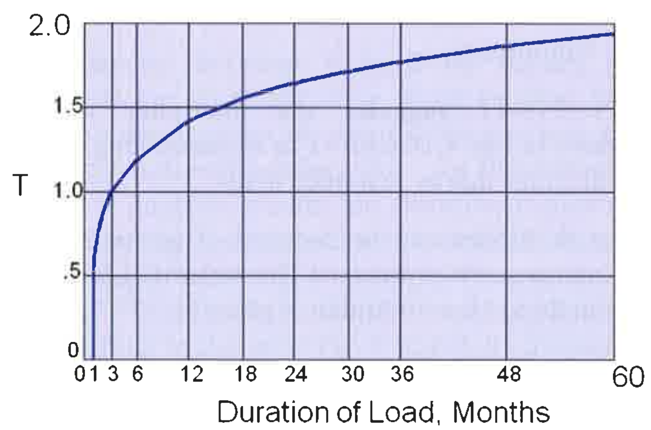


FIGURE 4.10.6.8A-1 Multiplier Factor for Long-Term Deflection (P257)

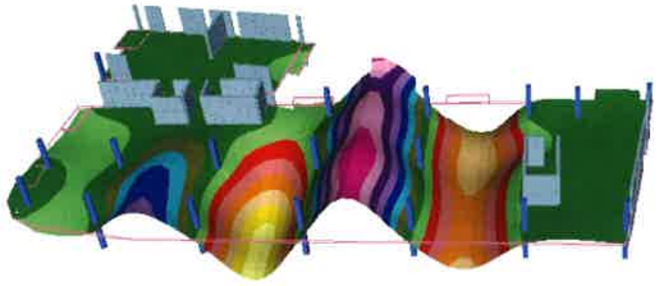


FIGURE 4.10.7-1 First Mode of Vibration of Parissa Floor (P388)

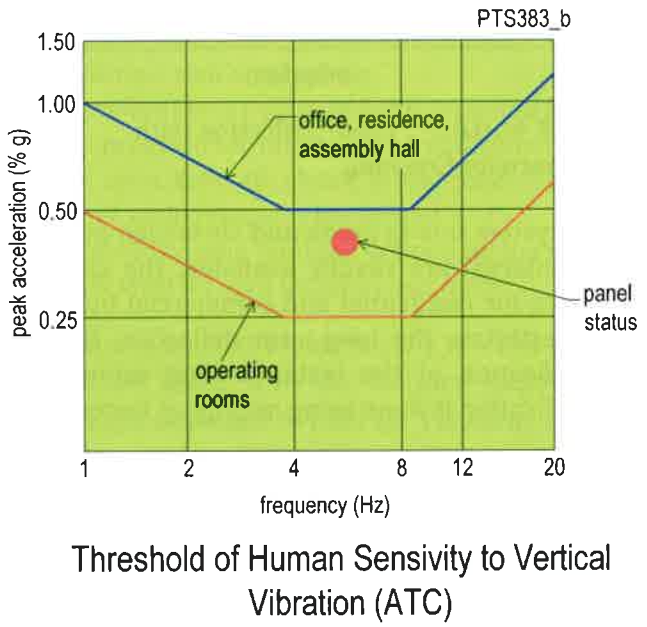


FIGURE 4.10.7-2 ATC Evaluation of Parissa Floor

Where

Δ_l = long-term deflection;
Δ_i = instantaneous deflection; and
C = multiplier.

ACI 318-11 suggests the multiplier factor shown in Fig. 4.10.6.8A -1 to estimate long term deflections due to sustained loads.

The multiplier can be reduced, if compression reinforcement is present. The factor (λ) for the reduction of the multiplier is given by:

$$\lambda = C/(1+50\rho') \qquad \text{(Exp 4.10.6.8A-2)}$$

Where, ρ' is the percentage of compression rebar at mid-span for simple and continuous members and at support for cantilevers.

The ACI 318 recommended multiplier factors apply to common construction. In practical structures there is typically a delay between the time the forms are removed; the time when superimposed load, such as floor cover is applied; and the concrete age when the structure is placed in service at which time live load is applied. The time-delay, as it is expounded henceforth, reduces the net deflection that is the objective of design check and is likely to impact the nonstructural members likely to be damaged. If no allowance is made for time delay in the application of design loads, the long-term deflection is likely to be more.

The load combination to be used for the long-term deflection of a floor system depends on the objective of the evaluation. ACI 318 recommends a load combination that reflects the "sustained" live load on the member. However, the code leaves the fraction of the design live load to be considered as "sustained" to the design engineer's judgment. The European code EC2 is more specific. Fractions of live load to be used for estimating long-term deflections are given in Section 4.10.1

B. Total Long-Term Displacement Subsequent to Removal of Forms: The simple, conservative first estimate load combination for long-term deflection is

$$(1.0\times SW + 1.0\times SDL + 1.0\times PT + 0.3\times LL)\times C$$

(Exp 4.10.6.8B-1)

Where,

SW = selfweight;
SDL = superimposed dead load, (floor cover and partitions);
PT = post-tensioning;
LL = design live load; and
C = long-term multiplier.

The above load combination is conservative as it assumes the application of superimposed loads as well as the application of sustained live load of the structure to take place at the time of removal of the supports below a cast floor. The factor 0.3 suggested for live load is for "sustained" load combination. When using EC2, the value of 0.3 should be substituted by ψ from Table 4.10.1-1.

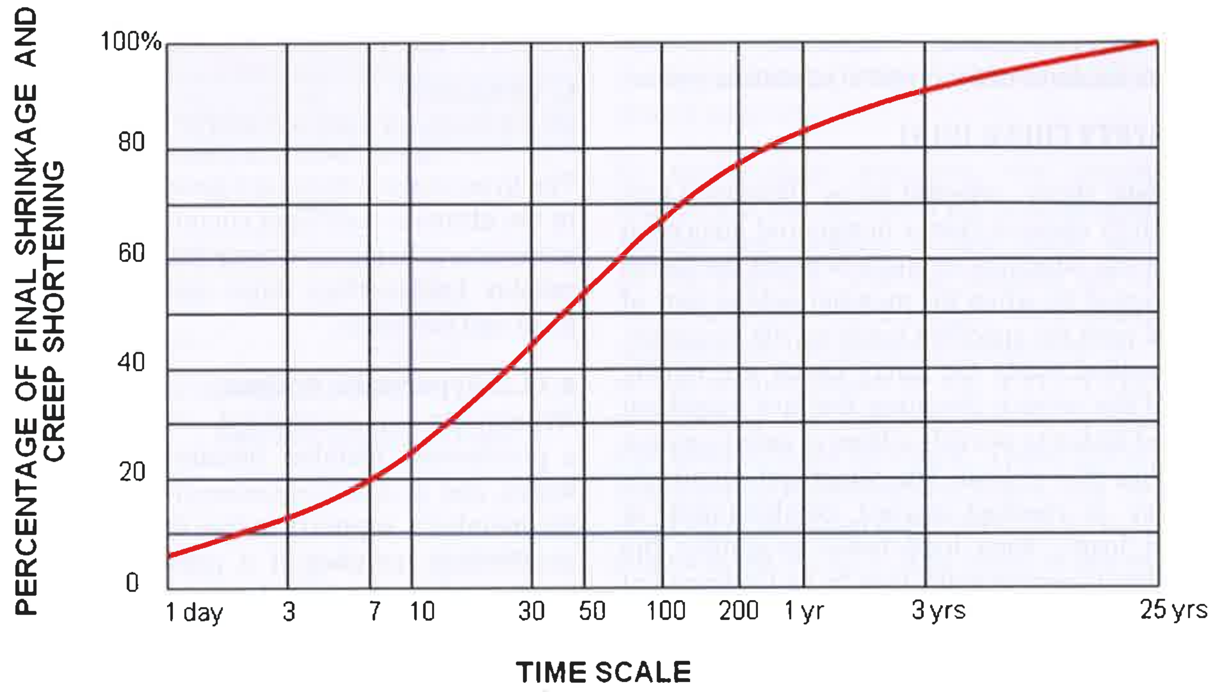


FIGURE 4.10.6.8C-1 Long-Term Shortening of Concrete Members with Time Due to Creep and Shrinkage (P569)

The significance of the above load combination is that it provides an upper-bound measure for the total deflection from the position of the forms at the time of concrete casting. The values are more applicable to parking structures and roofs, where a floor is placed in service in essentially in as-cast condition.

C. Detailed Combination of Long-Term Deflections: Detailed calculations generally apply, when the objective of estimating deflection is to determine its impact on non-structural members that are likely to be damaged. Graphs, such as the example reproduced in Fig. 4.10.6.8C-1 can be used to determine the increment of long-term deflection with respect to time for specific applications.

D. Incremental Estimate of Long-Term Deflections: When estimate of deflection over a given time period, as opposed to the final long-term deflection becomes necessary, graphs such as the one shown in Fig. 4.10.6.8C-1 can be used. The graph gives an estimate of incremental deflection over a given time as a fraction of the long-term deflection.

4.10.7 Vibration Control

Larger and thinner spans driven by application of higher strength concrete, desirability of large open spaces and architectural aspirations result in floor systems that are more susceptible to vibration. Vibrations are to be controlled, if they are perceived and found objectionable. The most common source of vibration in residential and commercial buildings is from footfall. Detail treatment of vibration evaluation of concrete floors is outside the scope of this book. Reference [ADAPT-TN 290, 2010] offers the background to the assessment and control of concrete floor vibrations resulting from foot drop, including the impact of vibrations on sensitive equipment.

Frequencies between 4 to 8 Hz (cycles per second) are most annoying when coupled with a peak acceleration of about 0.5%g, where g is the gravitational acceleration. To illustrate the kind of analysis results for vibration evaluation of a floor system, consider the panel of Parissa floor identified in Fig. 4.10.6.4-1. From reference [ADAPT-TN 290, 2010], the first mode of vibration of the floor is shown in Fig. 4.10.7-1. Its frequency is determined to be 5.97 Hz.[74] The example arrives at peak floor acceleration equal to 0.35g for foot

[74] Used ADAPT-Floor Pro software; www. Adaptsoft.com

drop. Figure 4.10.7-2 illustrates that the vibration of this floor is acceptable for office and residential occupancies, but not for hospital operating rooms.

4.11 SAFETY CHECK (ULS)

The safety check, referred to as Ultimate Limit State (ULS) ensures that a designated structural member has adequate strength to resist the forces it is assigned to, when the member acts as part of the load path for specified loads on the structure. The specified loads for safety check are mostly those of the service condition that are magnified by a load factor to provide a level of safety against possibility of overload. The structural member's adequacy is checked against combinations of factored loads. Each load factor magnifies the respective in-service value load by its likelihood of exceeding.

The two principal load cases for the safety check of a structure are "gravity" and "gravity plus lateral" conditions. The gravity case accounts primarily for the forces from selfweight, superimposed dead load (SDL), live load and prestressing. The lateral load case includes the effects of wind or earthquake, in addition to those specified for gravity design. There are other special cases, such as snow, temperature, water pressure, and time dependent effects, such as shrinkage. However, the focus of the material that follows is on gravity design of post-tensioned floor systems.

4.11.1 Load Combinations for Gravity Design

The primary load combinations for gravity design of selfweight, superimposed dead load, live load and prestressing are given below for the building codes covered. Selfweight and superimposed dead load are lumped together as "Dead Load."

A. ACI 318-11

U1 = 1.20DL + 1.60LL + 1.00HYP (Exp 4.11.1A-1)
U2 = 1.40DL + 1.00HYP (Exp 4.11.1A-2)

Where,

DL, LL, and HYP are the effects of dead load, live load and hyperstatic actions from prestressing respectively.

B. EC2 EN 1992-1-1:2004: There are several combinations. However, the basic combination for the common condition is as follows:

U1 = 1.35DL + 1.50LL + 1.00HYP
(Exp 4.11.1B-1)

C. TR43,2005

U1 = 1.35DL + 1.50LL + 0.90HYP (Exp 4.11.1C-1)

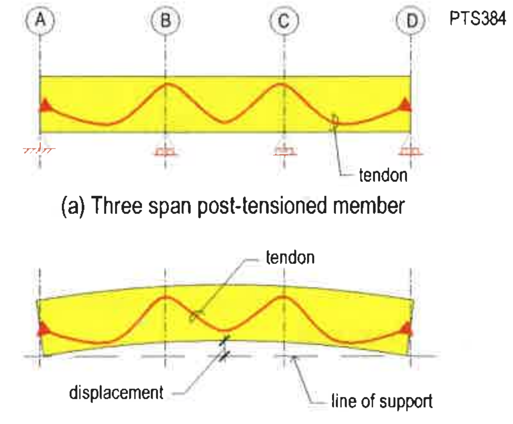
The hyperstatic actions are generally not factored in the ultimate state load combinations, since the parameters that govern their magnitude are more reliably known than those associated with the dead and live loads.

4.11.2 Hyperstatic Actions

Hyperstatic (or secondary) actions develop in a prestressed member because of prestressing forces, and as a consequence of the constraint of the member's supports to free deformation of the prestressed member. If a prestressed member is allowed to displace freely, as in the case of determinate structures or free standing precast members prior to alignment and installation, no hyperstatic (secondary) actions are generated. However, in most cast-in-place construction, where supports restrain the movement of the prestressed member, hyperstatic actions are generated and can be significant. For this reason they are included in the design protocol of prestressed members.

The development of hyperstatic actions and their significance on the safety of a post-tensioned member were discussed conceptually in Section 4.8.6. In what follows, we re-visit the formation of the hyperstatic actions in a more rigorous manner, and continue it with the computation of hyperstatic actions for different scenarios.

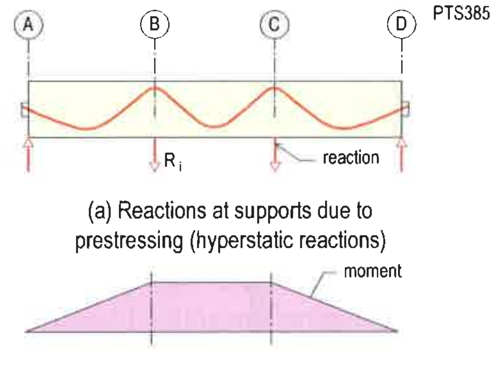
4.11.2.1 Hyperstatic Actions and Strength Demand: Consider the hypothetical case of a three-span post-tensioned member that has been cast and stressed prior to installation (Fig. 4.11.2.1-1). To eliminate the impact of self-weight from the discussion, assume that the member is resting freely on its side. Before installation, the tendon forces cause the member to camber as indicated by the curved soffit in Fig. 4.11.2.1-1(b). The camber is due solely to the flexing of the member under the action of its prestressing tendons. At installation, the member must be forced down to straighten, before it can be fixed to the aligned supports shown in Fig. 4.11.2.1-1(a). The forces at the supports necessary to keep the



(a) Three span post-tensioned member

(b) Member detached from supports

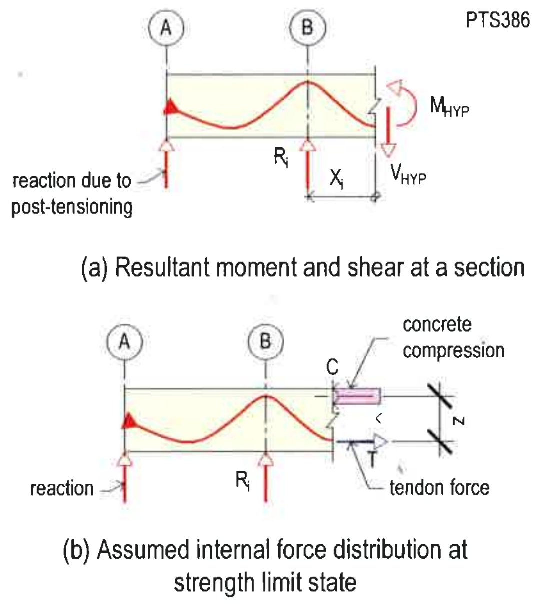
FIGURE 4.11.2.1-1 Member Subject to Post-Tensioning Only



(a) Reactions at supports due to prestressing (hyperstatic reactions)

(b) Moments due to prestressing support reactions (hyperstatic moments)

FIGURE 4.11.2.1-2 Hyperstatic (Secondary) Reactions and Moments



(a) Resultant moment and shear at a section

(b) Assumed internal force distribution at strength limit state

FIGURE 4.11.2.1-3 Sectional Actions From Prestressing and the Internal Distribution of Forces at Strength Limit State

member along the level line are the hyperstatic (secondary) reactions. Figure 4.11.2.1-2(a) shows the hyperstatic reactions of the member held on simple supports.

In cast-in-place members, the construction sequence is the reverse of the preceding example. Assume the member is cast while positioned on the supports, and the tendons are stressed afterwards (Fig. 4.11.2.1-1(a)). When the tendons are stressed, the supports prevent the formation of the camber shown in Fig. 4.11.2.1-1(b). The restraint of the supports results in additional reactions at these locations. These reactions are the hyperstatic (secondary) forces. The free-body diagram of the beam detached from its supports is shown in Fig. 4.11.2.1-2(a). Since, in this example, the supports are assumed hinged, no moments are developed at the connection of the member to the supports. The reactions from post-tensioning are in self-equilibrium.

$$\sum R_{hyp} = 0 \qquad \text{(Exp 4.11.2.1-1)}$$

$$\sum M_{hyp} = 0 \qquad \text{(Exp 4.11.2.1-2)}$$

Where R_{hyp} is the hyperstatic reaction and M_{hyp} is the hyperstatic moment. In this example, there are no hyperstatic moment reactions at the supports. Figure 4.11.2.1-2(b) illustrates the distribution of hyperstatic (secondary) moments in the member.

Pursuing the same example, in which only the post-tensioning forces are being reviewed, observe Fig. 4.11.2.1-3(a). At any section along the member, the hyperstatic reactions induce a hyperstatic moment (M_{hyp}) and a hyperstatic shear (V_{hyp}). There is no resultant horizontal force at the cut section for the roller-support example considered. From the statics of the free-body diagram of the cut member, the hyperstatic moment and shear are given by the following relationships.

$$V_{hyp} = \sum R_i \qquad \text{(Exp 4.11.2.1-3)}$$

$$M_{hyp} = \sum R_i X_i \qquad \text{(Exp 4.11.2.1-4)}$$

The hyperstatic shear and moment shown in Fig. 4.11.2.1-3(a) at the cut section are sustained by forces developed in the concrete and reinforcement over the cross section. At the strength limit state, the moment is assumed to be resisted by a compression block and a tensile force as shown in Fig. 4.11.2.1-3(b), for which the following relationships apply:

$$C = T \qquad \text{(Exp 4.11.2.1-5)}$$

$$M_{hyp} = Tz = Cz \qquad \text{(Exp 4.11.2.1-6)}$$

Where,

C = total compression force;
T = combined tension force due to the prestressing and nonprestressed reinforcement; and
z = internal lever arm of the section.

From the foregoing, it is evident that at the strength limit state, the section must develop an internal resistance associated with the hyperstatic moment. This resistance is in addition to that generated through the action of other loads, such as self-weight and live loading. Building codes require that the hyperstatic (secondary) actions be included in all strength demand load combinations.

4.11.2.2 Determination of Hyperstatic Actions: Hyperstatic actions can be calculated either directly or indirectly. For skeletal members, such as beams, and one- way floor systems, hyperstatic actions can be successfully calculated using both methods. For two-way floor systems that are modeled as isolated design strips, the indirect method involves an approximation. The degree of approximation varies with the complexity of the structure. For continuum members, such as floor slabs that are viewed as plates, the direct method must be used. Both methods are detailed in the following.

A. Direct Method: The direct method is based on the definition of hyperstatic actions as described in the preceding. Observe a typical frame as shown in Fig. 4.11.2.2A-1. The frame is subject to dead loading, live loading and post-tensioning. The reactions caused at the supports are due to dead, live and post-tensioning forces. Fig. 4.11.2.2A-2

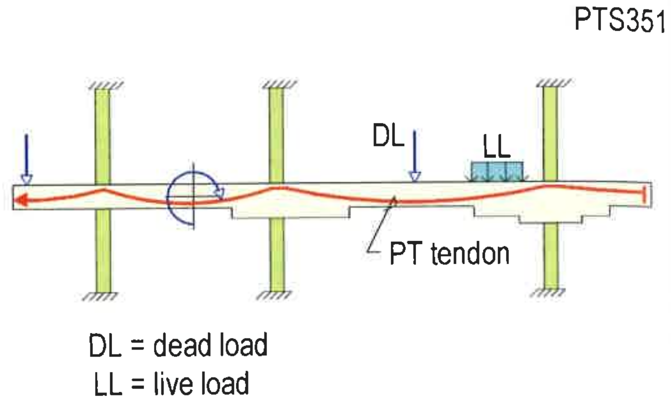


FIGURE 4.11.2.2A-1 Loaded Post-Tensioned Member

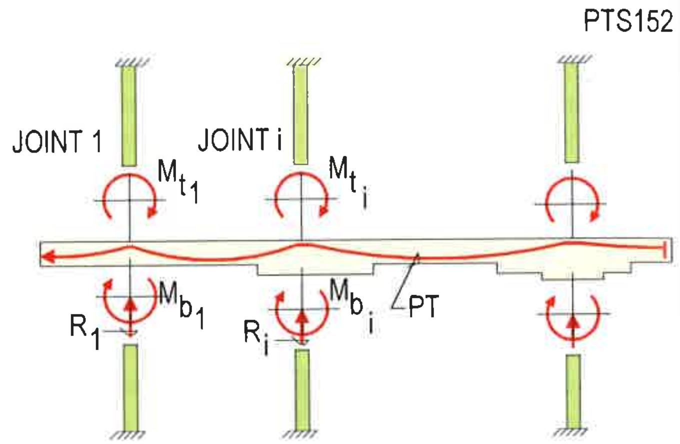
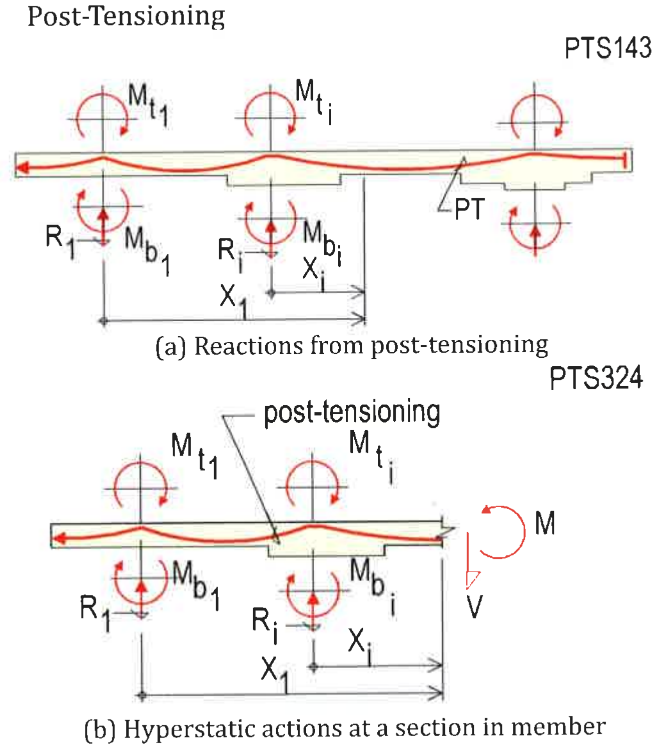


FIGURE 4.11.2.2A-2 Hyperstatic Reactions from Post-Tensioning



(a) Reactions from post-tensioning

(b) Hyperstatic actions at a section in member

FIGURE 4.11.2.2A-3 External and Internal Hyperstatic Actions

illustrates the free body diagram of the member in which only the post-tensioning and its reactions at the supports are shown. The free body diagram is complete with respect to post-tensioning forces. Balanced loads are not included in this diagram, since post-tensioning tendon is not removed. When in place, tendon force and the balanced loads neutralize one another.

By definition, the moments and reactions shown in Fig. 4.11.2.2A-2 are the hyperstatic actions, since these are induced by post-tensioning alone. For static equilibrium, the sum of all the hyperstatic actions on the frame must add to zero.

When "load balancing" is used, the procedure for computation of hyperstatic moments starts by representing the prestressing forces as balanced loads on the structure. Next, the frame is solved for the balanced loads. The reactions obtained are the hyperstatic actions on the frame. It is re-iterated that the central issue in this process is the correct representation of the balanced loading on the frame as a set of self-equilibrating external actions. Only where all the components of the balanced loading are properly represented and are in static equilibrium, will the calculated hyperstatic actions be in self-equilibrium and correct.

The hyperstatic reactions calculated above are next applied to the member to determine the distribution of the hyperstatic moment and shear along the member (Fig. 4.11.2.2A-3). At any distance Xi, as shown in the figure, the hyperstatic shear is simply the algebraic sum of all reactions, and the hyperstatic moment is the moment of all actions. The relationships are as follows:

$$V_{hyp} = \sum R_i \qquad \text{(Exp. 4.11.2.2A-1)}$$

$$M_{hyp} = \sum[M_{ti} + M_{bi}] + (R_i X_i) \qquad \text{(Exp. 4.11.2.2A-2)}$$

Where,

M_{ti}, M_{bi} and R_i = support reactions due to post-tensioning;
X_i = distance to the section under consideration.

B. Indirect Method: The indirect method is a procedure commonly used for the calculation of hyperstatic moments in skeletal structures. It is based on the following relationship:

$$M_{hyp} = M_{bal} - P \times e \qquad \text{(Exp 4.11.2.2B-1)}$$

Where,

e = eccentricity of post-tensioning/ prestressing with respect to the neutral axis of the section (positive if CGS is above the neutral axis, otherwise negative);
M_{hyp} = hyperstatic moment;
M_{bal} = balanced moment due to balanced loading; and
P = post-tensioning/prestressing force (positive).

In this relationship, moments causing tension at the bottom fiber are assumed positive. The hyperstatic reactions and shears are then calculated from the hyperstatic moments.

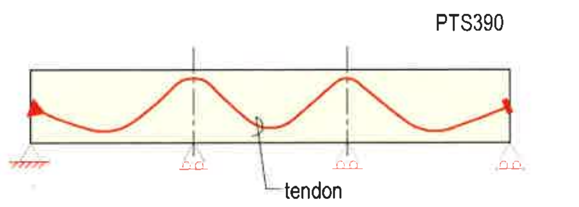
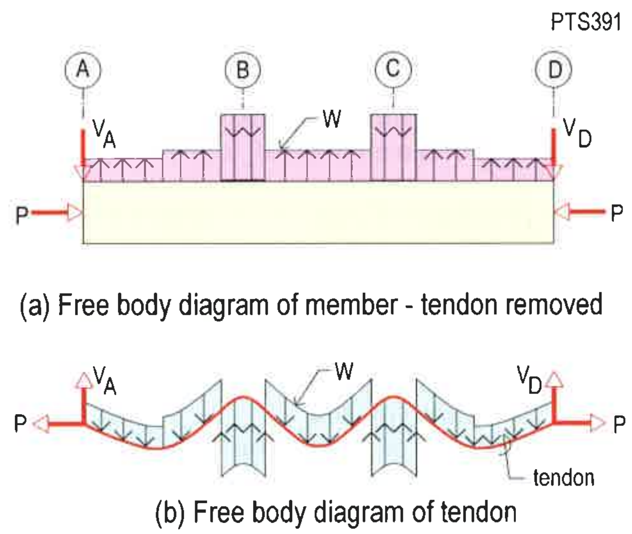


FIGURE 4.11.2.2B-1 Post-Tensioned Member on Supports



(a) Free body diagram of member - tendon removed

(b) Free body diagram of tendon

FIGURE 4.11.2.2B-2 Illustration of Force System Between Tendon and Member

The indirect method (Exp 4.11.2.2B-1) does not apply to members, such as floor systems, where prestressing force can disperse into a structure beyond the strip/section isolated for design. This will be explained in greater detail next.

Consider Fig. 4.11.2.2B-1. It illustrates a post-tensioned member supported on rollers. Following the load balancing procedure, we remove the tendon from its housing and replace it by the forces that the tendon exerted when in place. This is illustrated in Fig. 4.11.2.2B-2a. The loading shown in this diagram is the "balanced loading." In this example, it is comprised of upward and downward forces resulting from tendon segments, as well as a constant compression force *P*. Part (b) of the figure shows the forces on the extracted tendon. Evidently, the loads shown in part (a) and (b) of the figure are equal and opposite of one another.

The extracted tendon in part (b) is always statically determinate. By definition, it is a flexible member capable of resisting only a single direct tensile force *P*. The member in part (a) of the figure may or may not be statically determinate; its determinacy depends on its support conditions, which are not shown in the figure.

The force systems shown in parts (a) and (b) of the figure are both considered balanced loading. The balanced loading consists of all the forces due to prestressing that act on the member when tendon is extracted.

Figure 4.11.2.2B-3 shows a section of the member cut at distance "*a*" from the member end. The actions at the cut section are from prestressing forces only. These are: *P*, V_x, and M_p.

From part (a) of the figure, taking moments about *O*, the value of the moment M_p is given by:

$$M_p = \int (wdx)x + V_A \times a \qquad \text{(Exp 4.11.2.2B-2)}$$

Where,

M_p = primary moment;
w = intensity of balanced loading at distance *x* from the cut; and
a = distance of cut section from the member end.

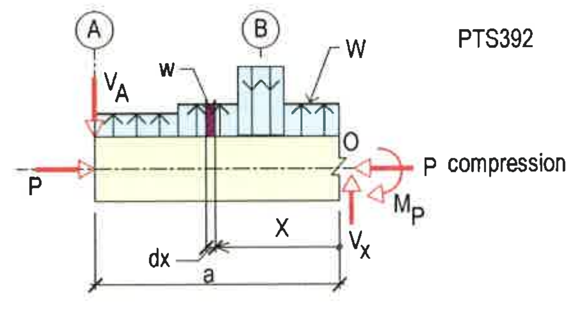
Considering the equal and opposite display of the balanced loading in part (b) of the figure, the value of the moment M_p about point *O* is given by:

$$P \times e = \int (wdx)x + V_A \times a \qquad \text{(Exp 4.11.2.2B-3)}$$

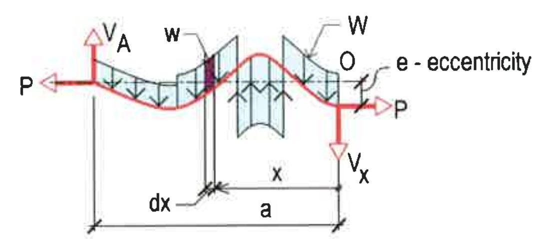
Comparison of the relationships (2) and (3) concludes that:

$$M_p = Pe \qquad \text{(Exp 4.11.2.2B-4)}$$

Consider now the condition, where the supports of the member do not allow the member's free deformation resulting from the balanced loading. Reactions develop at the supports as shown in Fig. 4.11.2.1-2. In this case, the free body diagram at a cut distance "*a*" from support "*A*" will also include



(a) Free body diagram of cut beam



(b) Free body diagram of cut tendon

FIGURE 4.11.2.2B-3 Post-Tensioning Forces over a Tendon Length "a"

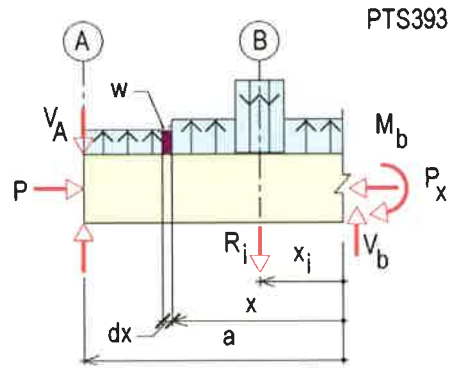


FIGURE 4.11.2.2B-4 Balanced Moment M_b Balancing the Primary and Secondary (Hyperstatic) Moments

the reactions from the balanced loading (Fig. 4.11.2.2B-4). The moment "M_b" at the section is due to post-tensioning forces only. The moment is referred to as "balanced moment."

From the statics of the cut section:

$$M_b = \left[\int (wdx)x + V_A \times a\right] + \Sigma R_i . x_i \qquad \text{(Exp 4.11.2.2B-5)}$$

Substituting for the value in the square brackets from 4.11.2.2B-2 and the last expression from 4.11.2.1-4, we obtain:

$$M_b = M_p + M_{hyp} \qquad \text{(Exp 4.11.2.2B-6)}$$

or

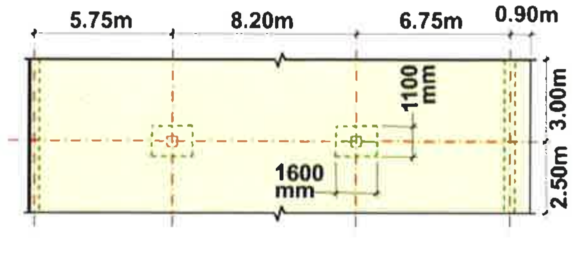
$$M_b = Pe + M_{hyp} \qquad \text{(Exp 4.11.2.2B-7)}$$

Expression 4.11.2.2B-7 is the indirect method for determination of hyperstatic moments in common use for skeletal members. Note that in arriving at this conclusion, we have implicitly assumed that $P = P_x$. In other words, no part of the post-tensioning force at the stressing end is diverted to adjoining members.

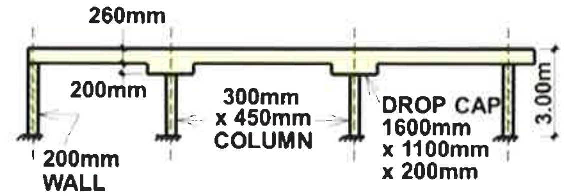
4.11.2.3 Example of a Design Strip: Traditionally, engineers treat the design strips extracted from a floor system in the same manner as "skeletal structures." The procedure is based on the assumption that the precompression from a tendon anchored within a given strip remains in the same strip. In other words, there is no dispersion of prestressing between adjacent strips.

Consider a design strip from a two-way floor system as detailed in Fig. 4.11.2.3-1. A three-dimensional view of the frame is shown in Fig. 4.11.2.3-2. We first use the direct method to compute the hyperstatic moments of this slab example. Next, we will use the indirect method.

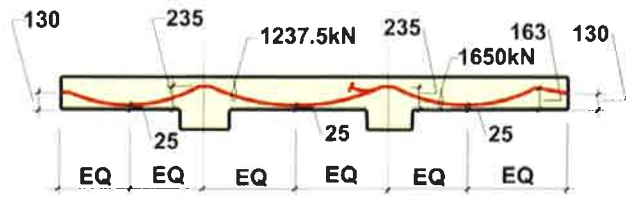
The balanced loads from the tendon are generated and used as applied loads to solve for the moments and other actions caused in the frame from prestressing forces.[75] The distribution of post-tensioning moments from the balanced loads is shown in Fig. 4.11.2.3 -3.



(a) PLAN



(b) ELEVATION



(c) TENDON GEOMETRY (Y-SCALE EXAGGERATED)

FIGURE 4.11.2.3-1 Geometry and Post-Tensioning Details of the Design Strip (P577)

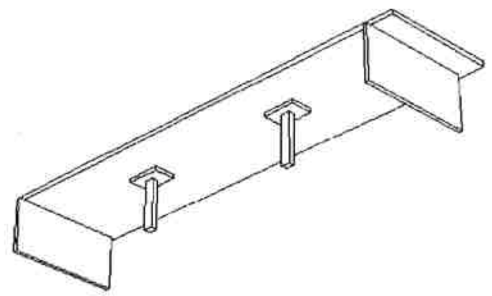


FIGURE 4.11.2.3-2 3D View of the Design Strip (P389)

The hyperstatic actions at the supports of the two-way slab example are shown in Fig. 4.11.2.3 -4(a). Note that the sum of the hyperstatic reactions is zero.

Reactions = 8.983 - 8.972 - 9.015 + 9.005
= 0.001 kN

[75] ADAPT-PT software and equivalent frame option were used to obtain the values quoted.

The hyperstatic actions constitute a self-equilibrating force system. Also note that in this case the third span has a different post-tensioning force than the first two spans. Added tendons extend from the tip of the cantilever at right to one-fifth point of the second span from the third support. The force in the added tendons is:

1650 - 1237.5 = 412.5 kN

The solution for hyperstatic actions in this case depends on the accurate representation of the balanced loading of all tendons, including the portion of the added tendons from the third support to where they are dead ended in the second span. Ignoring the contribution of these tendon parts leads to incomplete solution. See Fig. 4.11.2.3 -3 for the distribution of balanced moments in the slab frame.

A. Direct Method: The hyperstatic moments in the frame are determined using the hyperstatic actions at the supports and the statics of the structure.

Moment at left of support 2:

$$M_{hyp} = 8.983\,\text{kN} \times 5.75\,\text{m} = 51.652\ \text{kNm}$$

At right of support 2:

$$M_{hyp} = 51.652 - 9.834 = 41.818\ \text{kNm}$$

At mid-length of span 1:

$$M_{hyp} = 8.983 \times 2.875\ \text{m} = 25.826\ \text{kNm}$$

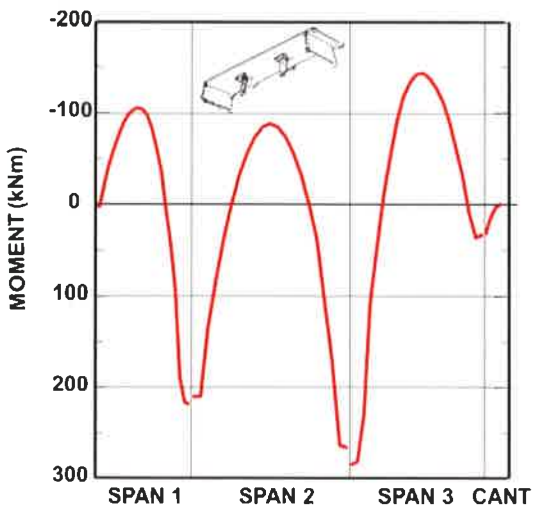
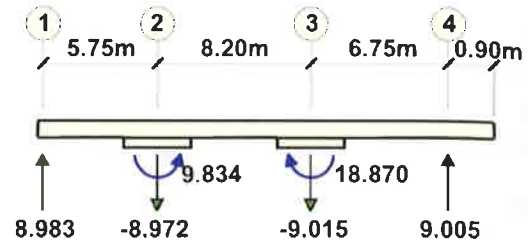
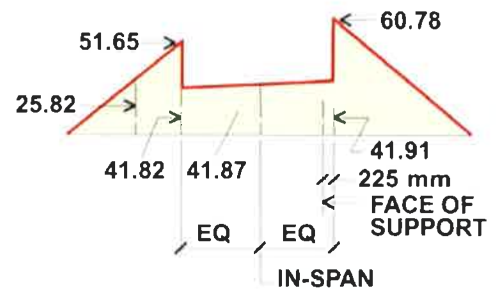


FIGURE 4.11.2.3-3 Distribution of Post-Tensioning Moments (P578)



(a) HYPERSTATIC ACTIONS AT SUPPORTS (UNITS IN kN AND kN-m)



(b) DISTRIBUTION OF HYPERSTATIC MOMENTS (UNITS kN-m)

FIGURE 4.11.2.3-4 Hyperstatic Reactions and Hyperstatic Moments (P579)

Moments at other locations are calculated in a similar manner. The complete distribution of hyperstatic moments is shown in Fig. 4.11.2.3-4 (b).

B. Indirect Method: Using the indirect method, the hyperstatic moments at the locations specified above are recalculated as follows:

$$M_{hyp} = M_{bal} - P \times e \qquad \text{(Exp 4.11.2.2B-1)}$$

The distribution of balanced moments (M_{bal}) is shown in Fig. 4.11.2.3-3. The section centroid is located 130 mm above the slab soffit at midspan; and is computed to be 299.37 mm above the drop cap soffit at the centerline of supports 2 and 3(Fig. 4.11.2.3-1).

At the center of span 1:

M_{bal} = -104.10 kNm (see Fig. 4.11.2.3-3)
P = 1237.50 kN
e = 25 - 130 = -105 mm

Note that eccentricity above neutral axis is taken as positive.

$$M_{hyp} = -104.10 + 1237.50 \times 105 / 1000 = 25.838$$
(= 25.826 kNm from Direct Method, OK)

At the left of the third support:

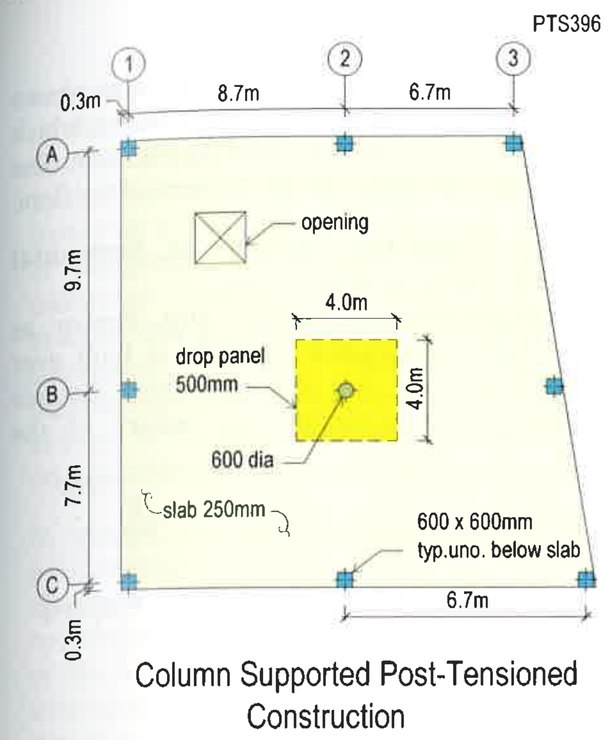


FIGURE 4.11.2.4-1

M_{bal} = 265.75 kNm
P = 1650 kN
e = 235 + 200 - 299.33 = 135.67 mm

M_{hyp} = $265.75 - 1650 \times 135.67 / 1000$ = 41.894 kNm
(= 41.91 kNm from Direct Method, OK)

Note that in this case, the assumptions made lead to an agreement between the results from the direct and indirect methods.

4.11.2.4 Hyperstatic Actions for Continuum Structures: The calculation of hyperstatic actions in continuum structures, such as slabs, shells and solids, presents a challenge. The actions generated in a continuum structure due to constraint of the supports distribute in two or three directions. For any given design section covering a finite length, the actions on the design section cannot readily be related to the position and force of the individual tendons within the same cross section. The one-to-one correlation that exists between tendon forces at a section of a skeletal member and the associated simplified expression of precompression from post-tensioning is not apparent in a continuum. The indirect method of computation explained in the preceding section does not apply.

In the case of continuum structures, the direct method of computation is the valid choice. This involves (i) the computation of reactions at the supports arising from post-tensioning; (ii) application of these reactions as applied loads to the continuum; and (iii) analysis of the continuum structure under the reaction loads from post-tensioning in order to determine the distribution of hyperstatic forces in the structure.

In other words, all the reactions at the supports due to prestressing must be computed. The computed reactions must then be applied to the continuum as a loading to determine the hyperstatic actions within the continuum. The concept and procedure is described through the example of a two-way post-tensioned floor system described next.

Consider the case of a simple floor system shown in Fig. 4.11.2.4-1. The slab is resting on eight square perimeter columns and a central round column. Other features of the slab include a column drop over the central support, and an opening. The tendon layout of the floor is displayed in Fig. 4.11.2.4-2. The tendons are banded along the grid lines 1, 2 and 3, and are distributed uniformly in the transverse direction. Other particulars of the floor are listed below.

Post-tensioning:

Effective Stress, f_{se} = 1200 MPa
Strand Area = 98 mm^2
Force assumed constant along tendon length

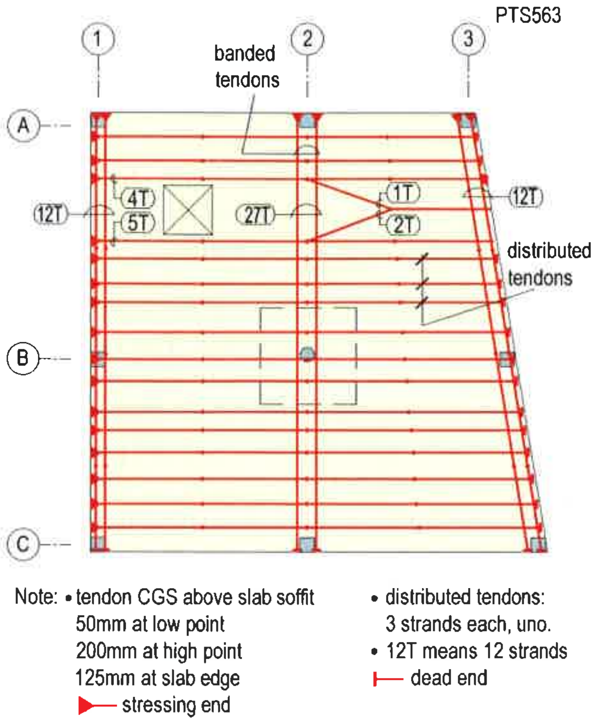
The floor is analyzed using a software developed for analysis and design of conventionally reinforced or post-tensioned floor systems[76]. A three-dimensional, computer generated view of the structure is shown in Fig. 4.11.2.4-3. For the tendon layout and associated pre-stressing, the computed hyperstatic reactions are entered in Fig, 4.11.2.4-4.

For a correct solution, the hyperstatic reactions from prestressing tendons must be in self-equilibrium. The following equilibrium check is performed to verify the validity of the solution shown in Fig. 4.11.2.4-4

Σ Secondary actions in the vertical direction

$= 1.05 + 20.3 + 2.18 + 47.8 - 140 + 38.8$
$\quad - 0.576 + 21.6 + 8.61\,\text{kN}$
$=$ - 0.236 kN ≅ 0 kN

[76] ADAPT Builder Floor Pro; www.adaptsoft.com



Tendon Layout in Slab

FIGURE 4.11.2.4-2

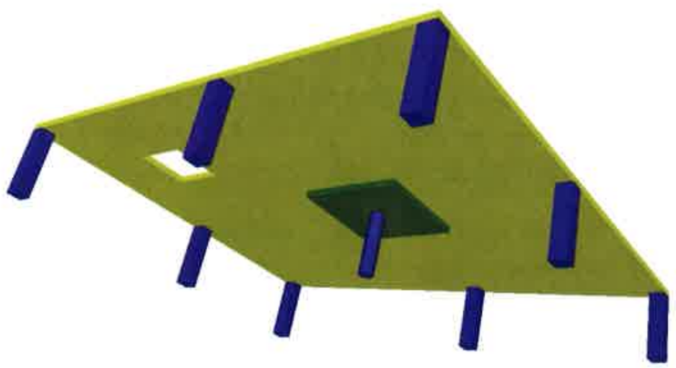


FIGURE 4.11.2.4-3 3D view of structure (P392)

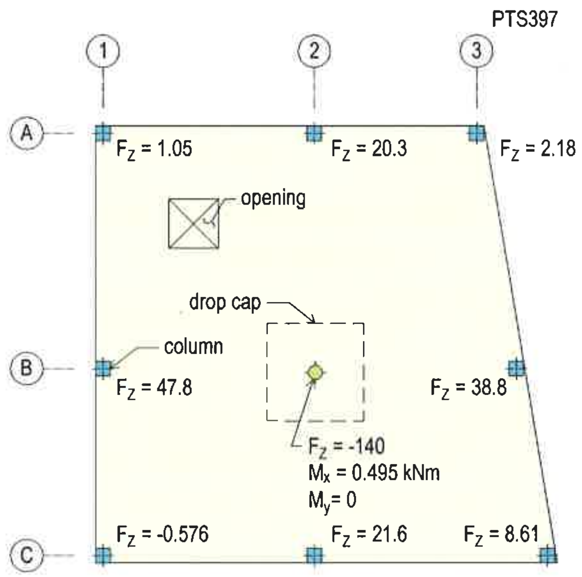
Vertical forces are in equilibrium.
Hyperstatic moments about gridline B:

$$\begin{aligned} M_{yy} &= (1.05+20.3+2.18)\,\text{kN}\times 9.7\,\text{m} \\ &\quad -(-0.576+21.6+8.61)\,\text{kN}\times 7.7\,\text{m}+0.495 \\ &= 0.554\ \text{kNm} \cong 0\ \text{kNm OK} \end{aligned}$$

Next, we use the hyperstatic reactions (shown in Fig. 4.11.2.4-4) as an applied loading to the continuum and determine the resulting actions within the continuum. For the floor slab under consideration, the distribution of hyperstatic moment for a section at the face of support is shown in Fig. 4.11.2.4-5. The figure also shows the total value of the hyperstatic moment, which is the integral of the displayed distribution. The total value is used for the safety check of the floor.

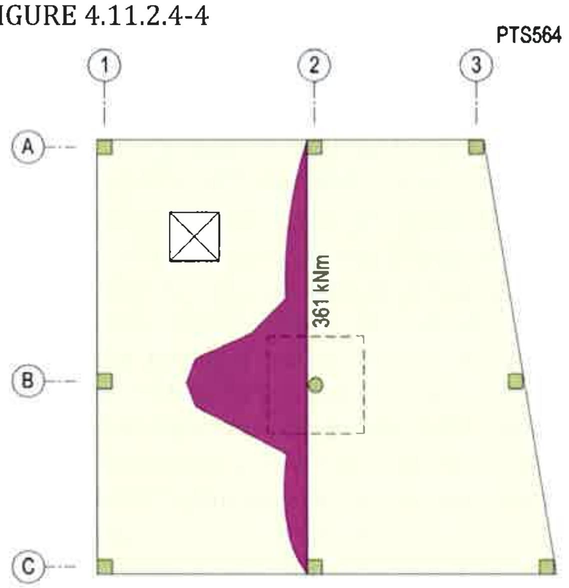
4.11.2.5 Hyperstatic Actions in Segmental (Phased) Construction

A. Introduction: A phased, also known as segmentally constructed structure is built over a period of time and in stages from discrete components. A distinguishing feature of the segmental construction is that, the components, or the partially assembled structure, are called upon to carry significant loading in a configuration by the interim structural system that is different from that of the completed structure. Examples of segmental construction include balanced cantilever bridges, incrementally launched bridges, and precast prestressed girders made continuous through splicing. Multi-story buildings are also constructed segmentally. The stresses in the completed structure can depend on the method and sequence of construction.



Plan of Floor Showing Reactions from Post-Tensioning at Far End of Supports (kN)

FIGURE 4.11.2.4-4



Distribution of Hyperstatic Moments Across the Floor at Face of Support, and its Total

FIGURE 4.11.2.4-5

In phased construction, some or all of the prestressing is applied before the structure assumes its final configuration. Since (i) the hyperstatic actions are caused by the resistance of the supports to the free movement of the structure, and (ii) the configuration of the structure and its support conditions change over time, the computation of hyperstatic actions can no longer can be based on the geometry and prestressing of the completed structure. In this case, an incremental computational procedure must be adopted. At each stage of construction, and for each application of prestressing, the increment in hyperstatic actions must be calculated. The hyperstatic actions due to each application or change in prestressing accumulate to yield the total of the hyperstatic actions at any given stage of construction.

The concept for the computation of hyperstatic actions in phased construction is described through the following illustrative example. To focus on the hyperstatic actions, without compromising the concept, the long-term effects of creep, shrinkage, relaxation, and aging of concrete are not included in the example. A simple member and prestressing pattern are selected.

B. Example of a Phased Construction: Figure 4.11.2.5-1 shows the construction stages of a two-span continuous beam. Each span consists of a precast, prestressed girder of rectangular cross section. The girders are transported to the site and installed with a gap over the central pier. A pre-installed duct in each precast girder is made continuous across the gap and the gap is cast to splice the two girders together. A post-tensioning tendon is passed through the duct. After the concrete of the splice gains adequate

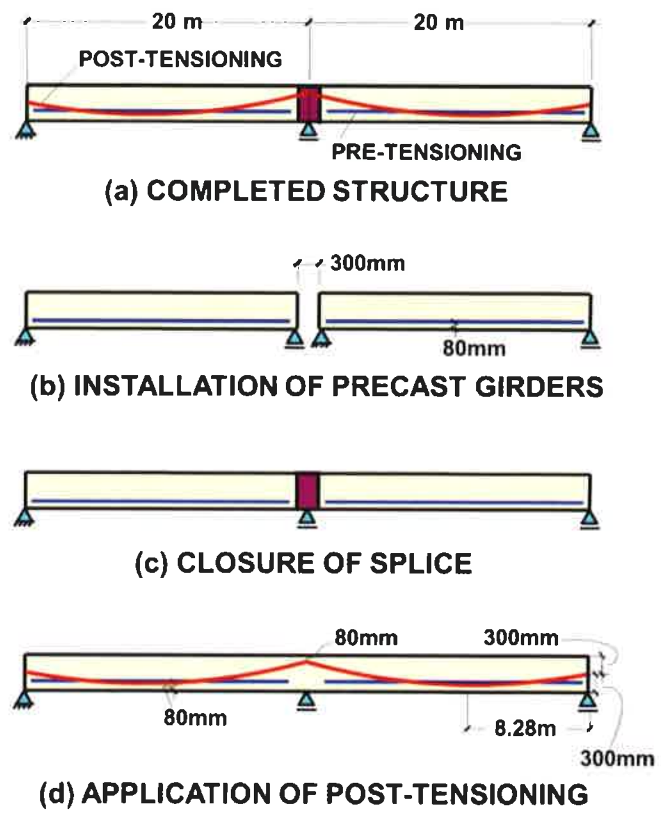


FIGURE 4.11.2.5-1 Segmentally Constructed Two-Span Member (P580; P381)

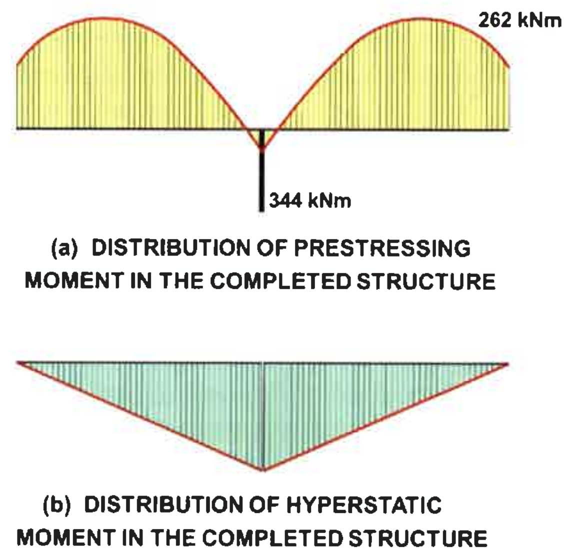


FIGURE 4.11.2.5 -2 Prestressing and Hyperstatic Moments (P581)

strength, the post-tensioning tendon is stressed, thereby achieving a continuous, two-span, post-tensioned member. In prototype construction, a topping slab is generally applied over the precast girders. Depending on the sequence of application of the topping and superimposed dead load, the stressing of the post-tensioning tendons may also be staged.

Additional details of the structure are:
Cross section: Rectangular 600 mm (23.62") deep, 250 mm (9.84") wide
Pre-tensioning tendons:
Effective force = 1193 kN, using 10 – 13 mm (0.5") seven wire strands
Location = parallel to member soffit; CGS 80 mm (3.15") from the girder soffit

Post-tensioning tendons:
Effective force = 1193 kN, using 10-13 mm (0.5") seven wire strands
Profile = simple parabola

The distribution of moment in the completed structure due to the action of the prestressing is shown in Fig. 4.11.2.5-2a. The non-zero values of moment at the first and last support are due to the eccentricity of the pretensioning strands at these locations with respect to the centroidal axis

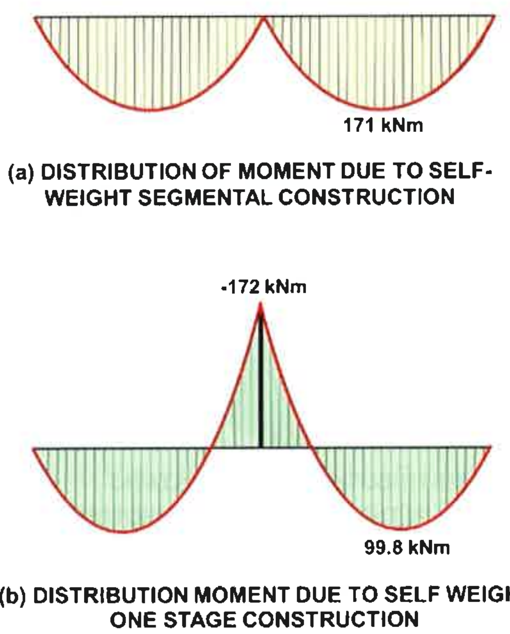


FIGURE 4.11.2.5-3 Moments Due to Segmentally and Monolithically Constructed Alternatives (P582)

of the member. The sharp rise in the prestressing moment over the central support is due to the fact that the pretensioning strands do not extend into the splice.

Figure 4.11.2.5-2b illustrates the hyperstatic moments for the completed structure, arising from the post-tensioning forces only. The hyperstatic moments are caused by the change in prestressing after the structure becomes indeterminate. In this example, the change is limited to the addition of the post-tensioned tendons, since the pretensioning was applied when the structure was determinate. In an actual structure, there is also a contribution to the hyperstatic moments from the stress loss in pretensioning, subsequent to the closure of the splice.

If the entire prestressing, consisting of the straight and the profiled strands were applied at one time, the resulting hyperstatic moments would have been different. The concept is illustrated, using the difference between the distributions of selfweight moments of a two span structure, where in one case the two spans are built disjointed over the common support, and the second case where the spans are built continuous. Figure 4.11.2.5-3a shows the bending moment in the completed structure for the segmentally constructed case. Note that the distribution is the same, as if the two spans were simply supported. However, in a one-stage construction (Fig. 4.11.2.5-3b), the distribution of moment is due to that of a two-span continuous beam. The difference is significant.

The following illustrates the computation of the hyperstatic actions. Note that, the computation includes only the forces from post-tensioning, since the pretensioning forces were applied before the structure became indeterminate.

The reactions due to post-tensioning alone will be vertical forces only, since the structure is assumed to be on rollers. The results are:

Location	Vertical Force	
First support	-4.12 kN	0.926 k
Middle support	8.24 kN	1.852 k
Right support	-4.12 kN	0.926 k
TOTAL	0	0

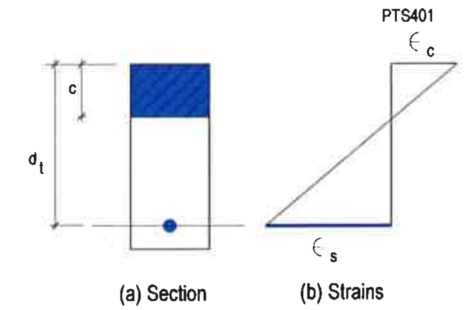


FIGURE 4.11.3A-1 Strain Distribution of a Section in Bending

From the definition of hyperstatic moments:
Moment over central pier = (reaction)(span length)
= 4.12 × 20= 82.4 kNm (60.77 k-ft)

4.11.3 Redistribution of Moments

A. Ductility in Design: Building codes include provisions for ductility of sections designed to resist bending. The ductility in bending gives a section the ability to continue rotation beyond the elastic limit of its reinforcement, albeit with little or no gain in resistance. The post-elastic deformation prior to failure of a ductile member is generally several times its elastic deformation. Building codes control the ductility by limiting the depth of the neutral axis in bending. This ensures that tension steel yields first, before compression zone (Fig. 4.11.3A-1). Failure initiated through post-elastic response of reinforcement leads to increased deflection that signals a member's weakness in strength.

For example, ACI 318-11, limits the depth of the neutral axis "c" to "$0.375d_t$" for basic bending of a section. Where "c" exceeds the maximum value, other provisions are triggered to ensure the acceptability of the section. In EC2 the following is recommended[77]:

$$c_{max} = 0.5h \text{ , for } f_{ck} \leq 50\,\text{MPa} \quad \text{and}$$

$$c_{max} = \left(1 - \varepsilon_{c3} / \varepsilon_{cu3}\right)h \quad \text{for } f_{ck} > 50 \text{ MPa}$$

Where,

$$\varepsilon_{c3} = \left[1.75 + 0.55\left[\left(f_{ck} - 50\right)/40\right]\right] \times 10^{-3}$$

$$\varepsilon_{cu3} = \left[2.6 + 35\left[\left(90 - f_{ck}\right)/100\right]^{4}\right] \times 10^{-3}$$

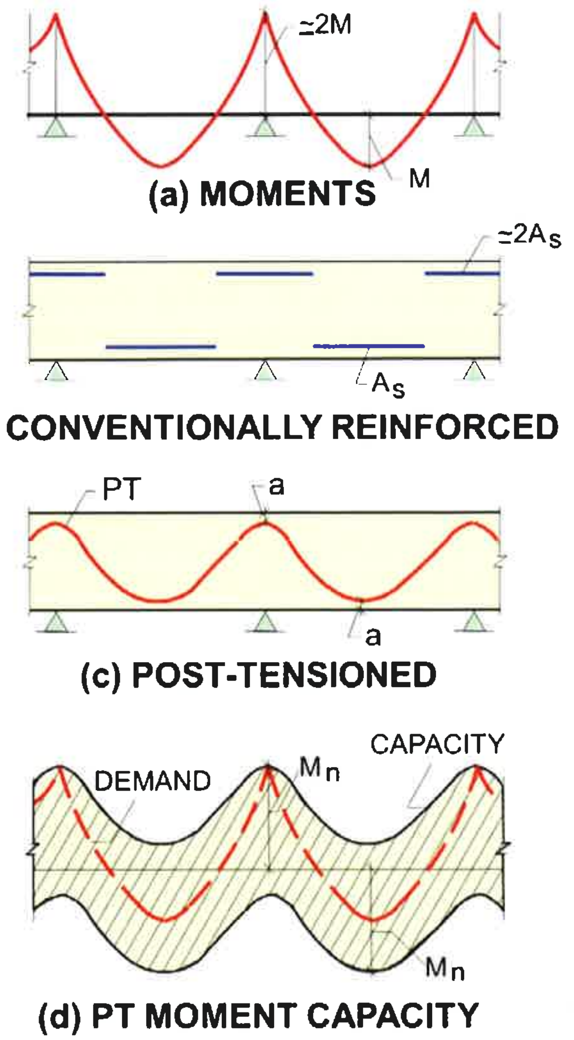


FIGURE 4.11.3B-1 Flexural Demand and Capacity in Conventionally Reinforced and Post-Tensioned Regular Members (P583)

B. Advantage of Moment Redistribution for Post-Tensioned Members: Moment redistribution is a procedure, whereby the calculated elastic moment profile of a given span can be raised or lowered at the face of each of its supports by a percentage not exceeding the code specified limit[78]. It is based on the post-elastic ability of a span to resist the loads it directly supports through a resisting mechanism other than that dictated by the linear elastic response.

Application of moment redistribution to post-

[77] EC2 (EN 1992-1-1-1; 2004), Table 3.1
[78] ACI 318-1, Section 8.4

tensioned members brings specific advantage, when compared to conventionally reinforced alternatives. The following explains.

For illustration, consider the typical case of a continuous multi-span member with equal spans under uniform loading. Using the common linear elastic analysis, the magnitude of the demand moment over the support is likely to be about twice that of midspan (Fig. 4.11.3B-1a). When designing a conventionally reinforced member, this large difference between the moment over the support and that at mid-span can be accommodated by increased rebar over the support ($2A_s$ at support; A_s at midspan). In the post-tensioned counterpart (part c), however, the presence of prestressing tendons, provides a base capacity (part d) for both the positive and negative moments over the entire length of a member. For an optimum design the demand at the support is best matched with the available capacity at the same location (part d). This leads to an over capacity at mid-span that is not utilized in design. The moment re-distribution option allows the demand moment to be lowered at the support and increased at midspan, thus allowing a more economical design, by reducing the difference between the values of the support and span moments.

C. Efficient Redistribution Procedure: The following describes the details of an efficient scheme for code-based redistribution of moments.

For a typical design strip the locations of permissible inelastic deformations are shown in Fig. 4.11.3C-1. According to the ACI 318-11 the negative moments at the end supports are not candidate for redistribution.

Figure 4.11.3C-2 shows a typical interior support. Figure 4.11.3C-2 (b) illustrates the critical regions to the left and right of the support where the negative moments may be adjusted. The heavy line in Fig. 4.11.3C-2a indicates the moments obtained from the elastic solution. The hatched regions above and below the line represent the permissible range for adjustment of elastically computed moments. The adjustments can be either upward or downward. The permissible adjustment range for each section depends on the geometry, and the amount of reinforcement at the same section. Details for the computation of the allowable percentage of re-distribution are in the respective codes.

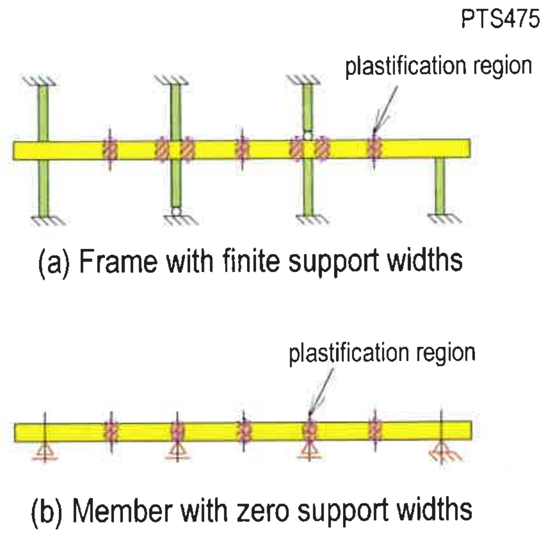


FIGURE 4.11.3C-1 Probable Locations of Post-Elastic Hinge Formations

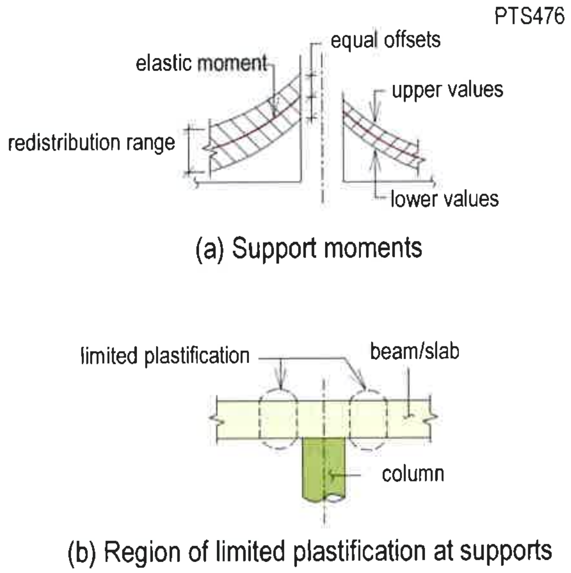


FIGURE 4.11.3C-2 Support Moments and Permissible Range of Moment Redistribution

The final economy of a design, with respect to minimizing the amount of reinforcement, depends on whether or not each design moment is adjusted up or down, and to what extent. Reduction of all negative moments to their respective maximum permissible extents does not necessarily yield the economically optimum design. In most cases the cross-sectional geometry of a member is the same on both sides of a support. Further, negative mild reinforcement required for one side of the support is commonly extended to the other side. Hence, the provided moment capacities of sections on each face of an interior support are likely to be the same, although they are subject to different elastic moment demands. For this reason, the redistribution procedure aims at:

- Readjusting the moments at each side of a support to become either equal, or as close to one another as permissible through the redistribution procedure; and
- Reducing the maximum negative moments at the supports.

Figure 4.11.3C-3 illustrates this concept. Note that the moment at the left of support is reduced by its maximum permissible value. The moment to the right of support is adjusted to be equal to the redistributed value to the left. Different conditions arise, depending on the relative magnitudes of the moments at each face of a support. Once the redistribution over the supports is finalized, the distribution of the moment in span is adjusted on the basis of static equilibrium of the respective span, using the adjusted support moments and the loads on the same span. Two examples of moment redistribution are shown in Fig. 4.11.3C-4.

4.11.4. Design for Strength

Going through the steps outlined in the foregoing sections we conclude with the demand actions at each of the design sections of a floor system. As illustrated in Fig. 4.9.5.5-1, at each design section, the demand for resistance is expressed by way of its six components, namely three moments and three forces acting at the centroid of the section. The design proceeds by establishing that the design section has adequate strength and ductility to resist the demand values. The base strength of a design section is established from the prestressing and non-prestressed reinforcement that crosses the face of the section, and is adequately anchored within the concrete beyond. The position, orientation, and the tendon forces determine the base capacity. Where the base capacity of a design section is not adequate, non-prestressed bonded reinforcement is added. Contrary to the conventionally reinforced concrete, where each design section is "designed," in a post-tensioned section first the "design capacity" is determined. Where the capacity is less than the design moment, reinforcement is added. This rests on the fact that the presence of post-tensioning provides a base capacity that in many locations exceeds the demand.

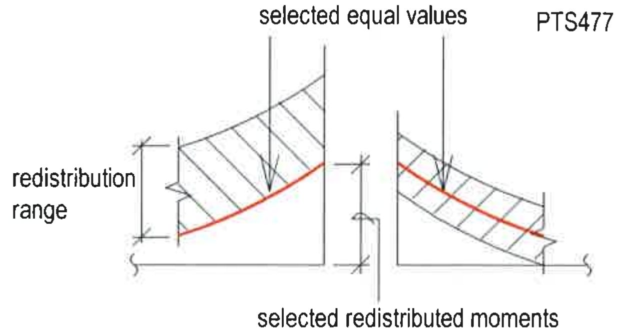


FIGURE 4.11.3C-3 Redistribution Based on Lowest Equal Moments

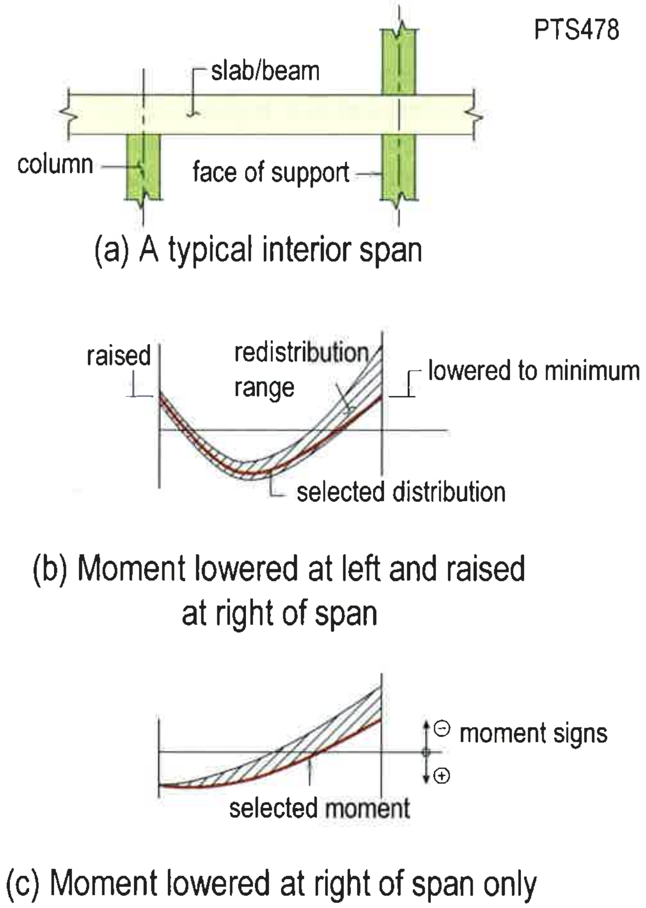


FIGURE 4.11.3C-4 Examples of Moment Redistribution

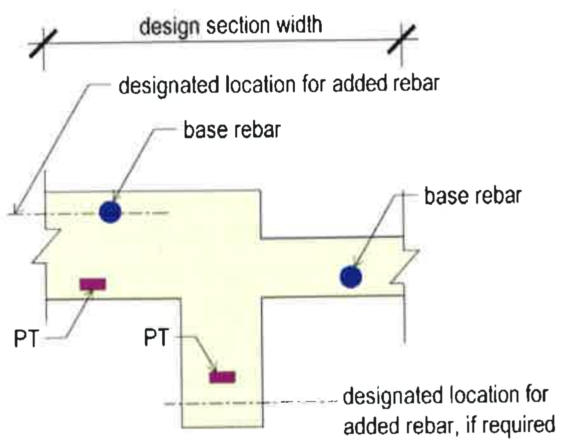


FIGURE 4.11.4A-1 Cross-Sectional Geometry of a Design Section Showing the Base Reinforcement, and Location of Added Rebar

In common building construction a typical floor system is generally deemed to be strong enough to resist the demand for inplane (N_{xy})[79] shear, and moment about the axis normal to the floor's plane (M_{zz}). Engineers do not carry a safety check for these two actions, unless for uncommon configurations and force demand.

The axial force (N_y) and the moment about the design section (M_{xx}) are combined and checked against the axial and moment capacity of a design section. Depending on the detailing of reinforcement, the torsion *Mxy* may also be considered along with the axial force and moment to perform an adequacy check for the combination of the three actions. Lumping the torsion M_{xy} with the other two (bending and axial) actions does not apply to the tendon layout in common practice for post-tensioned floors, nor to common styles of reinforcement layout in conventionally reinforced floors.

Design sections are checked for one-way shear (N_{zz}). In most cases the section is adequate. In transfer plates and the vicinity of planted columns on slabs, where force demand is high, a floor slab may be provided with virtual beams within the thickness of a slab to resist shear. This is explained in detail Chapter 4, Section 4.11.7.

Apart from the strength adequacy of design section, punching shear at column supports is also checked. Punching shear check is explained in Section 4.11.6.

The following describes briefly the basics and steps of design of a section subjected to bending and axial forces.

A. Design Section Features: Refer to Fig. 4.11.4A-1. It shows the cross-section of a design section that typically is constrained on its sides by adjoining sections. In the general case, in addition to post-tensioning tendons, the section can include one or more non-prestressed bars, pre-defined by designer as base reinforcement. The bars and tendons are can be located at arbitrary positions on the cross-section, and can exit the section at different angles. In addition, there are at least two pre-defined locations on the section, one at the top and one at the bottom, where designers add reinforcement, should the capacity of the section falls short of demand actions.

In this Section, we cover the design of the section for the combined actions of the demand moment and axial force expressed at the centroid of the section.

The objective of design is to determine whether the section in its as-is condition has the capacity to resist the demand moment and the axial force. And, if it does not, how much reinforcement needs to be added at the pre-defined locations identified on the section.

B. Design Section Strains: The following deformation assumptions are made. Refer to Fig. 4.11.4C-1.

❖ The displacement of a design section is assumed to follow uniaxial bending, even though the geometry and loads of the section may not be symmetrical about a normal to the floor's plane. The assumption is justified since design sections in floor systems are typically restrained on their sides by adjacent sections. The side restraint forces a design section to deform normal to the plane of the floor. This justifies the use of uniaxial relationships.

❖ The distribution of strain on the section is linear. To satisfy the design requirements, the following additional limitations are imposed on the assumed linear strain distribution.

[79] See Fig 4.6.1C-1 for sign convention

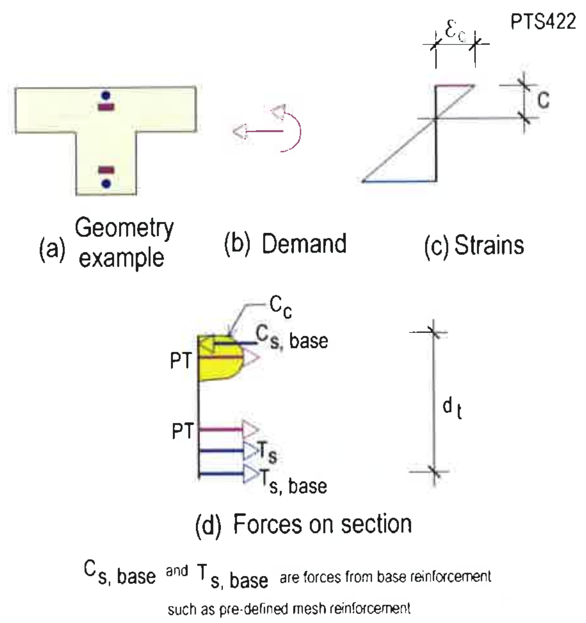


FIGURE 4.11.4C-1 Design Section Geometry, Reinforcement, Strains and Forces

- At strength limit, the strain in concrete at the compression fiber is the value stated in the respective code (ACI 318-11 ε_c = 0.003; EC2 ε_c = 0.0035).
- The depth of the neutral axis does not exceed a code specified value. (ACI 318 cmax <= 0.375 d_t; EC2 $c_{max} = 0.5h$, for $f_{ck} \leq 50\,\mathrm{MPa}$ and $c_{max} = (1 - \varepsilon_{c3} / \varepsilon_{cu3})h$ for $f_{ck} > 50$ MPa)[80] The strain in the reinforcement follows that of concrete at the location of reinforcement, except for unbonded tendons.

C. Design Section Forces: There will be a force distribution at the section (Fig. 4.11.4C-1d) associated with each strain distribution. In arriving at the forces the following assumptions are made:

❖ Force in concrete and the non-prestressed reinforcement follows the respective stress-strain curve of each material with the following modifications:
  - Concrete will resist compressive stresses only.
  - The stress in each bar is determined from the associated stress-strain curve. Since the bars are generally not normal to the section, the contributing resisting force is taken to be the component of the force normal to the section.
  - Using ACI 318-11, the forces considered in the force diagram (part d) are those directly derived from the associated stress-strain curves of the respective material with no reduction. The reliability of the capacity calculated on the basis of the forces shown is handled through the reduction of the computed capacity (nominal capacity) by a strength reduction factor ( ) to arrive at "design capacity."
  - Using EC2, the forces arrived at from the stress-strain curve are each reduced by the respective material factor (ϒ) to account for the reliability of the assumptions made for the characteristic material properties.

The forces on the section shown in part (d) of the figure are;

C_c = Compressive force of concrete;
F_{ps} = tensile force in prestressing tendon in tension zone at ULS of the section;
F'_{ps} = tensile force in prestressing tendon in compression zone at ULS of the section;
$T_{s,base}$ = tensile force in base reinforcement in tension zone;
$C_{s,base}$ = compressive force in base reinforcement in compression zone;
$T_{s,added}$ = tension force in tensile reinforcement added to meet demand values; and
$C_{s,added}$ = compressive force in reinforcement added to meet demand values.

❖ Forces from prestressing tendons depend on whether a tendon is bonded, or unbonded, and the extent of losses in prestressing force at the time the section is being evaluated.

4.11.5 Safety Against Cracking Moment

ACI 318-11[81] /IBC require that for members reinforced with bonded tendons the total amount of prestressed and nonprestressed reinforcement be adequate to resist a factored load at least 1.2 times the cracking load computed on the basis of the modulus of rupture of the section. In practice, this is viewed as the nominal moment capacity (M_n) of a section not to be less than 1.2 times its cracking moment (M_{cr}). Due to precompression

[80] ε_{c3} and ε_{cu3} can be read from Table 3.1 of EC2 (EN 1992-1-1:2004(E)); See also section on notations.
[81] ACI 318-11Section 18.8.2

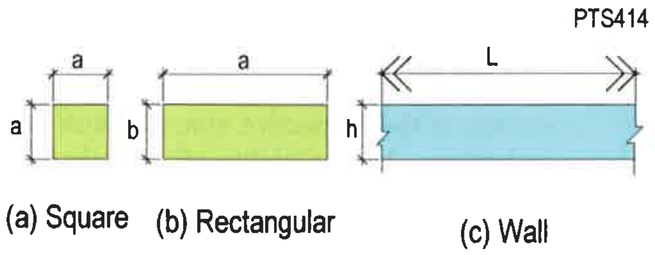


FIGURE 4.11.6.1-1 Selected Support Geometries

provided by prestressing, the cracking moments of post-tensioned sections are higher than conventionally reinforced members with the same amount of reinforcement. Once cracking initiates, the code requires that the available reinforcement be adequate to avoid rupture of the section. This requirement is waved for members reinforced with unbonded tendons, since the local demand on strain in tendon at location of cracking distributes over a longer length of unbonded tendons, thus avoiding over-straining of tendon to rupture at the critical section.

For prestressed members:

$$M_{cr} = \left(f_r + P/A\right)\times S \qquad \text{(Exp 4.11.5-1)}$$

Where, f_r is the modulus of rupture defined[82]

$$f_r = 7.5\sqrt{f'_c} \quad \text{(psi)} \qquad \text{(Exp 4.11.5-2)}$$

$$f_r = 0.62\sqrt{f'_c} \quad \text{(MPa)} \qquad \text{(Exp 4.11.5-2)}$$

Where, *P/A* is the average precompression, and *S* is the section modulus.

EC2 has a similar requirement for beams only[83] reinforced with unbonded tendons. Namely, the design capacity of a post-tensioned beam shall not be less than 1.15 times its cracking moment. The computation of the values is the same as given for ACI 318 in the preceding, with the difference that the cracking stress is expressed as follows:

$$f_{cr} = f_{ctm} = 0.3 f_c'^{2/3} \quad \text{(MPa)} \qquad \text{(Exp 4.11.5-3)}$$

Where f_{ctm} is the cracking stress.

4.11.6 Punching Shear

4.11.6.1 One Way and Two-Way Shear: Tests have shown that concrete slabs exhibit greater resistance intensity to shear, where the geometry

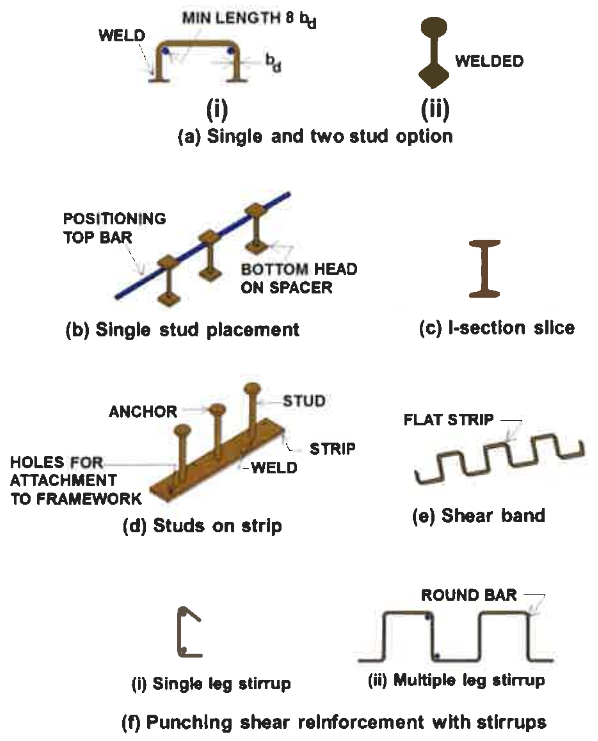


FIGURE 4.11.6.1-2 Common Shear Reinforcement Options (P588)

of a rectangular support is "square," and of about the same dimensions as slab thickness (Fig. 4.11.6.1-1a). As the support geometry becomes rectangular, with increase of the aspect ratio a/b, the unit resistance capacity[84] of concrete drops. At its limit, when a "column" support turns into a "wall," the intensity of slab's shear capacity is least, where the capacity is governed by the well-established one-way slab/beam shear. At its highest value, ACI 318-11 stipulates the concrete capacity for a square support to be twice that of the other extreme, namely elongated support or wall.

Under the designation of "punching shear," major building codes permit the designers to take advantage of the greater concrete capacity that square and near square supports provide, while leaving the one-way shear for longer supports, and where punching shear does not apply.

[82] ACI318-11, Section 9.5.2.3
[83] EC2 (EN 1992-1-1:2004 (E)), Section 9.2.1.1(4)
[84] Unit capacity is defined as strength per unit area of an assumed failure surface

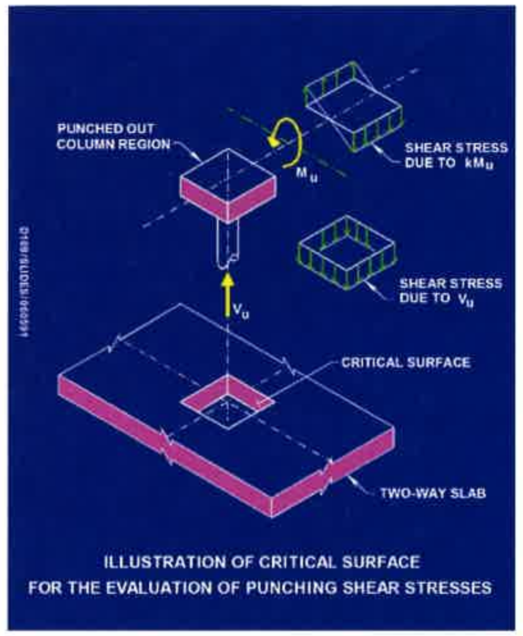


FIGURE 4.11.6.2-1 Assumed Distribution of Stress on a Notional Failure Surface (P587)

The design capacity of reinforcement commonly used for resisting shear (Fig. 4.11.6.1-2) remains unchanged, regardless of the support geometry. The capacity provided by each reinforcement is simply its cross-sectional area, times its characteristic material strength, reduced by a code-specified material or strength reduction factor. The essential difference between "punching" shear and "one-way" shear lies in the greater capacity that concrete provides.

The response of a slab in punching shear is multi-dimensional. It is governed by the features of the geometry, and reinforcement layout of the joint. The response, while understood, is considered too complex to be cast in a simplified form for use in everyday work of consulting engineers. For routine and everyday design, major building codes offer simplified alternatives, based on empirical formulas that are corroborated by test results.

A design that is based on the simple empirical formulas is deemed to meet or exceed the required capacity — thus, it is safe. One alternative to the application of the empirical formulas is the use of "strut-and-tie method," as suggested in ACI 318-11. Another alternative is to simply forego the advantages of higher concrete capacity for punching shear, and design the location using one-way shear.

4.11.6.2 – Empirical Design Model : The analytical model suggested by major building codes for the empirical formulas assumes a notional failure surface at a given distance from the face of support, along with a distribution of stress on the assumed surface, such as the one shown in Fig. 4.11.6.2-1, adopted by ACI 318[85]. EC2 suggests similar critical surfaces, but with rounded corners[86].

For the empirical formulas, (i) Using statics, the ordinates of the assumed stress distribution (Fig. 4.11.6.2-1) are calculated to bring the assumed distribution of stress in static equilibrium with the design shear; and (ii) using the results of extensive tests, the allowable values for the lead ordinates of the assumed stress distribution are calibrated to render the outcome of computations on the notional distribution to be safe. The safe application of the empirical punching shear formulas rests strictly on the support configurations, for which test results have validated the empirical models.

4.11.6.3 Generalization of Punching Shear Formulas: Design for punching shear is a "safety" requirement. A safe design can be achieved through the proper application of the established

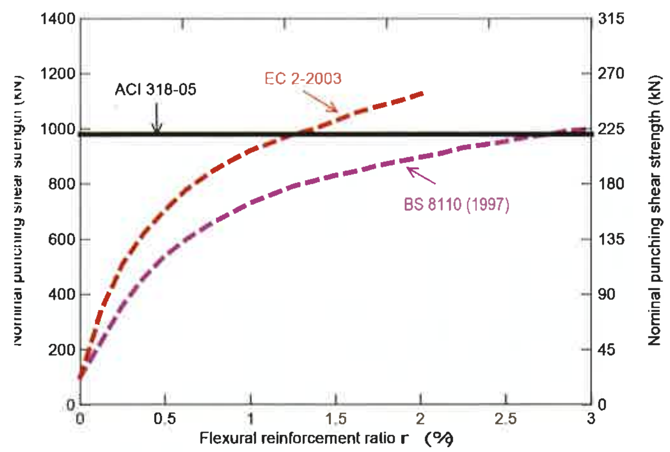


FIGURE 4.11.6.3-2 Nominal Punching Shear Strength in Relation to the Amount of Top Reinforcement[87] (PTS418)

[85] ACI 318-1; Section 11.11
[86] EC2 (EN 1992-1-1:2004E), Section 6.4
[87] The graph is derived for a 24" (610 mm) square column; 9" (230 mm) slab thickness with 7" (180 mm) effective depth; 4000 psi (28 MPa) concrete strength (28 day cylinder); and no punching shear reinforcement.

laws of mechanics, or reliance on test results for similar conditions. The punching shear design, when using the empirical formulas, falls in the latter category.

There are two central prerequisites for a safe design. These are:

(i) Selection of a stress (or generalized force) distribution that is in static equilibrium with the applied loads, coupled with availability of adequate resistance in the structure to meet the selected stress/force distribution.

(ii) The structure designed shall have, or be provided with, adequate ductility to deform on demand prior to failure to mobilize the resistance provided in the above designated distribution of stress (or generalized force).

The punching shear stress distribution adopted for the empirical formulas meets the first criterion, namely "static equilibrium," but does not satisfy the second condition. However, it still is a valid model, since its application is calibrated through test results of similar conditions.

It is well known among structural engineers that the failure mechanism in punching shear is in the form depicted in Fig. 4.11.6.3-1, where compression struts, in balance with the top reinforcement resist the applied shear V_u.

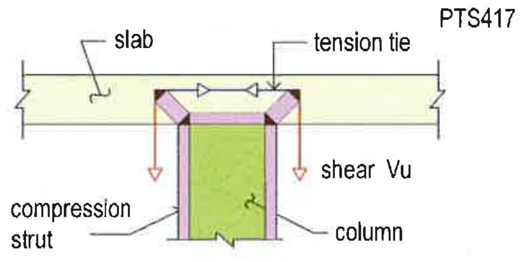
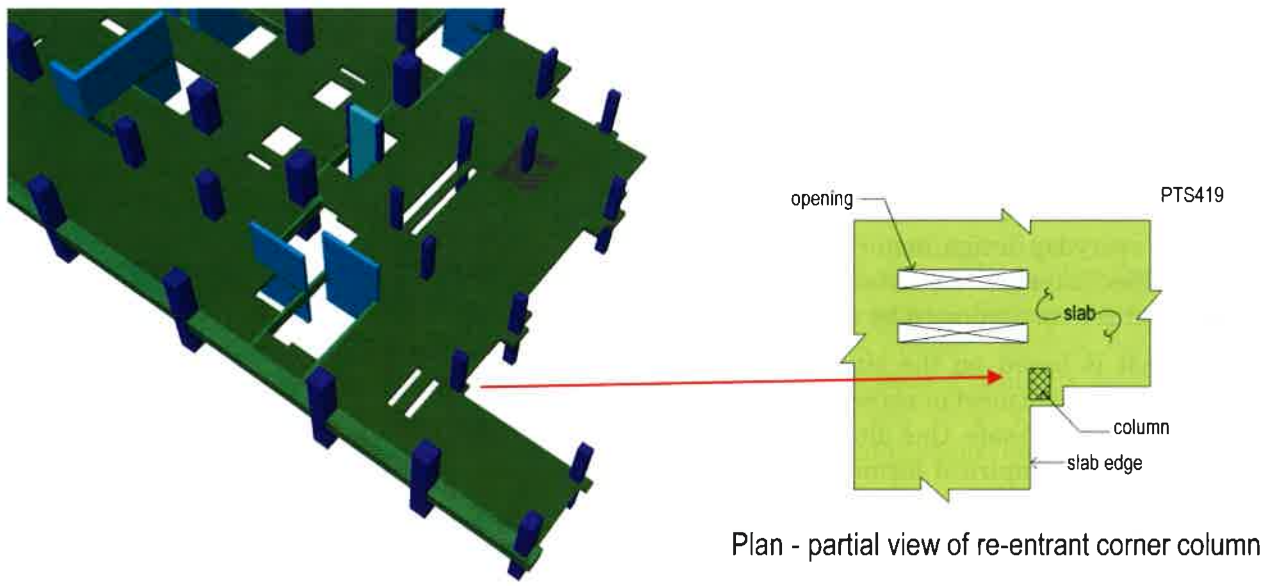


FIGURE 4.11.6.3-1 Punching Shear Resisting Mechanism

This mechanism, can meet both the safety prerequisites, namely (i) the forces in the struts and the tension in top bars can be calculated to be in static equilibrium with the applied force, and (ii) the mechanism shown in the figure is physically plausible, subsequent to cracking of concrete – the mechanism can develop.

Based on the foregoing, it is a mistake to make a logical argument, and extend the empirical formulas of the code for punching shear design to scenarios for which test results are either not available, or not used.

Figure 4.11.6.3-2 illustrates the significant differences in the determination of punching shear capacity of square column support among three building codes [Widianto et al, 2009].



(a) Partial 3D view of floor (P395)

(b) Plan – details of support

FIGURE 4.11.6.3-3 Example of a Floor System with Non-Standard Punching Shear Configurations (Jabal Omar, KSA)

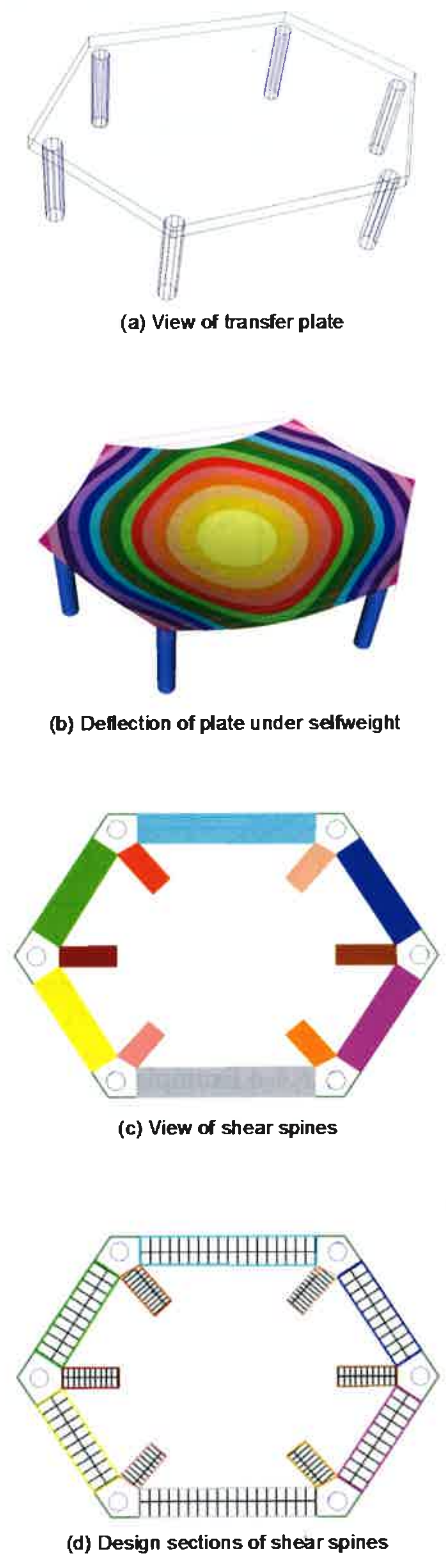
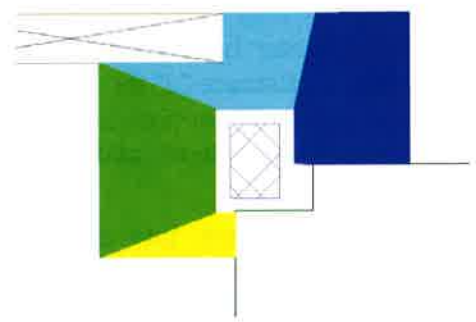


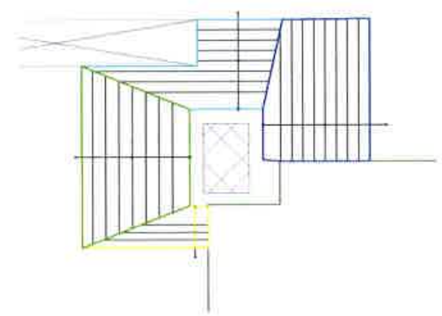
FIGURE 4.11.6.4-1. Floor System with One-Way Shear Spines (P589; P590;P591;P592)

As an example consider the partial view of an actual floor slab shown in Fig, 4.11.6.3-3.[88] Several of the column supports on the boundary, such as the one identified in part (b) of the figure, do not conform to the standard geometries defined in the code. These columns do not match the configurations for which test-based empirical formulas are suggested in the codes. The simplification of the geometry of the support to shapes that do not closely match the building codes extends the application of the empirical formulas beyond the range of their validity.

For irregular configurations that are not validated by test results, short of embarking on detailed analyses, such as strut-and-tie models, the alternative is to use the one-way shear approach, and forego the added concrete capacity that is available at the location on account of its multi-directional geometry. The attempts made in the literature to develop extended relationships for irregular shapes based on assumed none-



(a) Breakdown of the support vicinity into shear segments (P398)



(b) Illustration of design sections associated with each shear segment (P399)

FIGURE 4.11.6.4-2 Selection of Shear Segments Adjacent to Support

[88] Jabal Omar transfer plate, KSA

FIGURE 4.11.6.4-5 Reinforcement Installation of a Transfer Plate (Freyssinet HK; P705)

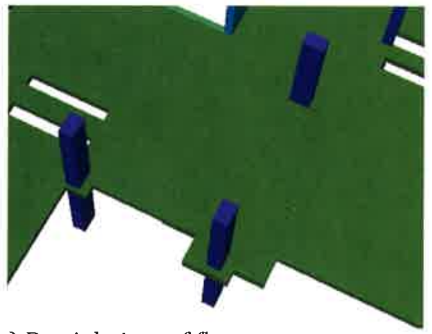
standard critical surfaces that cannot "develop at failure" and are not supported by test results are questionable, in particular when the necessity of top reinforcement is not considered.

4.11.6.4 One-Way Shear in Slab Design: In applying the one-way shear alternative to design of support locations the following criteria can be used.

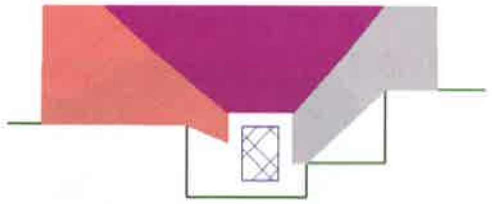
(i) Select a perimeter beyond the boundary of column support at distance "*d*" for conventionally reinforced slabs, or "*h/2*" for post-tensioned slabs, where "*d*" is the effective depth, and "*h*" is the slab thickness; and

(ii) break the perimeter into segments and associate with each segment an adequate region of the slab that can capture the entire shear flow into the support's extended perimeter. The slab segments can be in form of spines, as shown in Fig. 4.11.6.4-1 or wedge-shaped as illustrated in Fig. 4.11.6.4-2 for the non-standard column identified in Fig. 4.11.6.3-3. The spines or the wedges shall extend far enough from the face of support to where concrete section can resist the demand shear without reinforcement. In Fig. 4.11.6.4-1, on account of high perimeter loads, the spines are extended along the entire length of the perimeter.

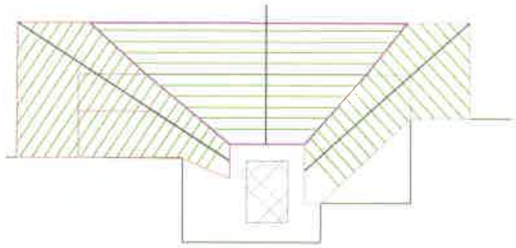
Figure 4.11.6.4-2 illustrates how the non-standard column shown in Fig. 4.11.6.3-3b may be analyzed using one-way slab shear. Starting at a code-specified distance from the face of the support, the region around the column is subdivided into fanned segments. Design sections are considered as illustrated in part (b) of the figure. For each design section the total one-way shear is calculated. Each design section is then evaluated against



(a) Partial view of floor (P670)



(b) Subdivision of support region in vicinity of column (P400)



(c) Identification of support lines and design strips (P401)

FIGURE 4.11.6.4-4 Example of an Irregular Perimeter Support and the Model for its Shear Design

the calculated one-way shear. One-way shear reinforcement is provided, where the computed capacity of a design section is not adequate.

The calculated reinforcement is generally provided by shear links, shear studs, or other alternatives shown in Fig. 4.11.6.1-2.

Another example of a non-standard perimeter support column and structural model for its safe shear transfer is shown in Fig. 4-11.6.4-4.

Figure 4.11.6.4-5 illustrates one example of a transfer plate, where the shear design and strength are provided through virtual beams within the depth of the slab.

4.11.6.5 Amount of Shear Reinforcement: The shear reinforcement generally consists of vertical bars, or shear studs, placed along each spine and across its width. The vertical bars may be encased in a U stirrup and closed at the top for thick slabs and transfer plates. For thin slabs, the bars are not encased (Fig. 4.11.6.5-1). The construction of such a cage within the body of a concrete slab is referred to as "virtual beam." The practice is more common for transfer plates, where slabs are thick and design shear values are generally high.

The mechanism for shear transfer through the vertical bars is illustrated in Fig. 4.11.6.5-2. The following conservative assumptions are made for an expeditious computation of shear capacity; (i) if the slab is post-tensioned, disregard the beneficial contribution of the vertical component of the tendons near the supports; (ii) also, disregard the contribution of precompression provided by post-tensioning; finally disregard the contribution of concrete in resisting shear.

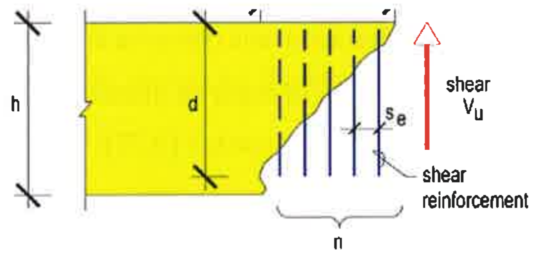


FIGURE 4.11.6.5-2 Partial View Illustrating the Shear Components

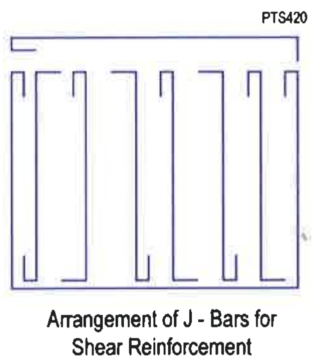


FIGURE 4.11.6.5-1 Arrangement of Vertical Reinforcement across the Width of a Shear Spine

The parameters for shear transfer through reinforcement are:

V_u = design shear (kN; k);
s_l = spacing of shear bars in direction of spine;
s_t = spacing of shear bars in transverse direction;
A_v = cross-sectional area of each bar;
f_y = yield stress of each bar;
γ = material factor, where applicable (non-US building codes);
n = number of bars across the assumed fracture surface; and
ϕ = strength reduction factor for US codes

$$\text{Shear capacity} = \frac{n\ A_v f_y}{\gamma} \qquad \text{(Exp 4.11.6.5-1) SI}$$

$$= \phi\left(n\ A_v f_y\right) \qquad \text{(Exp 44.11.6.5-1) US}$$

For one square unit of floor plan, the shear capacity will be:

$$= \left(\frac{A_v f_y}{\gamma}\right)\left(\frac{d}{s_l}\right)\left(\frac{1000}{s_t}\right) \qquad \text{(Exp 4.11.6.5-3) SI}$$

$$= \phi\left(A_v f_y\right)\left(\frac{d}{s_l}\right)\left(\frac{1000}{s_t}\right) \qquad \text{(Exp 4.11.6.5-4) US}$$

4.11.6.6 Openings and Punching Shear: Openings near the column supports, for mechanical or other installations, such as plumbing are inevitable. An opening, such as the one shown in Fig. 4.11.6.6-1 reduces the punching shear capacity of a column support. At the design stage, the precise location and size of the mechanical openings are not always known. Further, it is not unusual to change the position and size of a mechanical opening, at construction, subsequent to the structural design.

There are two options to account for the loss of strength from provision of an opening near a column support.

One option is to follow the code formula, whereby the punching shear design perimeters assumed around a column are adjusted to account for the opening. A view of such an adjustment is shown in Fig. 4.11.6.6-2a. In this option, the empirical code formula suggested for the design of the punching shear is modified to account for the loss of resisting surface caused by the opening.

The second option is to conclude the shear design of the region around the column, as if the region is solid (no opening). Next, add reinforcement around the sides of the opening in the amount necessary to compensate for the loss of design shear capacity resulting from the opening (Fig. 4.11.6.6-2b). Standard details and reinforcement tables can be provided on the construction drawings to account for the punching shear reinforcement on the sides of openings, depending on the size, concrete strength and the member thickness.

Borges et al [Borges, et al, 2013] have also included the option of added reinforcement around the openings in their experimental work. They have concluded that added shear reinforcement on the sides of an opening improves the shear capacity, but have not recommended a design proposal for the amount of reinforcement.

FIGURE 4.11.6.6-1 Opening Near a Column Support (P637)

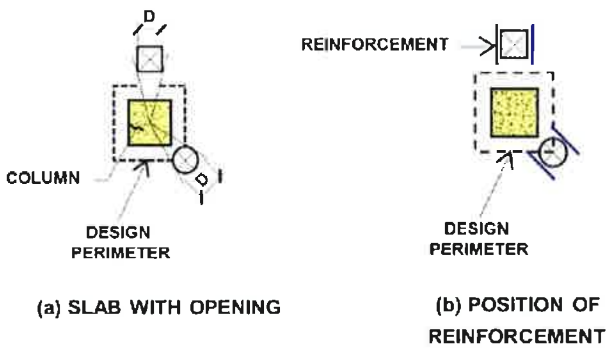


FIGURE 4.11.6.6-2 Options for Treatment of Openings (P671)

The following explains the treatment of openings, using the option for adding reinforcement.

A. ACI 318 : Conservatively, assume the maximum permissible punching shear stress applies for the loss of shear transfer through the opening.

The area of shear reinforcement needed is:

$$A_v = \frac{4dD\sqrt{f_c'}}{\phi f_y} \quad \text{(lb, in)} \qquad \text{(Exp 4.11.6.6A-1)}$$

$$A_v = \frac{0.33dD\sqrt{f_c'}}{\phi f_y} \quad \text{(N,mm)} \qquad \text{(Exp 4.11.6.6A-2)}$$

Where,

A_v = area of shear reinforcement;
D = diameter of the opening;
d = effective depth of the section;
f_c' = 28 day characteristic concrete strength;
f_y = yield strength of shear reinforcement; and
ϕ = strength reduction factor (0.75).

B. EC2: The loss of design shear capacity associated with an opening near a support, and the equivalent reinforcement to compensate for the loss can be conservatively estimated as follows:

$$A_v = \frac{(v_{min} + k_1\sigma_{cp})dD}{f_{ywd,ef}} \qquad \text{(Exp 4.11.6.6B-1)}$$

Where, the portion of shear that concrete is allocated to resist is

$$v_{min} = 0.035k^{3/2}f_{ck}^{1/2} \qquad \text{(Exp 4.11.6.6B-2)}$$

$$k = \left(1 + \sqrt{\frac{200}{d}}\right) \le 2 \qquad \text{(Exp 4.11.6.6B-3)}$$

Conservatively, assume k=2

f_{ck} = 28 day characteristic concrete strength (MPa);

k_1 = 0.1;

$$\sigma_{cp} = \frac{N_{ED}}{A} \quad \text{(MPa)} \qquad \text{(Exp 4.11.6.6B-4)}$$

Disregard the contribution from precompression σ_{cp} since precompression will be diverted to away from the opening.

$$f_{ywd,ef} = 250 + 0.25d \le f_{ywd} \quad \text{(MPa)}$$

The compensating reinforcement simplifies to:

$$A_v = \frac{(0.035 \times k^{3/2} \times f_{ck}^{1/2})dD}{250} = 0.0004 f_{ck}^{1/2} dD$$

C. Example:
Given:
Slab thickness: 8 in. (200 mm)
Effective depth; 6.5 in. (165 mm)
Concrete strength: 5000 psi (34.47 MPa)
Opening diameter: 4 in. (100 mm)
Yield stress of shear reinforcement: 60 ksi (413.69 MPa)

Required:
Reinforcement design for the opening

Using ACI 318

$$A_v = \frac{4 \times 6.5 \times 4\sqrt{5000}}{0.75 \times 60000} = 0.16\ in^2 \ (103\ mm^2)$$

Use 1#4 (13 mm) bar on each side of the opening. A_v = 0.2 in² > 0.16 in² (103 mm²) OK
Provide an added bar to make it 2 #4, one on each side of the opening.

Using EC2

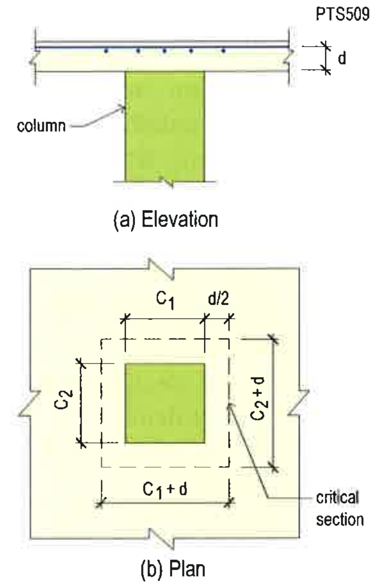
$A_v = 0.0004 f_{ck}^{1/2} dD$

$A_v = 0.0004 \times 34.47^{1/2} \times 165 \times 200 = 77\ mm^2$

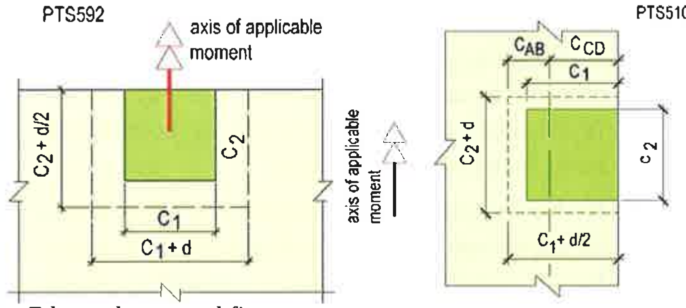
Use 1 -12 mm bar

A_s = 1 × 113 = 113 mm² > 77 mm² OK

4.11.6.7 ACI 318 Punching Shear Design: The punching shear response of a floor system and concept of its design, along with different design options were discussed in the first part of this Section. In the following, we review the prescriptive steps of the code for punching shear evaluation and design, followed by a numerical example. It is reiterated that the following procedure is empirical; calibrated with tests and deemed to produce safe designs. As such, the steps need not conform to the mechanics of solids.



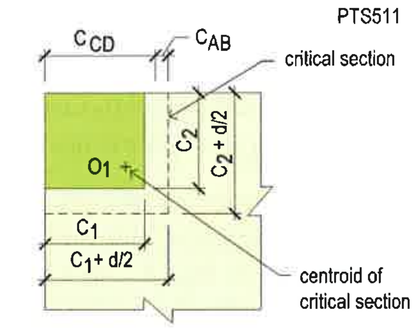
(a) Interior column



Edge column and first critical perimeter

End column and first critical perimeter

(b) End column and edge column; axis of applicable moment



(c) Corner column; first critical perimeter

FIGURE 4.11.6.7A Column Support Configuration Classifications

The calculation steps for punching shear design are:

- Determine the factored column moment (design moment M_u) and the factored shear (design shear V_u). In many instances, column reaction is conservatively used as design value for punching shear.
- Consider a fraction of the unbalanced moment (γM_u) to contribute to the punching shear demand. The unbalanced moment is conservatively taken as the difference between the upper and lower column moments at a joint.
- Using the code relationships, select an assumed failure surface ("critical section," also referred to as "design section") around the column and calculate a hypothetical maximum punching shear stress demand for the assumed surface.
- Using the proximity of the column to slab boundary, and the availability of precompression from prestressing calculate an "allowable" intensity of resistance of concrete to punching shear .
- If the calculated punching shear demand stress does not exceed the allowable resistance value, the section is considered safe. Terminate the design.
- If the hypothetical punching shear demand stress exceeds the allowable resistance value by a moderate amount, punching shear reinforcement may be provided to bring the combined contributions of concrete and added reinforcement to resist the demand.
- If the hypothetical punching shear stress demand exceeds the code stipulated maximum threshold, modify the parameters of design, such as column dimension, slab thickness, or material properties to bring the demand within the allowable range.

A. Geometry Parameters of Design: The code differentiates among interior, exterior and edge columns as outlined in Fig. 4.11.6.7A. In each instance, the figure also illustrates the first design perimeter from the face of the column. Subsequent design perimeters, if required, will each be taken a distance *"0.5d"* from the previous one.

Where the distance between the outward farce of a column and the slab edge exceeds 4d, the location is considered as edge/end condition.

B. ACI 318-11 Punching Shear Design Flow Chart. The following flow chart navigates through the punching shear check and design of the code, including the provision of punching shear reinforcement. The numbers next to each of the flow chart items are described following the flow chart itself.

The basic relationship for punching shear stress demand is:

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v \times M_u \times c}{J_c} \qquad \text{(Exp 4.11.6.7B-1)}$$

Where,

V_u = absolute value of the direct shear;
M_u = unbalanced column moment;
A_c = area of concrete of assumed critical section;
γ_v = fraction of the moment transferred by shear;
c = distance from centroidal axis of critical section to the perimeter of the critical section in the direction of analysis; and
J_c = property of assumed critical section analogous to polar moment of inertia of segments forming area A_c.

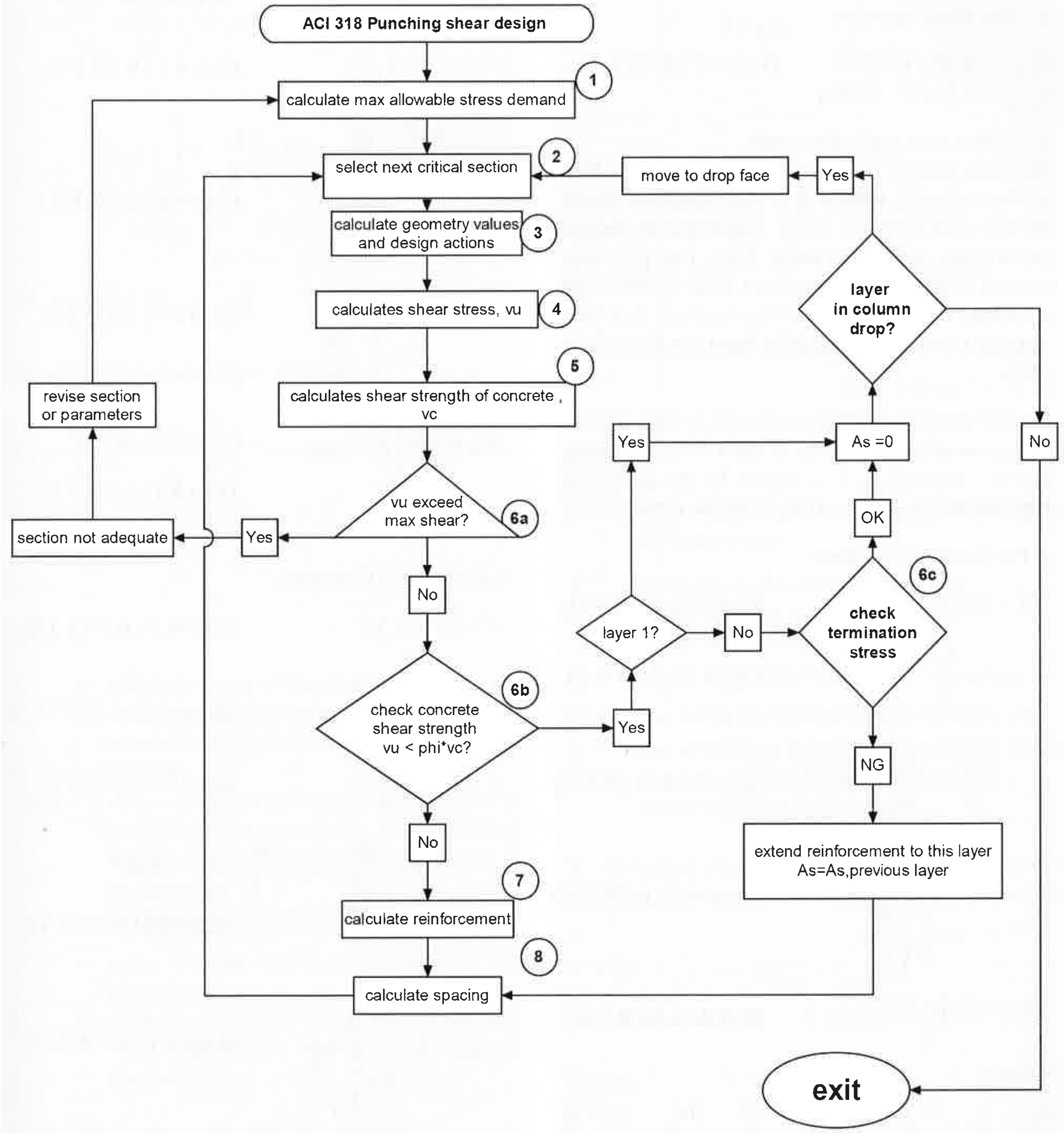
It is recognized that in the inherently biaxial response of a floor slab, there will be moments in both major directions. However, the above empirical relationship proposed in ACI 318 is calibrated to deliver safe designs by considering the moment of each direction separately, when calculating the design stress v_u. In other words, two separate computations are performed, one for each direction. The two stresses are not added.

The first critical shear failure plane is assumed at a distance *d/2* from the face of support. Where *"d"* is the effective depth of the section.

The following numbered parts refer to the items of the ACI punching shear flow chart.

1 - Maximum shear strength $v_{n,max}$
This is the maximum hypothetical demand shear stress that is allowed for, when using the code formula. Its value depends on whether the design section is going to be reinforced by shear studs, or stirrups.

ACI 318-11 Punching Shear Check and Design Flow Chart F122



a - For shear studs

$v_{n,\,max} = 8\sqrt{f'_c}$ (psi)

$v_{n,\,max} = 0.67\sqrt{f'_c}$ (MPa) (Exp 4.11.6.7B.1-1)

b - For shear stirrups

$v_{n,\,max} = 6\sqrt{f'_c}$ (psi) (Exp 4.11.6.7B.1-2)

$v_{n,\,max} = 0.50\sqrt{f'_c}$ (MPa)

2 - Select next critical section

The first design perimeter is *0.5d* from the face of the column, where d is the effective depth of the slab/column drop. Subsequent design perimeters are 0.5d away from the previous critical section. If the support is provided with a column drop, the first critical section that falls outside the drop will be 0.5d from the face of the drop.

3 - Calculate geometry values and design actions

The geometry properties of each critical section are its centroid, A_c, J_c, c, and γ. In the following expressions, b_1, b_2 are critical section dimensions.

a. For Interior Columns:

$$A = 2d\left(b_1 + b_2\right) \quad \text{(Exp 4.11.6.7B.3-1)}$$

$$c = c' = \frac{b_1}{2} \quad \text{(Exp 4.11.6.7B.3-2)}$$

$$J_c = \frac{b_1 d^3}{6} + \frac{d b_1^3}{6} + \frac{b_1^2 b_2 d}{2} \quad \text{(Exp 4.11.6.7B.3-3)}$$

$$\gamma_v = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} \quad \text{(Exp 4.11.6.7B.3-4)}$$

$$M_u = abs\left[\gamma_v M_{unbalanced}\right] \quad \text{(Exp 4.11.6.7B.3-5)}$$

Where,

b_1 = dimension of the critical section normal to the axis of column moment selected for consideration;

b_2 = dimension of the critical section normal to b_1; and

$M_{unbalanced}$ = is the fraction of factored column moment parallel to direction of b_2.

b. For end Columns:

$$A = \left(2b_1 + b_2\right)d \quad \text{(Exp 4.11.6.7B.3-6)}$$

$$c = c_{AB} = \frac{b_1^2}{2b_1 + b_2} \quad \text{(Exp 4.11.6.7B.3-7)}$$

$$c' = c_{CD} = b_1 - c \quad \text{(Exp 4.11.6.7B.3-8)}$$

$$J_c = \frac{b_1 d^3}{6} + \frac{d b_1^3}{6} + 2b_1 d\left(\frac{b_1}{2} - c\right)^2 + b_2 d c^2$$
(Exp 4.11.6.7B.3-9)

$$\gamma_v = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} \quad \text{(Exp 4.11.6.7B.3-4)}$$

$$M_u = abs\left[\gamma_v M_{unbalanced} - V_u\left(b_1 - c - \frac{c_1}{2}\right)\right]$$
(Exp 4.11.6.7B.3-10)

c. For corner Columns:

$$A = \left(b_1 + b_2\right)d \quad \text{(Exp 4.11.6.7B.3-11)}$$

$$c = c_{AB} = \frac{b_1^2}{2\left(b_1 + b_2\right)} \quad \text{(Exp 4.11.6.7B.3-12)}$$

$$c' = c_{CD} = b_1 - c \quad \text{(Exp 4.11.6.7B.3-8)}$$

$$J_c = \frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + b_1 d\left(\frac{b_1}{2} - c\right)^2 + b_2 d c^2$$
(Exp 4.11.6.7B.3-13)

$$\gamma_v = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} \quad \text{(Exp 4.11.6.7B.3-4)}$$

$$M_u = abs\left[\gamma_v M_{unbalanced} - V_u\left(b_1 - c - \frac{c_1}{2}\right)\right]$$
(Exp 4.11.6.7B.3-10)

d.For edge Columns

b_1 is perpendicular to axis of moment

$$A = \left(b_1 + 2b_2\right)d \quad \text{(Exp 4.11.6.7B.3-14)}$$

$$c = c' = \frac{b_1}{2} \quad \text{(Exp 4.11.6.7B.3-2)}$$

$$J_c = \frac{b_1 d^3}{12} + \frac{d b_1^3}{12} + 2b_2 d c^2 \quad \text{(Exp 4.11.6.7B.3-15)}$$

$$\gamma_v = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} \quad \text{(Exp 4.11.6.7B.3-4)}$$

$$M_u = abs\left[\gamma_v M_{unbalanced}\right] \quad \text{(Exp 4.11.6.7B.3-5)}$$

4- Calculate demand shear stress v_u

$$v_{u1} = \frac{V_u}{A_c} + \frac{\gamma_v \times M_u \times c}{J_c} \quad \text{(Exp 4.11.6.7B.4-1)}$$

$$v_{u2} = \frac{V_u}{A_c} - \frac{\gamma_v \times M_u \times c'}{J_c} \quad \text{(Exp 4.11.6.7B.4-2)}$$

Where,

V_u = absolute value of the direct shear;

M_u = absolute value of the unbalanced column moment about the centroid of the critical section;

c and c' = distances from centroidal axis of critical section to the perimeter of the critical section in direction normal to the axis of moment M_u ;

A = surface area of critical section;

γ_v = ratio of the moment transferred by shear; and

J_c = geometry constant of the critical section with respect to the axis of applied moment.

5 - Calculate shear strength of concrete v_c

❖ For RC members and prestressed members where outward face of a column is less than 4h from a slab edge, where h is the thickness of the slab:

$$v_c = \min\begin{cases}\left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f'_c} \\ \left(2 + \alpha_s\frac{d}{b_0}\right)\lambda\sqrt{f'_c} \\ 4\lambda\sqrt{f'_c}\end{cases} \quad \text{(lb in)}$$

(Exp.4.11.6.7B.5-1) US

$$v_c = \min\begin{cases}0.083\left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f'_c} \\ 0.083\left(2 + \alpha_s\frac{d}{b_0}\right)\lambda\sqrt{f'_c} \\ 0.083 \times 4\lambda\sqrt{f'_c}\end{cases} \quad \text{(MPa)}$$

(Exp.4.11.6.7B.5-1) SI

Where,

β = ratio of the larger to the smaller side of the critical section;

f'_c = strength of the concrete cylinder;

α_s = 40 for interior columns;
= 30 for edge and end columns;
= 20 for corner columns;

b_0 = the perimeter of the critical section; and

λ = a modification factor, 1.0 for normal weight concrete; 0.85 for sand lightweight; and 0.75 for all lightweight.

❖ For prestressed members where outward face of a column is more than 4h from a slab edge:

$$v_c = \left(\beta_p \lambda\sqrt{f'_c} + 0.3 f_{pc}\right) + V_p / b_0 d$$

(Exp.4.11.6.7B.5-3)

Where,

β_p = the smaller of 3.5 and $\left(\alpha_s d / b_0 + 1.5\right)$; (US)
= the smaller of 0.29 and $0.083\left(\alpha_s d / b_0 + 1.5\right)$ (SI)

V_p = the factored vertical component of all prestressing forces crossing the critical section. Conservatively it is often assumed as zero; and

$\sqrt{f'_c} \le$ 70 psi (5.8 MPa)

6 - Stress checks steps to validate adequacy of force transfer
Stress check is continued for one critical section after the other, until a critical section that does not require reinforcement is reached — when the computation will be terminated. If the support is provided with a column drop, at completion of the check within the drop, the stress check is continued beyond the face of the drop until the design is satisfied.

6a - Check demand stress against maximum allowable shear for the support

If $v_u > \varphi_v v_{n,max}$, demand for punching stress is excessive; the section is not adequate. Revert to alternative measures.

6b - Check adequacy of concrete alone to resist demand shear

If $v_u < \varphi_v v_c$, no punching shear reinforcement is required for this layer.

If $\varphi_v v_{n,max} > v_u > \varphi_v v_c$ provide punching shear reinforcement.

6c - Check termination stress

If the hypothetical demand shear stress (v_u) at a critical section is less than a code specified minimum value, the connection is deemed adequate and the punching shear design will terminate.

If $v_u \le \varphi_v 2\lambda\sqrt{f_c'}$ (lb,in); $v_u \le 0.083\varphi_v 2\lambda\sqrt{f_c'}$ (N,mm), no punching shear reinforcement is needed.

If $v_u > \varphi_v 2\lambda\sqrt{f_c'}$ (lb,in); $v_u > 0.083\varphi_v 2\lambda\sqrt{f_c'}$ (N,mm), extend shear reinforcement computed for the previous critical section to this section, and proceed with the evaluation of the next critical section.

This check is performed only if $v_u < \varphi_v v_c$ and the layer is not the first layer from the face of column.

Where,

φ_v is the strength reduction factor for shear (0.75)[89];

λ is a factor depending on the weight of concrete at the support (1 for normal weight concrete; 0.85 for sand lightweight; and 0.75 for all lightweight[90])

7 - Calculate shear reinforcement
A - For shear studs: For the first layer from the face of column, the reinforcement is calculated as the larger of A_v and $A_{v,min.}$

$$A_v = \frac{(v_u - \phi_v v_c) b_0 s}{\phi_v f_y} \qquad \text{(Exp 4.11.6.7B.7-1)}$$

$$A_{v,min} = \frac{2\sqrt{f_c'} b_0 s}{f_y} \qquad \text{(Exp. 4.11.6.7B.7-2)}$$
US

$$A_{v,min} = \frac{0.083 \times 2\sqrt{f_c'} b_0 s}{f_y} \qquad \text{(Exp. 4.11.6.7B.7-2)}$$
SI

For both RC and prestressed floors, where shear stud reinforcement is used, v_c shall not exceed $3\lambda\sqrt{f_c'}$ (lb-in), $[0.083 \times 3\lambda\sqrt{f_c'}$ (N,mm)]; if larger than the preceding cap, assume the cap value.

B - For stirrups:

For stirrups, the area of shear reinforcement (A_v) for the entire critical surface under design is:

$$A_v = \frac{(v_u - \phi_v v_c) b_0 s}{\phi_v f_y sin\alpha} \qquad \text{(Exp 4.11.6.7B.7-3)}$$

Where,

α = the angle of shear reinforcement with the plane of slab.

For both RC and prestressed floors, where stirrup shear reinforcement is used, v_c shall not exceed $2\lambda\sqrt{f_c'}$ (lb-in), $[0.083 \times 2\lambda\sqrt{f_c'}$ (N,mm)]; if larger than the preceding cap, assume the cap value.

8 - Arrangement of shear reinforcement
A - For shear studs:

89 ACI 318-11 Section 9.3.2.3
90 ACI 318-11 Section 8.6

The number of shear studs per rail (N_{shear_studs}) and the distance between the studs ($Dist_{shear\_studs}$) are given by:

$$N_{shear\_studs} = \frac{A_v}{A_{shear\_stud} \times N_{rails}} \qquad \text{(Exp 4.11.6.7B.8-1)}$$

$$Dist_{shear\_studs} = \frac{d/2_{slab}}{N_{shear\_stud}} \qquad \text{(Exp 4.11.6.7B.8-2)}$$

The spacing between the column face and the first peripheral line of shear reinforcement shall not exceed $d/2$. The spacing between adjacent shear reinforcement elements, measured on the perimeter of the first peripheral line of shear reinforcement, shall not exceed $2d$.

Where,

A_v = required area of shear reinforcement;

A_{shear_stud} = area of one stud;
N_{rails} = number of rails; and
d = effective depth of critical section.

B - For stirrups (links):
In case of shear links, the number of shear links[91] (N_{shear_links}) along a critical section and the distance between the links ($Dist_{shear\_links}$) are given by:

$$N_{shear\_links} = \frac{A_v}{A_{shear\_link}} \qquad \text{(Exp. 4.11.6.73.8-3)}$$

$$Dist_{shear\_links} = \frac{b_0}{N_{shear\_links}} \qquad \text{(Exp 4.11.6.7B.8-4)}$$

The first layer of stirrups shall be placed at $d/2$ from the column face and the successive layers are at $d/2$ from the previous layer. The spacing between the adjacent stirrup legs in the first line of shear reinforcement shall not exceed $2d$ measured in a direction parallel to the column face.

4.11.6.8 EC2 Punching Shear Design; Using EC2[92] the punching shear resistance of a column support in a two-way floor system should be checked/designed at the face of support, and at a first critical section (control section), as illustrated in Fig. 4.11.6.8-1 for an interior support, and in Fig. 4.11.6.8-2 for supports adjacent to a slab edge. If the section at the first critical support is not adequate, punching shear reinforcement is added. The design for an inadequate control section is continued at control sections beyond the one reinforced. The design is terminated at a control section for which no reinforcement is required. EC2 does not differentiate between the end and edge columns, since the presence of an unbalanced moment at a joint is accounted for through a universal multiplier that accounts for moments, as opposed to computation of hypothetical stresses resulting from an unbalanced moment[93].

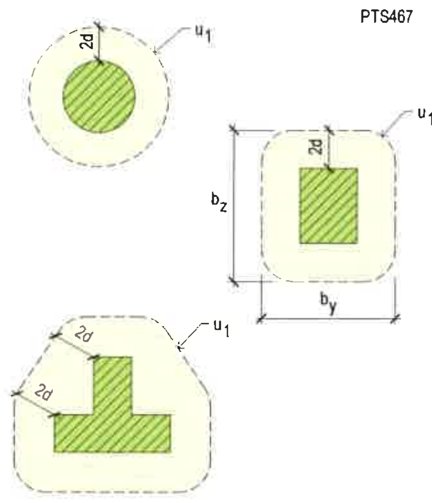


FIGURE 4.11.6.8-1 First Critical (Control) Section at Interior Supports

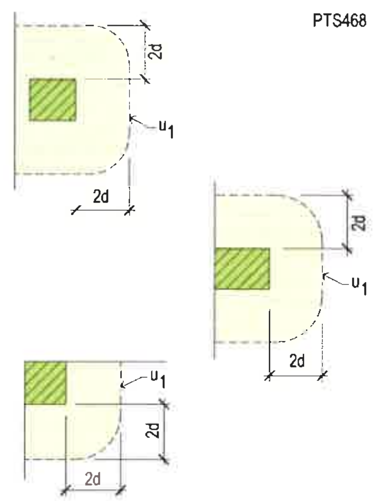
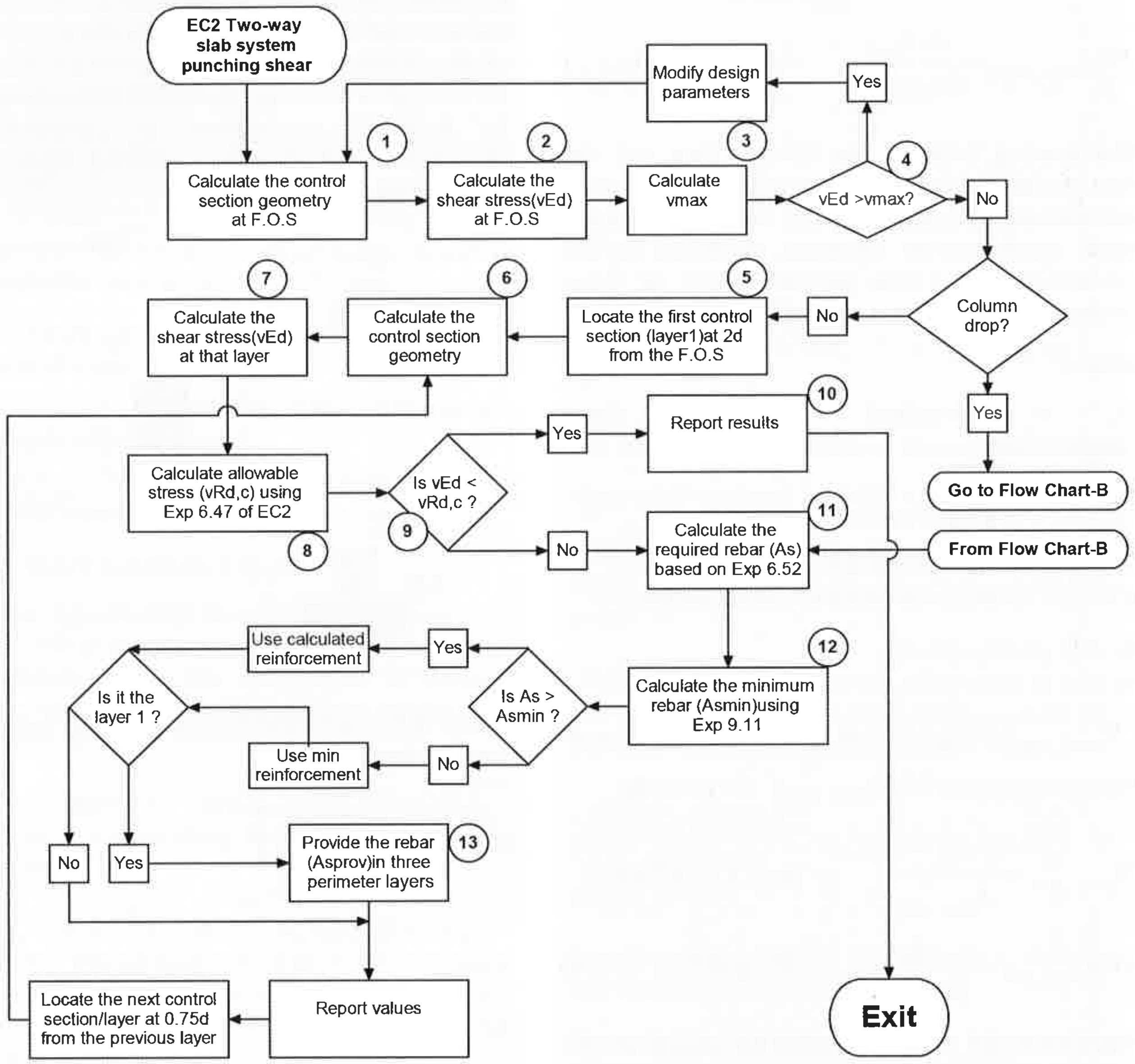


FIGURE 4.11.6.8-2 First Critical (Control) Section at Supports near a Slab Boundary

91 A "link" is considered as one vertical bar normal to the plane of the floor.
92 EC2(EN 1992-1-2004€), Section 6.4
93 Procedure adopted in ACI 318-11, Section 11.11.7

EC2 Punching Shear Flow Chart

F1123



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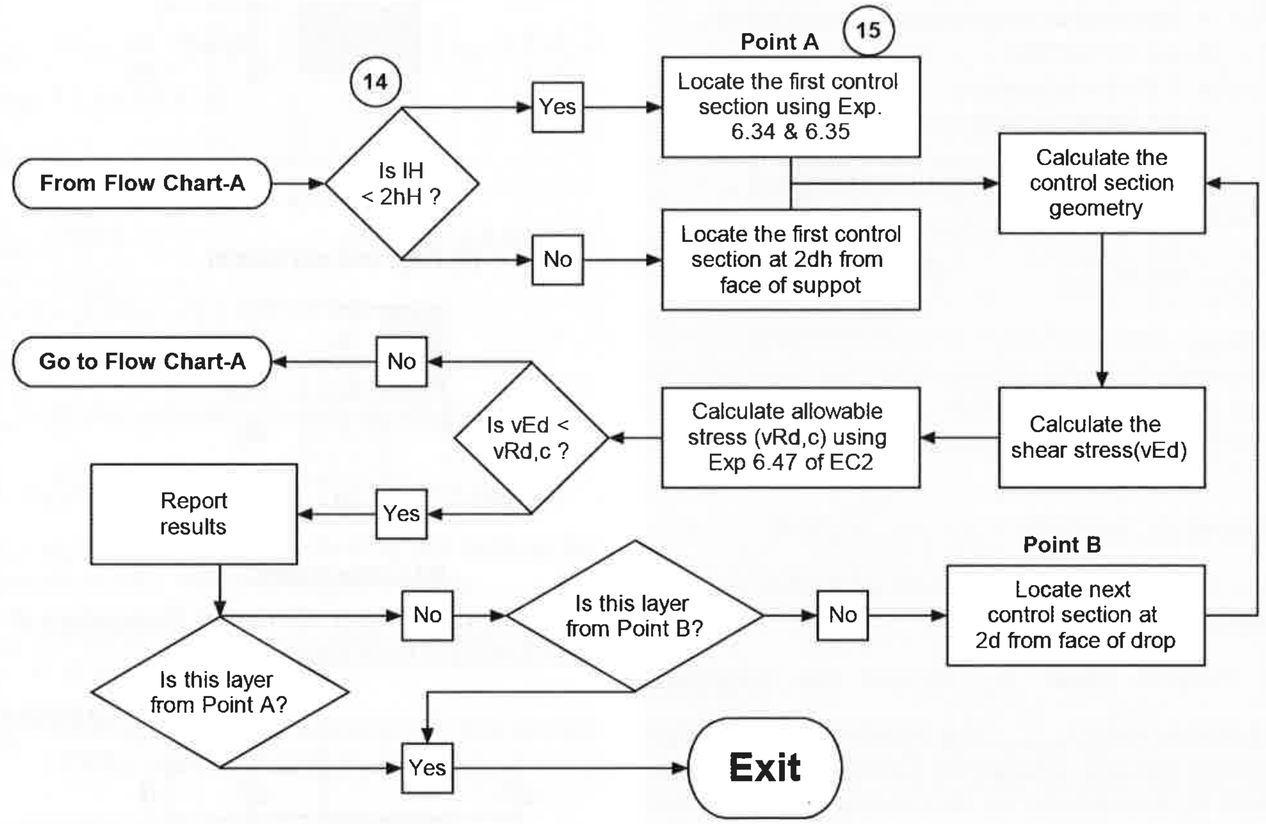
EC2 Punching Shear Flow Chart
for Support with Column Drop
Flow Chart B



A. EC2 Punching Shear Flow Chart: The following flow chart walks you through the adequacy check and design for punching shear of a column support in a two-way floor system. The numbers next to the items of the flow chart refer to the explanations that follow the flow chart. A numerical example in Section 4.11.6.9 and 10 illustrate the application of the flow chart. The following numbered parts refer to the items of EC2 punching shear floor chart.

1 - Calculate the control section geometry at face of support

For two-way action, shear resistance should be checked at the face of the column, when using EC2 code[94]. Perimeter of the column is calculated as following[95].

For an interior column, $u_0 = 2(c_1 + c_2)$
(Exp 4.11.6.8A.1-1)

For an edge column, $u_0 = c_2 + 3d \le c_2 + 2c_1$
(Exp 4.11.6.8A.1-2)

For a corner column, $u_0 = 3d \le c_1 + c_2$
(Exp 4.11.6.8A.1-3)

Where,

c_1, c_2 = column dimensions; and
d = effective depth of the slab.

2 - Calculate demand shear at face of support

Demand shear is calculated using the following relationship[96], where the demand shear force can conservatively be assumed as difference between the upper and lower column factored reactions.

[94] EN 1992-1:2004, section 6.4.1(4)
[95] EN 1992-1:2004, section 6.4.5(3)
[96] EN 1992-1:2004, section 6.4.5(3), Eqn 6.53

$$v_{Ed} = \frac{\beta V_{Ed}}{u_0 d} \quad \text{(MPa)} \qquad \text{(Exp 4.11.6.8A.2-1)}$$

Where,

V_{Ed} = factored demand shear force [N]; and
β[97] = 1.5 for corner;
= 1.15 for interior;
= 1.4 for end/edge.

3 - Calculate the maximum allowable shear resistance $v_{Rd,max}$[98].

$$v_{Rd,max} = 0.5 \upsilon f_{cd} \qquad \text{(Exp 4.11.6.8A.3-1)}$$

Where,

$$\upsilon = 0.6\left[1 - \frac{f_{ck}}{250}\right] \qquad \text{(Exp 4.11.6.8A.3-2)}$$

Where, f_{ck} is in MPa.

4 - Check whether maximum allowable shear is exceeded

If demand shear v_{Ed} exceeds the maximum allowable value $v_{Rd,max}$, the punching shear design cannot proceed. Change the parameters of design, such as dimensions, or alternative load paths for safe shear transfer.

5 - Select first control section
The first control section to be investigated shall be located at $2d$ from the periphery of the column, where d is the effective depth of the slab/column drop[99]. Subsequent sections are $0.75d$ away from the previous control section. For columns near a slab edge the control perimeter is shown in Fig. 4.11.6.8A-1.

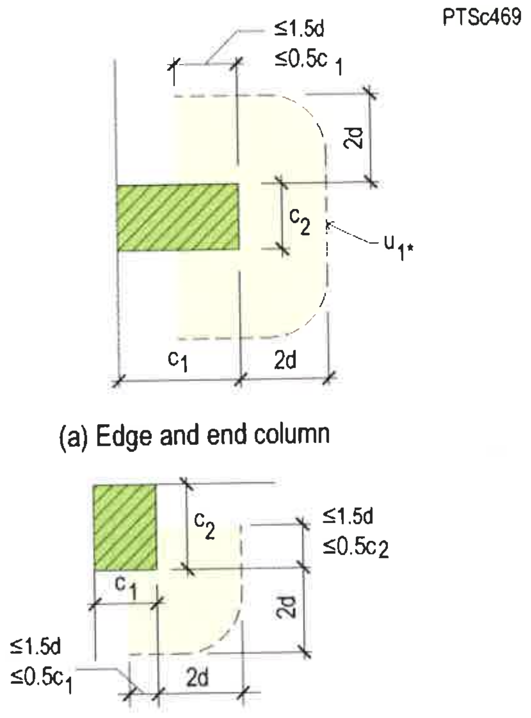
6 - Calculate the control section properties
For a column or column drop with dimensions "c_1" and "c_2" and a control section which is at distance "c" from the face of column or column drop, area "A" is given by[100]:

Interior column:

$$A = 2(c_1 + c_2) + 2\pi c \qquad \text{(Exp. 4.11.6.8A.6-1)}$$

Edge column: (c_1 is parallel to the axis of moment)

$$A = 2c_1 + c_2 + \pi c \qquad \text{(Exp. 4.11.6.8A.6-2)}$$



(a) Edge and end column

(b) Corner column

FIGURE 4.11.6.8A -1 Geometry Parameters of the Supports near Slab Boundary

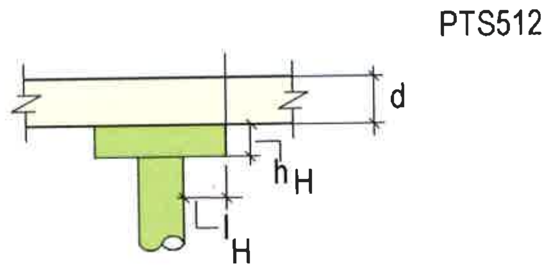


FIGURE 4.11.6.8A-2 Parameter Definition for Column Drops

Corner Column:

$$A = c_1 + c_2 + \frac{\pi}{2} c \qquad \text{(Exp. 4.11.6.8A.6-3)}$$

End column:

$$A = 2c_1 + c_2 + \pi c \qquad \text{(Exp. 4.11.6.8A.6-4)}$$

7 - Calculate demand shear stress at control section
The hypothetical demand shear stress (v_{Ed}) for the control section is:

[97] EN 1992-1:2004, section 6.4.3(6), Note
[98] EN 1992-1:2004, section 6.4.5(3), Note
[99] EN 1992-1:2004, section 6.4.2(1)
[100] EN 1992-1:2004, section 6.4.2, Figure 6.13 &6.15

$$v_{Ed} = \frac{\beta V_{Ed}}{A} \qquad \text{(Exp. 4.11.6.8A.7-1)}$$

8 - Calculate allowable hypothetical shear stress

$$v_{Rd,c} = C_{Rd,c} k \left(100 \rho_1 f_{ck}\right)^{1/3} + k_1 \sigma_{cp} \geq \left(v_{min} + k_1 \sigma_{cp}\right)$$
(Exp. 4.11.6.8A.8-1)

Where,

$$C_{Rd,c} = 0.18 / \gamma_c \ ; \qquad \text{(Exp. 4.11.6.8A.8-2)}$$

$$k = 1 + \sqrt{(200/d)} \leq 2.0 \text{ , } d \text{ in mm}$$
(Exp. 4.11.6.8A.8-3)

f_{ck} = 28 day cylinder strength of concrete;

$$\rho_1 = \sqrt{\rho_{ly} \times \rho_{lz}} \leq 0.02 ; \qquad \text{(Exp. 4.11.6.8A.8-4)}$$

ρ_{ly}, ρ_{lz} = bonded tension steel in two orthogonal directions over effective support width;

$v_{min} = 0.035 k^{3/2} \times f_{ck}^{1/2}$; (Exp. 4.11.6.8A.8-5)
k_1 = 0.1; and

σ_{cp} = average of normal stress on control section (MPa, positive if compression).

9 - Compare demand with allowable
Stresses are calculated at the critical(control) sections and compared against the allowable values:

If $v_{Ed} < v_{Rdc.}$ no punching shear reinforcement is required for this layer.

If $v_{Ed} > v_{Rdc.}$ provide punching shear reinforcement.

10 - Report results
If demand is less than the allowable value, report the outcome and exit.

11 - Calculate the required punching shear reinforcement
Where demand exceeds the allowable value, calculate the punching shear reinforcement (A_s) as detailed below[101]:

$$A_s = \frac{\left(v_{Ed} - 0.75 v_{Rd,c}\right) u \times d \times s_r}{1.5 \times d \times f_{ywd,ef} \, sin\alpha}$$
(Exp.4.11.6.8A.11-1)

Where,

$$f_{ywd,ef} = 250 + 0.25d \leq f_{ywd} \quad \text{(MPa)}$$
(Exp. 4.11.6.8A.11-2)

f_{ywd} = design strength of punching shear reinforcement;
s_r = spacing of perimeters of shear reinforcement in the radial direction;
u = perimeter of the control section;
α = the angle of shear reinforcement with the plane of slab [90°]; and
d = effective depth.

12 - Calculate the minimum reinforcement
Where punching shear reinforcement is required by design, its amount shall not be less than the minimum area given in the following[102]:

$$_{s\ min} \quad \frac{0.08\sqrt{f_{ck}} \times s_r \times s_t}{(1.5 sin\alpha \quad cos\alpha) f_{yk}}$$
(Exp. 4.11.6.8A.12-1)

Where,

s_r = spacing of shear links in the radial direction;
s_t = spacing of shear links in the tangential direction;
α = the angle of shear reinforcement with the plane of slab [90°]; and
f_{yk} = characteristic yield strength of reinforcement.

13 - If design is for the first control layer
If the reinforcement calculated was for the first control layer, assume the same reinforcement for two additional layers beyond it as "minimum reinforcement". Place this reinforcement in three layers between the face of support and first critical layer.

Flow Chart B – Flow Chart for Support with Column Drop
The geometry of a column drop is defined as shown in Fig. 4.11.6.8A-2. If a column drop/drop panel exists[103], stresses are checked both within the cap and outside the cap if the width of the cap on one side of the column (l_H) is greater than two times the depth of the cap below the slab ($2h_H$).

[101] EN 1992-1:2004, section 6.4.5, Exp. 6.52
[102] EN 1992-1:2004, section 9.4.3, Eqn 9.11
[103] EN 1992-1:2004, section 6.4.2(8-11)

14 - Check geometry of column drop

If column drop/panel exists[104], stresses are to be checked both within the drop and outside the drop, if the extension of the drop beyond the face of column (l_H) is greater than twice the depth of the drop below the slab soffit ($2h_H$).

15 - Locate the first control section

If $l_H < 2h_H$, punching shear stress check is only required for the section outside the cap.

The control section from the centroid of the column is located using the following relationship[105]:

$$r_{cont} = \text{lesser of}\left[2d + 0.56\sqrt{(l_1 l_2)} \text{ and } 2d + 0.69 \times l_1\right]$$

(Exp. 4.11.6.8A.15-1)

Where,

$l_1 = c_1 + 2l_{H1};$ (Exp. 4.11.6.8A.15-2)

$l_2 = c_2 + 2l_{H2}$, $l_1 \le l_2$; and (Exp. 4.11.6.8A.15-3)

c_1, c_2 = column dimensions.

B - Arrangement of Shear Reinforcement: If shear reinforcement is provided, the spacing between the reinforcement in the radial direction should not exceed 0.75d[106]. Further, at least two control perimeters within a distance 2d from the face of support shall be provided with shear reinforcement. The spacing of shear reinforcement around a perimeter should not exceed 1.5d within the first control perimeter (2d from the loaded area), and should not exceed 2d for perimeters outside the first control perimeter, where part of the perimeter is assumed to contribute to the shear capacity.

Shear reinforcement can be in the form of shear studs or shear stirrups (links). In case of shear links, the number of shear links[107] (N_{shear_links}) along a control section and the distance between the links ($Dist_{shear\_links}$) are given by:

$$N_{shear\_links} = \frac{A_s}{A_{shear\_link}} \quad \text{(Exp. 4.11.6.8B-1)}$$

$$Dist_{shear\_links} = \frac{u}{N_{shear\_links}} \quad \text{(Exp. 4.11.6.8B-2)}$$

Where,

A_s = required area of shear reinforcement; and

A_{shear_link} = area of the single shear link.

For shear studs:

If shear studs are used, the number of shear studs per rail (N_{shear_studs}) and the distance between the studs ($Dist_{shear\_studs}$) are given by:

$$N_{shear\_studs} = \frac{A_s}{A_{shear\_stud} \times N_{rails}} \quad \text{(Exp. 4.11.6.8B-3)}$$

$$Dist_{shear\_studs} = \frac{s}{N_{shear\_stud}} \quad \text{(Exp. 4.11.6.8B-4)}$$

Where,

A_{shear_stud} = area of one shear stud;

N_{rails} = number of rails; and

s = spacing between the critical sections.

4.11.6.9 Punching Shear Example with Post-Tensioning: The following numerical example demonstrates the application of the procedure and the flow charts outlined in the preceding Section to punching shear design of post-tensioned column supported floor systems. The geometry, the amount and arrangement of post-tensioning and non-prestressed reinforcement are shown in Figs. 4.11.6.9-1 through 4.11.6.9-3.

Given:

Geometry

Slab thickness, h = 8 in (203 mm)

Column dimensions = 24" (610 mm) square

Material

❖ Concrete, $f'_c (f_{ck})$ = 5000 psi (34.47 MPa)

Reinforcement

❖ Post-Tensioning:

System, both systems bonded and unbonded are considered

104 EN 1992-1:2004, section 6.4.2(8-11)

105 EN 1992-1:2004, section 6.4.2, Exp. 6.34 & 6.35

106 EN 1992-1:2004, section 9.4.3

107 A "link" is considered as one vertical bar normal to the plane of the floor.

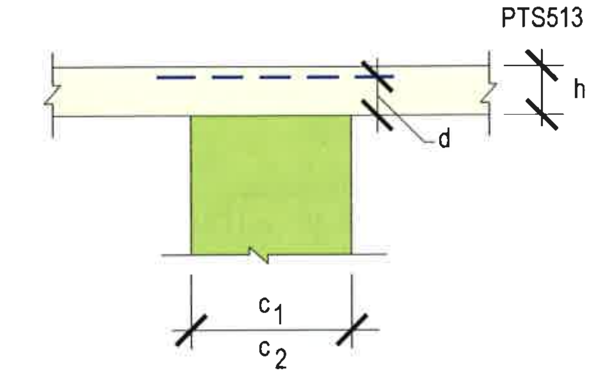


FIGURE 4.11.6.9-1 Elevation of Column Support

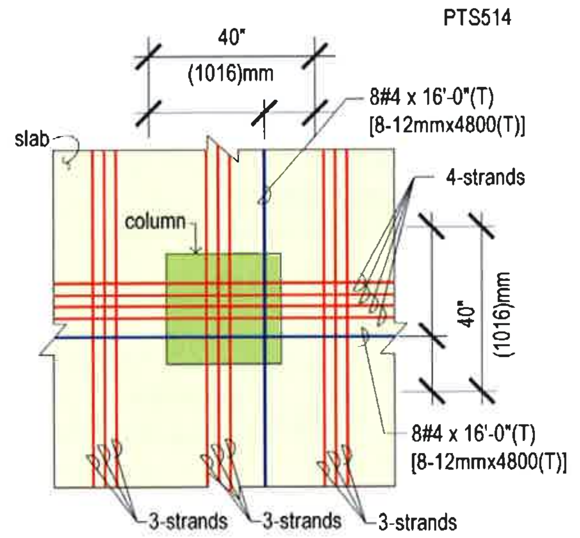


FIGURE 4.11.6.9-2 Plan - Column Support and Reinforcement in Slab

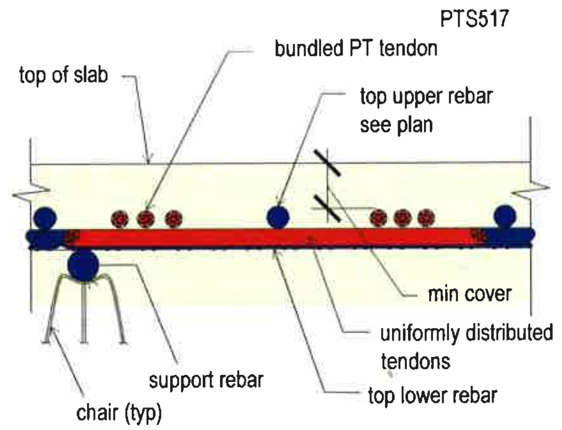


FIGURE 4.11.6.9-3 Plan - Column Support and Reinforcement in Slab; Section through Banded Tendons over Support

Average precompression = 125 psi (0.86 MPa)

Strand diameter = ½ in (13 mm)

Strand area = 0.153 in^2 (99 mm^2)

Modulus of elasticity = 28000 ksi (193054 MPa)

Ultimate strength of strand, f_{pu} = 270 ksi (1860 MPa)

Average effective stress (f_{se}) =175 ksi (1200 MPa)

Tendon layout, as shown Fig. 4.11.6.9-2 and 3

❖ Non-prestressed reinforcement:

Yield stress, f_y = 60 ksi (413.69 MPa)

Modulus of elasticity = 29,000 ksi (200,000 MPa)

Top bars 8#4 x 16'-0" (8-13 mm x 4.9 m; area of each bar taken as 129 mm^2) in each direction. Cover from the top first layer of reinforcement consisting of the banded tendons and non-prestressed reinforcement is 0.75" (20 mm). Below the top layer are the distributed tendons and non-prestressed reinforcement in direction of distributed tendons (Fig. 4.11.6.9-3).

Cover

❖ Non-prestressed Reinforcement

Cover to top bars = 0.75 in (20 mm)

❖ Prestressed Reinforcement

Top cover = 0.75 in (20 mm) for outer layer

Factored Demand Actions

Mu_{xx} = 150 k-ft (203.37 kNm)

Mu_{yy} = 180 k-ft (244.04 kNm)

Vu = 250 kip (1112.05 kN)

A. ACI-Based Punching Shear Design Example

The first critical shear failure plane is assumed at a distance *d/2* from the face of support, where "*d*" is the effective depth of the section. Subsequent sections are *d/2* away from the previous critical section.

Punching shear stress is calculated using the following relationship[108]:

$$v_u = \frac{V_u}{A} + \frac{\gamma M_u c}{J_c}$$

At d/2 from the face of support:

❖ Section properties:

Column width, c_1 = 24 in (610 mm)

Column depth, c_2 = 24 in (610 mm)

108 ACI 318M-11, Section R11.11.7.2

Rebar used #4 bars, diameter = 0.5 in (13 mm)
Top cover to rebar = 0.75 in (20 mm)
d_r = 8-0.75-0.5 =6.75 in (171 mm)

Since top bars in one direction are placed above the top bars in the other direction, *d* value in this case is measured from the bottom of the slab to the bottom of the top layer of rebar.

$b_1 = c_1 + d$ = 24+6.75 =30.75 in (781 mm)
$b_2 = c_2 + d$ = 24+6.75 =30.75 in (781 mm)

$A_c = 2d\left(c_1 + c_2 + 2d\right)$

$= 2\times6.75\left(24+24+2\times6.75\right)$ = 830.25 in² (5.356 e+5 mm²)

$$J_c = \frac{b_1 d^3}{6} + \frac{b_1^3 d}{6} + \frac{b_1^2 b_2 d}{2}$$

$$= \frac{30.75\times6.75^3}{6} + \frac{30.75^3\times6.75}{6} + \frac{30.75^2\times30.75\times6.75}{2}$$

= 1.324e+5 in⁴ (5.512 e+10 mm⁴)

$$\gamma_v = 1 - \frac{1}{1+\frac{2}{3}\sqrt{\frac{b_1}{b_2}}} = 0.40$$

❖ Stress due to direct shear:

$$\frac{Vu}{A_c} = \frac{250}{830.25} = 0.301\,\text{ksi}\left(2.08\,\text{MPa}\right)$$

❖ Stress due to bending:

$$\text{M}_{\text{xx,stress}} = \frac{\gamma_v \times M_{xx} \times b_2}{2\times J_c}$$

$$= \frac{0.4\times150\times12\times30.75}{2\times1.324\times10^5} = 0.084\,\text{ksi}\left(0.58\,\text{MPa}\right)$$

$$\text{M}_{\text{yy,stress}} = \frac{\gamma_v \times M_{yy} \times b_1}{2\times J_c}$$

$$= \frac{0.4\times180\times12\times30.75}{2\times1.324\times10^5} = 0.100\,\text{ksi}\left(0.69\,\text{MPa}\right)$$

❖ Total stress:
Total stress is stress due to shear plus stress due to bending. Note that in ACI 318 empirical procedure, the stresses from moments in the two directions xx and yy are not combined, when checking for punching shear design. Each is handled separately.

Total stress in xx-direction
= 0.301 + 0.084 = 0.385 ksi (2.65 MPa)
Total stress in yy-direction
= 0.301 + 0.100 = 0.401 ksi (2.76 MPa)

❖ Allowable stress[108]:

$$V_c = \left[\left(\beta_p \lambda \sqrt{f_c'} + 0.3 f_{pc}\right) b_0 d + V_p\right]$$

Where,

ϕ = 0.75

β_p is the smaller of 3.5 or $\left(\alpha_s d / b_0 + 1.5\right)$
α_s = 40 for interior column
b_0 = Perimeter of the critical section

= $2\times\left(30.75+30.75\right)$ = 123 in (3124 mm)
d = 6.75 in (171 mm)

$$\beta_p = \left(\alpha_s d / b_0 + 1.5\right) = \left(\frac{40\times6.75}{123} + 1.5\right) = 3.70$$
>3.5 ∴ use 3.5

λ = 1 for normal weight concrete
f_{pc} = P/A = 125 psi (0.86 MPa)

$\phi\, v_c = 0.75\left(3.5\times1\times\sqrt{5000} + 0.3\times125\right)$
= 213.74 psi (1.47 MPa)

Allowable Stress = 0.214 ksi (1.47 MPa)

Note that in the evaluation of allowable stresses, the term corresponding to the vertical component of tendon force (V_p) is conservatively disregarded.

❖ Stress ratio:

Stress ratio = Actual/Allowable
= 0.401/0.214= 1.87

Punching shear stress exceeds the permissible value. Provide shear reinforcement.

❖ Check maximum allowable shear stress[110]:

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109 ACI 318M-11, Section 11.11.2.2, equation 11.34
110 ACI 318-11, Section 11.11.5.1

$\phi v_n \le \phi 8\lambda\sqrt{f_c'} = 0.75\times8\times1\times\sqrt{5000}$
=424.26 psi (2.93 MPa) > 401 psi (2.76 MPa)
OK

❖ Shear Reinforcement Design:

$$A_v = \frac{\left(v_u - \varphi v_c\right) b_0 s}{\varphi f_y sin\alpha} \ge A_{vmin}$$

$\varphi v_c = \varphi 3\lambda\sqrt{f_c'} = 0.75\times3\times1\sqrt{5000}$
$= 159.10\,\text{psi}\left(1.10\,\text{MPa}\right)$

$$A_v = \frac{\left(0.401-0.159\right)123\times6.75/2}{0.75\times60}$$
$= 2.23\,\text{in}^2\left(1439\,\text{mm}^2\right)$

$$A_{vmin} = \frac{2\sqrt{f'_c}\, b_0 s}{f_y} = \frac{2\sqrt{5000}\times123\times6.75/2}{60\times1000}$$
$= 0.98\,\text{in}^2\left(633\,\text{mm}^2\right)$

Assume 0.625" diameter studs, area of 1 stud =
= 0.31 in² (200 mm²)
Number of rails per side = 3
Number of sides = 4

$$\text{Number of studs, } \text{N}_{\text{studs}} = \frac{2.23}{0.31\times3\times4} = 0.60 \approx 1$$

$$\text{Distance between the studs} = \frac{s}{N_{stud}} = \frac{6.75/2}{1}$$
$= 3.38\,\text{in}\left(86\,\text{mm}\right) \approx 3.35\ \text{in}\left(85\ \text{mm}\right)$

Check the shear stress on successive perimeters by following the same procedure until a section with
no shear rebar is required. v_u shall not exceed $2\varphi\lambda\sqrt{f_c'}$ at the critical section located d/2 outside the outermost line of shear reinforcement that surround the column.

The output of detailed calculations performed for other layers is summarized in Table 4.11.6.9C-1. The general arrangement of the studs is shown graphically in Fig. 4.11.6.9C-1. For both the unbonded and bonded systems, a total of 10 studs spaced at 3.35"(85 mm) are required.

Comment:
Using ACI 318, the amount and arrangement of top reinforcement, be it non-prestressed or prestressed does not impact the computed punching shear capacity of a column supported slab. Likewise, ACI 318 does not differentiate between bonded and unbonded tendons in computation of design capacity for punching shear. The same is not true for EC2. As you will note in the following example, the computed punching shear capacity based on EC2 depends on the amount and location of both the prestressed and non-prestressed top reinforcement. Also, the computed capacity depends on whether the post-tensioning is bonded or unbonded. On the other hand, using EC2, contrary to ACI 318 the value of joint moment does not enter the design of punching shear capacity. In EC2, the presence of joint moment is accounted for by way of a constant magnification factor, irrespective of the value of joint moment.

B. EC2- Based Punching Shear Design Example
First, the demand shear is checked at the face of support, to determine whether the material and geometry of the joint can be designed to resist the demand actions. Once the adequacy of material and geometry are verified, the design for the necessity and amount of shear reinforcement begins. For shear design, the first critical shear plane is assumed at a distance 2*d* from the face of support, where "*d*" is the effective depth of the section. Subsequent sections are *0.75d* away from the previous critical section.

Factored actions at the joint are:

Mu_{xx} = 203.37 kNm (150 k-ft)
Mu_{yy} = 244.04 kNm (180 k-ft)
Vu = 1112.05 kN (250 kip)

At face of the support:
❖ Section properties:
Column width, c_1 = 610 mm (24 in)
Column depth, c_2 = 610 mm (24 in)
Slab depth, h = 203 mm (8 in)
d_r = 203 -19-13 =171 mm (6.73 in)

$u_0 = 2(c_1 + c_2) = 2\times\left(610+610\right)$
= 2440 mm (96.06 in)

$u_0 d = 2440\times171 = 417240$ mm² (646.72 in²)

❖ Stress due to direct shear and moment[111]:

$$v_u = \frac{V_u \beta}{u_0 d}$$

β = 1.15 for interior column

$$v_u = \frac{1112.05\times10^3\times1.15}{417{,}240} = 3.07\,\text{MPa}\,(0.445\text{ ksi})$$

❖ Allowable stress:

$$v_{Rd,max} = 0.5 v f_{cd}$$

$$v = 0.6\left[1-\frac{f_{ck}}{250}\right] = 0.6\times\left[1-\frac{34.47}{250}\right]$$

$= 0.52\,\text{MPa}\,(75.42\text{ psi})$

$v_{Rd,max} = 0.5\times0.52\times34.47/1.5$

$= 5.97\,\text{MPa}\,(0.866\,\text{ksi}) > 3.07\,\text{MPa}\,(0.445\text{ ksi})$

❖ Stress ratio:
Stress ratio = Actual/Allowable
= 3.07/5.97 =0.51< 1 OK

At 2d from the face of support:

Using Unbonded tendons

❖ Section properties:
Column width, c_1 = 610 mm (24 in)
Column depth, c_2 = 610 mm (24 in)

$u_0 = (2c_1 + 2c_2 + 4\pi d)$

$= (2\times610 + 2\times610 + 4\times\pi\times171)$

= 4589 mm (180.67 in)

$u_0 d$ = 4589×171 = 784,719 mm² (1216.31 in2)

❖ Stress due to direct shear and moment:

$$v_u = \frac{V_u \beta}{u_0 d}$$

β= 1.15 for interior column

$$v_u = \frac{1112.05\times10^3\times1.15}{784{,}719} = 1.63\,\text{MPa}\,(0.236\text{ ksi})$$

❖ Allowable stress[112]:
The allowable stress depends on the amount of top reinforcement. The presence and amount of effective top reinforcement is reflected through the parameter ρ [113].

$$V_{Rd,c} = C_{Rd,c} k (100 \rho_1 f_{ck})^{1/3} + k_1 \sigma_{cp} \geq (v_{min} + k_1 \sigma_{cp})$$

where,

$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.5 = 0.12$

$$k = 1+\sqrt{\frac{200}{d}} = 1+\sqrt{\frac{200}{171}} = 2.08 > 2 \quad \text{use 2}$$

$\rho l = \sqrt{\rho l_1 \times \rho l_2} \leq 0.02$

$A_{s1} = 8\times129 = 1032\,\text{mm}^2\,(1.60\text{in}^2)$

$A_{s2} = 8\times129 = 1032\,\text{mm}^2\,(1.60\text{in}^2)$

For unbonded tendons, the area of prestressing tendons will not be included, hence:

$A_{ps1,2} = 0$

$$\rho l_1, \rho l_2 = \frac{A_{sl1,2} + A_{ps1,2}(f_{pk}/f_{yk})}{bd}$$

$$= \frac{1032+0}{(610+3\times171\times2)\times171} = 0.004$$

$\rho l = \sqrt{0.004\times0.004} = 0.004 < 0.02$

$$\sigma_{cp} = \frac{N_{ED}}{A} = 0.86\,\text{MPa}\,(125\,\text{psi})$$

$k_1 = 0.1$

$v_{min} = 0.035 k^{3/2} f_{ck}^{1/2} = 0.035\times2^{3/2}\times34.47^{1/2}$
= 0.58 MPa (84.12 psi)

$(v_{min} + k_1\sigma_{cp}) = 0.58 + 0.1\times0.86$

$= 0.67\,\text{MPa}\,(97.18\text{ psi})$

---

111 EN 1992-1:2004, section 6.4.5(3), Eqn 6.53

112 EN 1992-1:2004, section 6.4.4, Eqn 6.

113 EN 1992-1:2004, section 6.4.4(1) ρ ly, ρ lz relate to the bonded tension steel in y- and z- directions respectively. The values ρ ly and ρ lz should be calculated as mean values taking into account a slab width equal to the column width plus 3d each side."

$$v_{Rd,c} = \left[0.12\times2\times(100\times0.004\times34.47)^{1/3} + 0.1\times0.86\right]$$

= 0.66 MPa (95.73 psi) < 0.67 MPa (97.18 psi)

$v_{Rd,c}$ = 0.67 MPa (0.097 ksi)

❖ Stress ratio:
Stress ratio= Actual/Allowable
= 1.63/0.67 =2.43 > 1

Punching shear stress exceeds the permissible value. Provide shear reinforcement.

In EC2 presence of column moment is accounted for through β factor, but the value of the moment does not enter the computations.

❖ Shear Reinforcement Design:

$$A_{sw} = \frac{(v_{Ed} - 0.75 v_{Rd,c}) u \times d \times s_r}{1.5\times d\times f_{ywd,ef}\,sin\alpha} \geq A_{sw,min}$$

$$A_{sw,min} = \frac{0.08\sqrt{f_{ck}}\times s_r\times s_t}{(1.5 sin\alpha + cos\alpha) f_{yk}}$$

$s_r = 2d$ = 342 mm (13.46 in)

$s_t = 1.5d$ = 257 mm (10.12 in)

$f_{ywd,ef}$ = 250+0.25d = 292.75 MPa (42.46 ksi)

< (413.69/1.15) = 359.73 MPa (52.18 ksi)

$$A_{sw} = \frac{(1.63 - 0.75\times0.67)\times4589\times171\times342}{1.5\times171\times292.75}$$

$= 4030\text{ mm}^2\,(6.25\text{ in}^2)$

$$A_{sw,min} = \frac{0.08\sqrt{34.47}\times342\times257}{1.5\times413.69}$$

$= 67\,\text{mm}^2\,(0.10\text{ in}^2) < 4030\text{ mm}^2\,(6.25\text{ in}^2)$

A_{sw} = 4030 mm² (6.25 in²)

Assume 16 mm diameter studs, area of 1 stud = 201 mm² (0.31 in²)
Number of rails/side = 3
Number of sides = 4

$$\text{Number of studs, } N_{studs} = \frac{4030}{201\times3\times4} = 1.67 \approx 2$$

There should be at least 3 studs within the critical section at 2d from the face of support.

Number of studs, N_{studs} = 3

$$\text{Distance between the studs} = \frac{s}{N_{stud}} = \frac{2\times171}{3}$$

$= 114\,\text{mm}\,(4.5\text{ in}) \approx 110\text{ mm}\,(4.3\text{ in})$

Check the shear stress on successive perimeters at 0.75d by following the same procedure until a section with no shear rebar is required.

The output of the detailed calculation performed for other layers are given in Table 4.11.6.9C-1, with the general arrangement of the rails and studs shown in Fig. 4.11.6.9C-1. For the design based on EC2, a total of 12 studs per rail is required; first 3 studs at 110 mm (4.33") and the remaining at 125 mm (4.92").

Using grouted tendons

Using grouted tendons, similar to non-prestressed top bars, only the area of the strands that cross a pre-defined distance from the face of support, as explained above, will be accounted for.

❖ Section properties:
Column width, c_1 = 610 mm (24 in)
Column depth, c_2 = 610 mm (24 in)

$u_0 = (2c_1 + 2c_2 + 4\pi d)$

$= (2\times610 + 2\times610 + 4\times\pi\times171)$
= 4589 mm (180.67 in)

TABLE 4.11.6.9C-1 Punching Shear Reinforcement

Code	Number of studs per rail	
	Unbonded tendon	Bonded tendon
ACI	10 studs@3.35" (85 mm)	10 studs@3.35" (85 mm)
EC2	12 studs; first 3 @110 mm, (4.33"); remainder at 125 mm (4.92")	7 studs, first 3 studs @110 mm (4.33"); remainder at 125 mm (4.92")

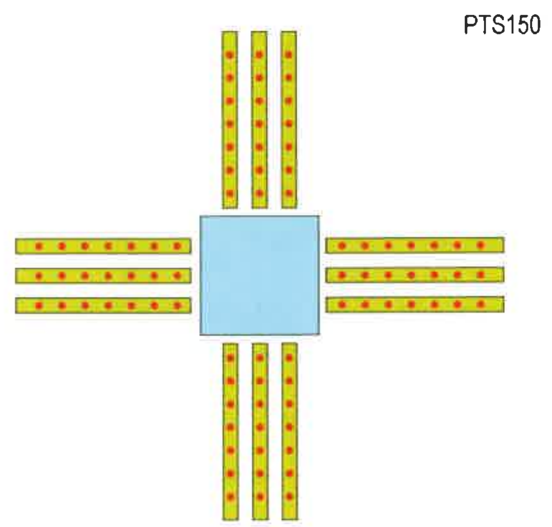

FIGURE 4.11.6.9C-1 General Arrangement of Shear Stud Reinforcement

$u_0 d$ = 4589×171 = 784,719 mm² (1216.31 in2)

❖ Stress due to direct shear and moment:

$$v_u = \frac{V_u \beta}{u_0 d}$$

β= 1.15 for interior column

$$v_u = \frac{1112.05\times10^3\times1.15}{784,719} = 1.63\,\text{MPa}\,(0.236\ \text{ksi})$$

❖ Allowable stress:

$$V_{Rd,c} = C_{Rd,c}k\left(100\rho_1 f_{ck}\right)^{1/3} + k_1\sigma_{cp} \ge \left(v_{min} + k_1\sigma_{cp}\right)$$

Where,

$$C_{Rd,c} = 0.18/\gamma_c = 0.18/1.5 = 0.12$$

$$k = 1+\sqrt{\frac{200}{d}} = 1+\sqrt{\frac{200}{171}} = 2.08 > 2, \text{use} 2$$

$$\rho l = \sqrt{\rho l_1 \times \rho l_2} \le 0.02$$

A_{s1} = 8×129 = 1032 mm² (1.60 in²)

A_{s2} = 8×129 = 1032 mm² (1.60 in²)

For the first critical perimeter 2d (342 mm) from the face of support, the strands to be included are:
In one direction 4
In the transfer direction 4×3 = 12

Hence

A_{ps1} = 4×99 = 396 mm² (0.61in²)

A_{ps2} = 12×99 = 1188 mm² (1.84 in²)

$$\rho l_1 = \frac{A_{sl1} + A_{ps1}\left(f_{pk}/f_{yk}\right)}{bd}$$

For computation of *bd* in the above relationship, EC2 defines[114] column side plus *3d* on each side.

$$= \frac{1032 + 396\times(1860/413.69)}{(610+3\times171\times2)\times171} = 0.01$$

$$\rho l_2 = \frac{A_{sl2} + A_{ps2}\left(f_{pk}/f_{yk}\right)}{bd}$$

$$= \frac{1032 + 1188\times(1860/413.69)}{(610+3\times171\times2)\times171} = 0.023$$

$$\rho l = \sqrt{0.01\times0.023} = 0.015 < 0.02$$

$$\sigma_{cp} = \frac{N_{ED}}{A} = 0.86\,\text{MPa}\,(125\,\text{psi})$$

k_1 = 0.1

$$v_{min} = 0.035k^{3/2}f_{ck}^{1/2} = 0.035\times2^{3/2}\times34.47^{1/2}$$

= 0.58 MPa (84.12 psi)

$$\left(v_{min} + k_1\sigma_{cp}\right) = 0.58 + 0.1\times0.86$$

$$= 0.67\,\text{MPa}\,(97.18\ \text{psi})$$

$$v_{Rd,c} = \left[0.12\times2\times(100\times0.015\times34.47)^{1/3} + 0.1\times0.86\right]$$

= 0.98 MPa (142.15 psi) > 0.67 MPa (97.18 psi)

$v_{Rd,c}$ = 0.98 MPa (0.142 ksi)

❖ Stress ratio:
Stress ratio= Actual/Allowable
= 1.63/0.98 = 1.66 > 1

Punching shear stress exceeds the permissible value. Provide shear reinforcement.

❖ Shear Reinforcement Design:

$$A_{sw} = \frac{\left(v_{Ed} - 0.75v_{Rd,c}\right)u\times d\times s_r}{1.5\times d\times f_{ywd,ef}\sin\alpha} \ge A_{sw,min}$$

114 EN 1992-1:2004, Section 6.4.4

$$A_{sw,min} = \frac{0.08\sqrt{f_{ck}}\times s_r\times s_t}{(1.5\sin\alpha + \cos\alpha)f_{yk}}$$

s_r = 2*d* = 342 mm (13.46 in)

s_t = 1.5*d* = 257 mm (10.12 in)

$f_{ywd,ef}$ = 250+0.25*d* = 292.75 MPa (42.46 ksi)
< (413.69/1.15) = 359.73 MPa (52.18 ksi)

$$A_{sw} = \frac{(1.63 - 0.75\times0.98)\times4589\times171\times342}{1.5\times171\times292.75}$$

$$= 3199\ \text{mm}^2\,(4.96\ \text{in}^2)$$

$$A_{sw,min} = \frac{0.08\sqrt{34.47}\times342\times257}{1.5\times413.69}$$

$$= 67\ \text{mm}^2\,(0.10\ \text{in}^2) < 3199\ \text{mm}^2\,(4.96\ \text{in}^2)$$

$$A_{sw} = 3199\ \text{mm}^2\,(4.96\ \text{in}^2)$$

Assume 16 mm diameter studs, area of 1 stud = 201 mm² (0.31 in²)
Number of rails/side = 3
Number of sides = 4

$$\text{Number of studs, } N_{studs} = \frac{3199}{201\times3\times4} = 1.33 \approx 2$$

There should be at least 3 studs within the critical section at 2*d* from the face of support.

Number of studs, N_{studs} = 3

$$\text{Distance between the studs} = \frac{s}{N_{stud}} = \frac{2\times171}{3}$$

$$= 114\,\text{mm}\,(4.49\ \text{in}) \approx 110\ \text{mm}\,(4.33\ \text{in})$$

Check the shear stress on successive perimeters at 0.75*d* by following the same procedure until a section where no shear rebar is required.

The output of detailed calculations performed for other layers is summarized in Table 4.11.6.9C-1. The general arrangement of the studs is shown graphically in Fig. 4.11.6.9C-1. For the grouted system, a total of 7 studs per rail are required; first 3 studs at 110 mm (4.33") and the remaining at 125 mm (4.92"). For the unbonded system a total of 12 studs per rail are required.

C. Comparison of ACI and EC2 results
For the design based on EC2, the first three studs are within the distance 2d from the face of support. The remainder of the studs has a slightly different spacing from the studs beyond distance 2d (Table 4.11.6.9C-1).

4.11.7 One-Way Shear
One-way shear is checked for one-way slabs, and beams. It is also checked in column-supported floors, where shear transfer to column is achieved through virtual beams within the thickness of slab, in lieu of transfer through punching shear. Design for one-way shear is relatively straight forward. It is detailed well in ACI 318[115] and EC2[116], as well as text books, such as references [Collins et al, 1997, Nawy 1997]. The details of one-way shear design will not be repeated in this book. A numerical example for a post-tensioned beam frame using the provisions of the building codes covered in this book is given in Chapter 7.

The following provides a brief overview, and refers to the respective sections of the code for details. Refer to Fig. 4.11.7-1. The figure illustrates a typical distribution of design shear between two adjacent supports. The design shear is broken down to a maximum of four regions as outlined below.

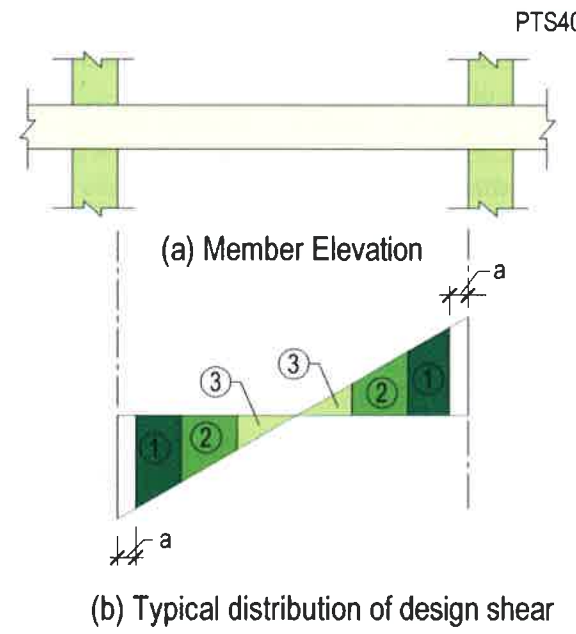

FIGURE 4.11.7-1 One-Way Design Shear Designations

115 ACI 318-11 Chapter 11
116 EC2 (EN 1992-1-1:2004(E)), Section 9.2.2

- The first region extends a distance "*a*" from the face of support. The reinforcement to be provided over this region is to be the same as required by design at the section distance "*a*" from the face of support. For conventionally reinforced members, ACI 318-11 recommends "a" to be a distance "d/2" from the face of support, where "*d*" is the effective depth of the member. For prestressed members, the recommended distance is "*h*/2", where "*h*" is the depth of the section at the face-of-support.

  EC2[117] recommends the distance "*a*" to be " *d*" from the face of support for both conventionally reinforced members and prestressed members. In addition, EC2 also requires to verify that the shear at the face of support does not exceed $V_{Rd,max}$.

- Region marked "1" is where the demand shear is greater than the shear capacity provided by concrete alone. Hence, shear reinforcement needs to be added to increase the capacity not to be less than the shear demand.

  Design shear ≥ Concrete design capacity

- Region marked "2" is where shear demand is less than the section's capacity provided by concrete alone, but exceeds a code-specified threshold. ACI 318 stipulates a threshold equal to 50% of the section capacity.

Code defined fraction of concrete capacity < design shear < concrete design capacity

In this case, if the member is viewed as a "beam," ACI 318 -11 requires a minimum amount of reinforcement[118]. If the member is designed as a "one-way slab," ACI 318-11 recommends no reinforcement, however.

EC2 also suggests providing minimum reinforcement[119] for only beams as in ACI 318.

- Region 3 is where the demand shear is less than a threshold of the section's concrete design capacity. In this case the ACI 318-11 recommends a threshold of 50%. No reinforcement is recommended either for slabs or beams if the demand shear is less than 50% of design capacity.

EC2 treats this region the same as region 2. For beams, minimum reinforcement is recommended for the entire region where design shear is less than concrete design capacity. For slabs, none is required.

Numerical Example for one-way shear is given in Chapter 7, Section 8.5.

4.12 INITIAL CONDITION; TRANSFER OF PRESTRESSING

In addition to adequacy check for in-service performance (SLS), and safety against overload (ULS), prestressed members are checked at transfer of prestressing (jacking), for the following reasons:

- At time of stressing, the prestressing tendon is at its highest force value;
- concrete is generally green, not having reached its in-service design strength; and
- design live and other loads are not generally present to balance the uplift effects of prestressing should it exceed selfweight.

As a result, at force transfer, a post-tensioned member may experience stresses that can cause to cracking or excessive creeks. For this reason, building codes require the evaluation of the member at force transfer, and specify remedial measures, should the transfer stresses be critical.

4.12.1 Load Combinations

Building codes covered are not specific on the details of the load combination at stressing. It is left to the judgment of the design engineer for the condition at hand. The load combination commonly used is:

1.00 Selfweight + 1.15 Prestressing
(Exp 4.12.1-1)

The above load combination is based on the premise that at stressing the long-term stress losses in prestressing have not taken place, and only the selfweight of the member is available to resist the prestressing forces.

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117 EC2 (EN 1992-1-1:2004(E)), Section 6.2.1(8)
118 ACI 318-11 , Section 11.4.6
119 EC2 (EN 1992-1-1:2004(E)), Section 6.2.1(4) and Section 9.2.2

4.12.2 Extreme Fiber Stress Check

Using the "initial condition" load combination, along with design strips and design sections, the hypothetical extreme fiber stresses are calculated and compared with the allowable values of the code. The breakdown of a floor system into design strips and the computation of hypothetical fiber stresses follow the same procedure outlined in Chapter 3.

If at any section along a member the hypothetical tensile stress exceeds the allowable value, rebar should be added in an amount necessary to resist the entire tensile force developed at that section. The computation of the force in the tensile zone of concrete is based on Fig. 4.12.2-1.

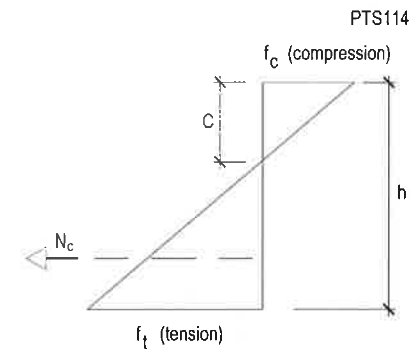

FIGURE 4.12.2-1 Tensile force N_c in Tension Zone of a Section

$$N_c = 0.5\left(\frac{f_t^2}{f_t + f_c}\right)\text{bh} \qquad \text{(Exp 4.12.2-1)}$$

The reinforcement stipulated to resist the tensile force is given by:

- Based on ACI 318 [120]

$$A_s = \frac{N_c}{0.6 f_y} \qquad \text{(Exp 4.12.2-2)}$$

- EC2 is not explicit on the amount of reinforcement. The recommendations of ACI 318, given above may be used.

This reinforcement is to be placed as close to the farthest tensile fiber of the section as practical. The treatment of high tensile values through addition of rebar is based on the premise that locations subject to high tension at transfer of prestressing are likely to either have a lesser tensile stress when in service, or be in compression, once design loads are applied. The tensile stresses under service loads are checked separately, and kept within allowable limits. The "initial condition" is a transient situation.

If at a design section the hypothetical compressive stress exceeds the allowable value, the jacking of tendons has to be delayed, until concrete gains added strength, and the hypothetical stresses fall within the allowable thresholds.

The allowable stresses for the initial condition are listed in Table 4.12.2-1.

TABLE 4.12.2-1 Allowable Stresses for Initial (Transfer) Condition (T156)

		Tension	Compression
ACI 318 [121]	Simply* supported	$6\sqrt{f'_{ci}}$	$0.7 f'_{ci}$
	Other	$3\sqrt{f'_{ci}}$	$0.6 f'_{ci}$
EC2		$f_{ct,eff}$	$0.6 f'_{ci}$ [122]

*End of simply supported members

Where,

f'_{ci} = concrete cylinder compressive strength at stressing; and

$f_{ct,eff} = 0.3 f_{ci}^{2/3}$

The reason for the limit on compressive stresses is to avoid undue creep of concrete at a young age. Young concrete, having lower modulus of elasticity undergoes larger initial elastic deformation and greater creep. For this reason, both ACI and EC2 recommend strict limits on the allowable compressive stress in concrete at stressing. If the computed stress exceeds the allowable, the stressing has to be delayed, or the initial stressing force reduced, until the stresses are within range.

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120 ACI 318-11, Section 18.4.1
121 ACI 318-11, Section 18.4.1
122 EC2(EN 1992-1-1:2004(E)), Section 5.10.2.2(5)

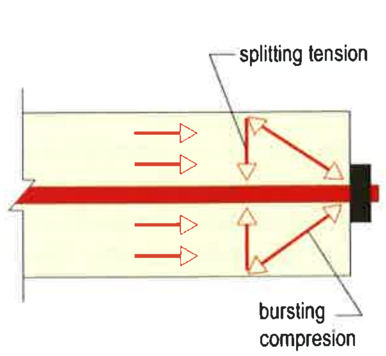


FIGURE 4.12.3-1 Dispersion of Force behind Anchorage

4.12.3 Bursting Reinforcement

Reinforcement is added behind both the dead and live stressing anchors, in order to avoid bursting of concrete under high local forces; and at the same time, to avoid cracks due to splitting of concrete in tension. Figure 4.12.3-1 illustrates the conceptual dispersion of jacking force in a member. It includes both bursting compression and splitting tensile forces.

Notwithstanding the availability of formulas to determine the amount and disposition of reinforcement behind an anchorage device, in practice hardware suppliers provide the detailing of reinforcement for the anchorage castings they supply. The bursting reinforcement suggested by each supplier is deemed to match the configuration, and force of the proprietary anchorage assembly the supplier provides.

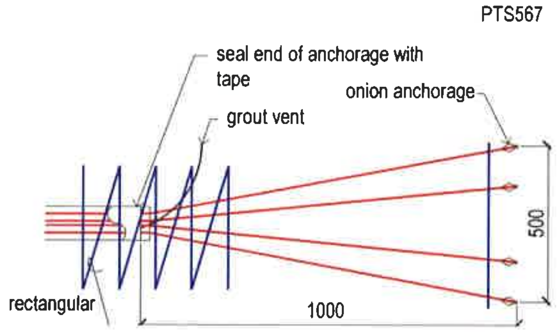
(a) Bursting steel being arranged behind dead end of a grouted flat duct tendon (P593)

(b) Bursting steel arranged behind dead end of multi strand transfer plate (Macau; Freyssinet) (P596)

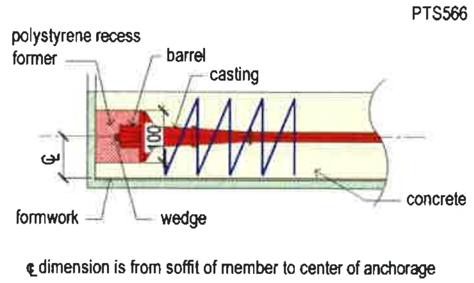
(c) Banded unbonded tendons in place with bursting bars (P594)

(d) View of a pre-assembled cage for bursting reinforcement of grouped unbonded tendons (P595)

FIGURE 4.12.3-2 Examples of bursting Reinforcement



(a) Dead end of four strand flat duct (mm)



(b) Section- Live end of tendon at slab edge

FIGURE 4.12.3-3 Examples of bursting Reinforcement for Mono-Strand Grouted Tendons

(a) Bursting of slab behind grouped tendons (US, P672)

(b) Bursting of slab behind grouped tendons (South Korea P673)

FIGURE 4.12.3-4 Examples of Bursting of Concrete at Stressing

Fig. 4.12.3-2 illustrates several examples of bursting reinforcement. Figure 4.12.3-3 is another example of commercially used bursting reinforcement. Fig. 4.12.3-4 a and b illustrate two examples where improper positioning of bursting reinforcement and early stressing have resulted in local failure at jacking end.

4.12.4 Application of Initial Condition (Transfer) Stress Check

At stressing, cast-in-place post-tensioned members are likely supported on casting forms. In most floor slab construction, the uplift (percentage of selfweight balanced) from the post-tensioning is less than the weight of the member. In such cases, the application of the prestressing force will not lift the member off its bed (Fig. 4.12.4-1). Consequently, the shape of the member under jacking force remains to be that of its casting bed. Bending stresses develop where a member is free to follow the curvature associated with the applied forces. Since the member does not curve (lift), bending stresses will not develop. Note that bending stresses are generated as a result of member's curvature under load. Hence, at stressing, the distribution of stress in a post-tensioned member will be uniform compression away from the immediate vicinity of the stressing anchors. The bending stresses calculated for code compliance of the initial condition should be performed with concrete strength at the time, when the forms are removed, and the member is free to flex. While the member is on its bed, bending stresses will not be generated.

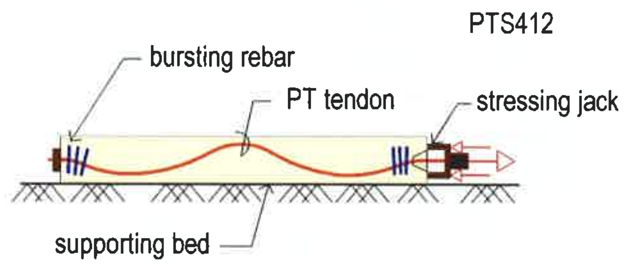


FIGURE 4.12.4-1 Post-Tensioned Member on Stressing Bed

4.12.5 Stage Stressing

In transfer plates, and mat foundations, where a floor is designed to carry the weight of multi-levels of construction above, stage stressing of post-tensioned tendons may become necessary. Tendons designed to counteract the weight of the entire superstructure may be excessive, if tendons are all activated prior to the application of adequate load from the superstructure. Figure 4.12.5-1 illustrates a transfer plate for a multi-story building, where in line with the progress of construction, at selected stages of construction a fraction of tendons were stressed. For each stage, the process is to select a number of favorably positioned tendons; stress the selected tendons to their maximum design force, and finish the stressed tendons. At a subsequent stage, a new group of tendons will be selected and stressed to full design force, until the entire floor system is post-tensioned.

(a) Transfer plate to support multi levels of construction above (P369a)

(b) Tendons at slab edge were subject to stage construction (P369b)

FIGURE 4.12.5-1 Stage Stressing of a Transfer Plate
(Erwada; Dubai; P639a, P639b)

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